Particle-vibration coupling in halo nuclei

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G.Gori et al., nucl-th/0301097
Aim of the talk

To discuss the role of core polarization in halo nuclei.

To show that, based on limited phenomenological input, it is possible to provide a quantitative calculation of the basic features of $^{11}$Be, $^{12}$Be, $^{10}$Li, $^{11}$Li and of the spectroscopic factors, in reasonable agreement with experiment.
The parity inversion problem in $^{11}\text{Be}$

<table>
<thead>
<tr>
<th></th>
<th>$^{11}\text{Be}$</th>
<th>$^{10}\text{Be}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separation energy</td>
<td>0.5 MeV</td>
<td>6.8 MeV</td>
</tr>
<tr>
<td>Lowest excited state</td>
<td>0.32 MeV,</td>
<td>3.4 MeV</td>
</tr>
<tr>
<td>Radius</td>
<td>3 fm</td>
<td>2.3 fm</td>
</tr>
</tbody>
</table>

Good situation for the mean field approximation:

*But! The quantum numbers of the ground state are not those predicted by the mean field (1p1/2)*

*but 2s1/2, in the next shell!!*

Stronger spin-orbit force for halo states?

(N. Vinh Mau, *Nucl. Phys. A592(95)33*)

(F. Nunes et al., *Nucl. Phys. A596(96)171*)
Experimental systematics

Mean-field results with Skyrme force
(Sagawa, Brown, Esbensen PLB 309(93)1)
A. SELF ENERGY

\[ E_{\text{shift}} = -2.5 \text{ MeV} \]

\[ S_{1/2} \]

\[ d_{5/2} \]

\[ 2^+ \]

\[ \hbar \Omega_2 = 3.4 \text{ MeV} \]

\[ \beta_2 \sim 1.0 \]

\[ S_{1/2} \]

B. PAULI BLOCKING OF GROUND STATE CORRELATIONS

\[ n \otimes {}^{10}\text{Be} \]

\[ P_{1/2} \]

\[ P_{3/2} \]

\[ \Rightarrow E_{\text{shift}} = +2.5 \text{ MeV} \]
Fig. 4.29. Representation of the results of Bernard and Nguyen Van Giai [66] for the neutron quasiparticle energies in the valence shell of $^{208}$Pb. The observed values are plotted on the left-hand side and the results of the Skyrme III–Hartree–Fock approximation on the right-hand side. The middle column gives the quasiparticle energies, see section 4.6.5.

Fig. 4.30. Representation of the results of Bonsignori et al. [315] for the neutron quasiparticle energies in the valence shell of $^{208}$Pb. The notation is the same as in fig. 4.29.
COLLECTIVE SURFACE VIBRATIONS

\[ R(\vec{r}) = |R_0|^2 + \sum_{\alpha} \alpha^* \chi_\alpha(\vec{r}) \chi_\alpha(\vec{r}) \]

\[ \beta = 2, 3, 4, 5, 6 \]

\[ \hbar \omega = \frac{1}{2} \sqrt{\frac{2}{B}} \]

\[ \beta_a = \sqrt{2 + \frac{1}{4} \sqrt{\frac{2}{B}} \}

The values of \( \hbar \omega \) and \( \beta_a \) are taken from experiment or alternatively from an RPA calculation.

THE MODEL HAMILTONIAN

\[ H = \frac{p^2}{2m} + U(\vec{r}, \vec{x}) + H_{\text{cell}} \]

where

\[ U(\vec{r}, \vec{x}) = U_0 \left( \frac{r}{x + \sum \alpha \chi_\alpha \chi_\alpha} \right) + U_0(r) - \frac{1}{2} \frac{\partial U_0}{\partial r} \sum \alpha \chi_\alpha \chi_\alpha \]

\[ \Rightarrow H = \frac{p^2}{2m} + U_0(r) + H_{\text{cell}} + H_{\text{pv}} \]

With

\[ H_{\text{pv}} = \frac{\partial U_0}{\partial r} \sum \alpha \sqrt{\frac{\hbar \omega_a}{2 \xi_a}} \left[ \omega_a^+ + (-1)^a \omega_a^- \right] \chi_\alpha(r) \]
Pauli-blocking correlations

The $^{10}$Be core itself is not a simple Slater determinant assumed in mean field:

There are ground state correlations, that is, mixing of configurations $\rightarrow$ partial occupation of orbits that in a pure mean field description are completely empty.

When the halo neutron is added to form $^{11}$Be, the extra neutron will partially block the single-particle orbits available for $^{10}$Be to correlate: the binding energy decreases.

The effect is strongest when associated to the lowest "empty" orbit, that is to the $1p_{1/2}$ orbit.

In this way Sagawa, Brown ed Esbensen explained the parity inversion in $^{11}$Be.
Elaborating on the $^{11}$Be calculation

New elements of our calculation:

Standard Woods-Saxon potential including spin-orbit according to Bohr and Mottelson

We include the (discretized) continuum for s-, p- and d-orbits:

Schrödinger equation solved with reflecting boundary conditions at a variable radius $R \rightarrow \infty$.

The calculations have been carried out using the Nuclear Field Theory: ...

*a systematic and fully consistent scheme for the particle-vibration coupling. It allows a coherent treatment of configuration mixing, .

Pauli-blocking correlations*
\[(\text{Saxon-Woods + spin-orbit B + M Vol. I})\]

\[U_0 = -565 \text{MeV} + \frac{N-Z}{A} 33 \text{MeV}\]

150 MeV

40 fm

150 keV

(hundred's of s.p. states)

\[\text{Figure 2-30} \quad \text{Energies of neutron orbits calculated by C. J. Veje (private communication).}\]
Fermionic degrees of freedom:
• s1/2, p1/2, d5/2 Wood-Saxon levels up to 150 MeV

Bosonic degrees of freedom:
• 2+ and 3- QRPA solutions with energy up to 50 MeV;
  residual interaction: multipole-multipole separable with the coupling constant tuned to reproduce E(2+)=3.36 MeV e 0.6<β2<0.7

Effective, energy-dependent matrix (Bloch-Horowitz)
\[ \begin{align*}
\text{b}|_{\text{limit}} &= \langle b \mid -i \frac{\partial}{\partial x} Y_n \mid a \rangle \sqrt{\frac{\hbar \omega}{C_1}} \\
\text{c}|_{\text{limit}} &= \frac{\langle b_{\text{limit}} \mid Y_{2a} \rangle^2}{\mathcal{E}_a - (\mathcal{E}_b + \hbar \omega)} \\
\text{f}|_{\text{limit}} &= \frac{\langle c_{\text{limit}} \mid Y_{2a} \rangle^2}{\mathcal{E}_a - (2\mathcal{E}_a - \mathcal{E}_b + \hbar \omega)}
\end{align*} \]
Admixture of $d_{5/2} \times 2^+$ configuration in the $1/2^+$ g.s. of $^{11}\text{Be}$ is about 20%.

9Be(11Be,10Be+ $\gamma$) X

T. Aumann et al.
PRL 84(2000)35

p(11Be,10Be)d

S. Fortier et al.
Particle-vibration coupling in $^{11}\text{Be}$

NFT ground state

$$| \frac{1}{2} + \rangle = \sqrt{0.87} | s_{1/2} \rangle + \sqrt{0.13} | d_{5/2} \otimes 2 + \rangle$$


$$| \frac{1}{2} + \rangle = \sqrt{0.84} | s_{1/2} \rangle + \sqrt{0.16} | d_{5/2} \otimes 2 + \rangle$$

$^{11}\text{Be}(p,d)^{10}\text{Be}$ in inverse kinematic detecting both the ground state as well as the $2^+$ excited state of $^{10}\text{Be}$. 
Good agreement between theory and experiment concerning energy and spectroscopic factors

<table>
<thead>
<tr>
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<th>Theory</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>particle-vibration +Argonne</td>
<td>mean field</td>
</tr>
<tr>
<td>$E_{9/2^+}$</td>
<td>-0.504 MeV</td>
<td>-0.48 MeV</td>
<td>$\sim$ 0.14 MeV</td>
</tr>
<tr>
<td>$E_{9/2^-}$</td>
<td>-0.18 MeV</td>
<td>-0.27 MeV</td>
<td>-3.12 MeV</td>
</tr>
<tr>
<td>$S[1/2^+]$</td>
<td>0.77</td>
<td>0.87</td>
<td>1</td>
</tr>
<tr>
<td>$S[1/2^-]$</td>
<td>0.96</td>
<td>0.96</td>
<td>1</td>
</tr>
</tbody>
</table>

Experimental Spectroscopic Factors from

B. Zwieglinski et al., Nucl.Phys.A315(1979) 124
CORE + 2neutrons

COLLECTIVE SURFACE VIBRATIONS

$R(\phi) = R_0 \left[ 1 + \sum_{n} \alpha_n^* X_n(\phi) \right] \quad \lambda = 2^+, 3^+, 4^+, 5^-$

$H_{\text{coll}} = \frac{1}{2} \sum_{n \neq m} \left( B_{\lambda n} \alpha_n^* \alpha_m^* + C_{\lambda n} \alpha_n^* \alpha_m \right)$

$\beta_\lambda = \sqrt{2\lambda + 1} \sqrt{\frac{\hbar \omega}{2C_\lambda}}$

MODEL HAMILTONIAN

$H = \frac{\vec{p}_1^2}{2m} + \frac{\vec{p}_2^2}{2m} + \frac{\vec{p}_{\text{core}}^2}{2M_{\text{core}}} + U(\vec{r}_1; \vec{r}_2) + U(\vec{r}_1; \alpha) + U(\vec{r}_2; \alpha) + H_{\text{coll}}$

with $\vec{p}_{\text{core}} = -\vec{p}_1 - \vec{p}_2$

$H = \frac{\vec{p}_1^2}{2\mu} + \frac{\vec{p}_2^2}{2\mu} + U_{\text{nm}}(\vec{r}_1; \vec{r}_2) + U(\vec{r}_1; \alpha) + U(\vec{r}_2; \alpha) + H_{\text{coll}} + \frac{\vec{p}_1 \cdot \vec{p}_2}{M_{\text{core}}}$

where

$U(\vec{r}; \alpha) = U_0 \left( \frac{r}{1 + \sum \alpha_n^* X_n} \right)$

$\leq U_0(r) - \nabla r \cdot \frac{\partial U_0}{\partial r} \sum \alpha_n^* X_n(\vec{r}) \quad \alpha \neq 1.$
Bloch–Horowitz perturbation method

> Total Space: \{ \begin{array}{c} \uparrow \uparrow \\ \downarrow \downarrow \\ \uparrow \uparrow \\ \downarrow \downarrow \end{array} \}

> Model Space: \{ \begin{array}{c} \uparrow \uparrow \\ \downarrow \downarrow \end{array} \}

Model Space states must not appear as intermediate states

\[ a^3 \uparrow \downarrow a = \left[ a^3 \uparrow \downarrow a \right] / [E_g - E_{\text{Int}}] \]
Fermionic degrees of freedom:
- two particle states coupled to zero angular momentum on s1/2, p1/2, d5/2 Woods-Saxon levels up to 150 MeV

Bosonic degrees of freedom:
- 1-, 2+ and 3- QRPA solutions up to 50 MeV, associated to a multipole-multipole separable interaction with coupling constant tuned to reproduce $E(1^-)=2.7 \text{ MeV}$ and $B(E1)=0.052 \text{ e}^2\text{fm}^2$, $E(2^+)=2.1 \text{ MeV}$ and $0.6<\beta_2<0.7$

12Be

Effective, energy-dependent matrix
The pairing energy between the valence neutrons originates mostly from polarization effects, and not by the nucleon-nucleon bare interaction (Argonne potential)
Spectroscopic factors from \(^{12}\text{Be},^{11}\text{Be} + \gamma\) reaction to \(\frac{1}{2}^+\) and \(\frac{1}{2}^-\) final states:

\[ S[\frac{1}{2}^-] = 0.42 \pm 0.10 \quad S[\frac{1}{2}^+] = 0.37 \pm 0.10 \]

A. Navin et al.,
PRL 85(2000)266
\[ S[1/2^-] = |\langle ^{11}\text{Be}|a_{1p1/2}|^{12}\text{Be}\rangle|^2 \approx |T_{1/2^-}|^2. \]

\[ T_{1/2^-} = \sum_{n_{1p1/2}} \xi_{n_{1p1/2}}^* \left\{ \sum_{p, p'} \xi_p \xi_{pp'} x_{np_{1/2}} \right\} + \sum_{p, \lambda, \lambda', \lambda''} \xi_{p\lambda} \xi_{\lambda'\lambda''} x_{np_{1/2}}. \]

\[ \xi_p = \frac{\xi_p}{\sqrt{N(^{11}\text{Be})}}, \quad \xi_{it'} = \frac{\xi_{it'}}{\sqrt{N(^{12}\text{Be})}}, \]

\( \xi_i \) are obtained diagonalizing the energy-dependent matrix.
Spectroscopic factors measure the overlap between $^{11}$Be and $^{12}$Be

Good agreement between theory and experiment concerning energy and spectroscopic factors

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<td>particle-vibration + Argonne</td>
</tr>
<tr>
<td>$S_{2n}$</td>
<td>-3.673 MeV</td>
<td>-3.58 MeV</td>
</tr>
<tr>
<td>$s^2, p^2, d^2$</td>
<td>23%, 29%, 48%</td>
<td>0%, 100%, 0%</td>
</tr>
<tr>
<td>$S[1/2^+]$</td>
<td>0.42±0.10</td>
<td>0.31</td>
</tr>
<tr>
<td>$S[1/2^-]$</td>
<td>0.37±0.10</td>
<td>0.57</td>
</tr>
</tbody>
</table>
Particle-vibration coupling in $^{10}$Li and $^{11}$Li
Comparison with experiment

<table>
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</tr>
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<tbody>
<tr>
<td></td>
<td></td>
<td>particle-vibration +Argonne</td>
</tr>
<tr>
<td>( ^{10}_3 \text{Li}_7 ) (not bound)</td>
<td>s</td>
<td>0.1 – 0.2 MeV</td>
</tr>
<tr>
<td></td>
<td>p</td>
<td>0.5 – 0.6 MeV</td>
</tr>
<tr>
<td>( ^{11}_3 \text{Li}_8 ) (bound)</td>
<td>( S_{2n} )</td>
<td>0.294 ± 0.03 MeV</td>
</tr>
<tr>
<td></td>
<td>( S^2, p^2 )</td>
<td>50 % , 50 %</td>
</tr>
<tr>
<td></td>
<td>( \langle r^2 \rangle^{1/2} )</td>
<td>3.55 ± 0.1 fm</td>
</tr>
<tr>
<td></td>
<td>( \sigma_{\perp} )</td>
<td>48 ± 10 ( \frac{\text{MeV}}{c} )</td>
</tr>
</tbody>
</table>
Spatial correlations between
the two halo neutrons
ADVANTAGES OF THE MODEL:

- Standard mean-field potential, without adjustments of the spin-orbit force or l-dependent terms; the same parametrization for Li and Be isotopes

- Coupling with continuum states is taken into account

- Bare interaction between the valence neutrons, without ad-hoc density dependent terms

- Limited amount of phenomenological input: strength of low-lying vibrations

- Consistent treatment of the Pauli principle (Nuclear Field Theory)

- SOME LIMITATIONS OF THE MODEL:

- One-phonon configurations

- Harmonic vibrations of the core

- Tamm-Dancoff treatment of pairing vibrations
Pairing gaps in uniform matter calculated with effective and bare interactions look similar...

However, in $^{120}$Sn Argonne potential reproduces experiment only taking into account renormalization effects.