A Regge Model for Nucleon-Nucleon Scattering Amplitudes

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Outline

Objective and Motivation

Regge Phenomenology

Application to NN Scattering

Progress

Summary and Outlook
Our goal is to calculate the nucleon-nucleon scattering amplitudes.

- Utilize isospin \( I \) to describe proton-proton (pp) and proton-neutron (pn)
- Full spin dependence, in order to describe all observables.
  - Helicity basis, \( \lambda_i = \vec{S} \cdot \hat{p} \)
  - \((++; ++), (++; --), (+--), (++; +--), (+--; --)\)
- Ability to extrapolate to high energies where pn data is sparse \((s > 6 \text{ GeV}^2)\).

We need a relativistic, fully spin dependent model.
Motivation

- Describe final state interactions in electrodisintegration of the deuteron.
- A calculation by Jeschoneck and Van Orden, utilize the scattering amplitudes as input.
  - Data is available with high energy nucleons.
Why Regge Theory?

- We need to model this process at mid to high energies \((s > 6 \text{ GeV}^2)\).
- Many methods are ineffective at these energies.
- Regge phenomenology has had great historical success and scales to high energies.
- We need a parameterization method.
  - Fit to available data (mostly low energy).
  - Confidence to extrapolate to higher energies.

Regge theory allows us to construct a relativistic, fully spin dependent model, over a large energy range.
Concepts of Regge Analysis

Analyze $T(s, t)$ with continuous, complex angular momentum.

$s$ - channel

$N + N \rightarrow N + N$

$t$ - channel

$N + \bar{N} \rightarrow \bar{N} + N$

$s = 4E_{cm}^2$

$t = -2\vec{p}^2(1 - \cos(\theta))$

$\cos(\theta) = 1 + \frac{2t}{s-4m^2}$

$t$ - channel analysis gives approximation to $s$ - channel process

$T^{NN\rightarrow NN}_{Regge} (s, t) \rightarrow \lim_{s \rightarrow \infty} T^{NN\rightarrow NN} (s, t) \propto \left( -1 - \frac{2s}{t - 4m^2} \right)^{J \rightarrow \alpha(t)}$
Regge Parameters

- The Regge trajectories, $J \rightarrow \alpha(t) = \alpha_0 + \alpha' t$, interpolate between physical mesons.

- Regge exchanges should have good quantum numbers, PGI.

For full angular dependence, we use additional trajectories.
Regge Poles

- Regge exchanges characterized by residue \( \beta(t) \) and trajectory \( \alpha(t) \).

\[
R(s, t) = \xi(t) \beta(t) \left( -1 + \frac{2s}{4m^2 - t} \right)^{\alpha(t)}
\]

\[
\xi(t) = \begin{cases} 
  e^{-i\pi(\alpha(t)+\delta)/2} & \zeta_{PG} = +1 \\
  -ie^{-i\pi(\alpha(t)+\delta)/2} & \zeta_{PG} = -1
\end{cases}
\]

\[
\alpha(t) = \alpha_0 + \alpha't
\]

\[
\beta(t) = (B_1 + B_2t)e^{ct}
\]
Applying Regge Theory to Nucleon-Nucleon Scattering

- Regge exchanges are in the crossed channel, and the helicity crossing relations are complicated.

Utilizing the Fermi invariants makes crossing trivial.

\[ \hat{T} = F_I^S(s, t)I^{(1)} \cdot I^{(2)} - F_I^P(s, t)(i\gamma_5)^{(1)} \cdot (i\gamma_5)^{(2)} \]
\[ + F_I^V(s, t)\gamma^{\mu(1)}\gamma^{\mu(2)} + F_I^A(s, t)(\gamma_5\gamma^\mu)^{(1)}(\gamma_5\gamma^\mu)^{(2)} \]
\[ + F_I^T(s, t)\sigma^{\mu\nu(1)}\sigma^{\mu\nu(2)} \]

- Spin dependence explicitly dealt with.
- Analytically continuing scalar functions trivial.
Overview of the Calculation

- t-channel exchange
- Partial Wave Sum
- Initial/final states of good PGI
- Reggeization
- Matrix elements of F.I.
- Compare
- Relate Regge exchanges and F.I.
- Analytically continue to s c.m. frame.
s-channel helicity amplitudes in terms of the invariants.

\[
T_{i \rightarrow pp}^{pp \rightarrow pp} = \sum_{j} \left\{ C_{ij}^{t} \left[ F_{j}^{0}(s, t) + F_{j}^{1}(s, t) \right] - C_{ij}^{u} \left[ F_{j}^{0}(s, u) + F_{j}^{1}(s, u) \right] \right\}
\]

\[
T_{i \rightarrow pn}^{pn \rightarrow pn} = \sum_{j} \left\{ C_{ij}^{t} \left[ F_{j}^{0}(s, t) - F_{j}^{1}(s, t) \right] - 2C_{ij}^{u} F_{j}^{1}(s, u) \right\}
\]

\[
i \rightarrow (++; ++), (++; +-), (+-; + -), (++; --), (+-; --)
\]

\[
j \rightarrow S, V, T, P, A
\]
Total Cross Section for Proton-Proton and Proton-Neutron

\[ \sigma(GeV^{-2}) \]

\[ s(GeV^2) \]
Proton-Proton High Energy Differential Cross Section

\[ \frac{d\sigma}{dt} (GeV^{-4}) \]

\( t (GeV^2) \)

\( x10^{-9} \ s =3852.7 \\
\( x10^{-8} \ s =3844.0 \\
\( x10^{-7} \ s =2809.0 \\
\( x10^{-6} \ s =2788.9 \\
\( x10^{-5} \ s =1992.7 \\
\( x10^{-4} \ s =961.0 \\
\( x10^{-3} \ s =932.7 \\
\( x10^{-2} \ s =552.2 \\
\( x10^{-1} \ s =550.4 \\
\( x10^{0} \ s =377.1 \)
Proton-Proton Differential Cross Section

$\frac{d\sigma}{dt} (\text{GeV}^{-4})$

$\theta$ (Degrees)

$+ x2^0 \ s = 5.7$
$+ x2^{-1} \ s = 5.7$
$+ x2^{-2} \ s = 6.3$
$+ x2^{-3} \ s = 6.3$
$+ x2^{-4} \ s = 7.3$
$+ x2^{-5} \ s = 7.3$
$+ x2^{-6} \ s = 8.8$
$+ x2^{-7} \ s = 10.4$
$+ x2^{-8} \ s = 11.0$
$+ x2^{-9} \ s = 11.3$
$+ x2^{-10} \ s = 12.2$
$+ x2^{-11} \ s = 12.3$
Proton-Neutron Differential Cross Section

Proton-Neutron Differential Cross Section

\[ \frac{d\sigma}{dt}(\text{GeV}^{-4}) \]

\[ \theta (\text{Degrees}) \]

\[ x_2^0 \quad s = 5.5 \]
\[ x_2^{-1} \quad s = 5.6 \]
\[ x_2^{-2} \quad s = 5.7 \]
\[ x_2^{-3} \quad s = 5.9 \]
\[ x_2^{-4} \quad s = 5.9 \]
\[ x_2^{-5} \quad s = 7.7 \]
\[ x_2^{-6} \quad s = 7.8 \]
\[ x_2^{-7} \quad s = 8.3 \]
\[ x_2^{-8} \quad s = 8.7 \]
\[ x_2^{-9} \quad s = 9.5 \]
\[ x_2^{-10} \quad s = 9.6 \]
\[ x_2^{-11} \quad s = 10.2 \]
\[ x_2^{-12} \quad s = 11.3 \]
\[ x_2^{-13} \quad s = 12.4 \]
Proton-Proton Polarization

Proton-Proton Polarization ($A_N$)

$P$ vs. $\theta$ (Degrees)

$+ s = 5.5$
$+ s = 7.5$
$+ s = 9.5$
$+ s = 11.6$
Proton-Neutron Polarization

Proton-Neutron Polarization ($A_N$)

$P$ vs. $\theta$ (Degrees)

- $s = 5.4$
- $s = 9.5$
- $s = 11.3$
- $s = 13.2$
Proton-Proton Double Spin Observables

Proton-Proton Double Polarization Observables

\[ A_{XXpp} \]

\[ A_{XZpp} \]

\[ A_{ZZpp} \]

\[ A_{YYpp} \]

\[ D_{pp} \]

\[ D_{Tpp} \]
Proton-Neutron Double Spin Observables

Proton-Neutron Double Polarization Observables

AZXpn

AYYpn

DTpn

AZZpn

Dpn

\(5.4 \leq s \leq 5.6\)

\(5.4 \leq s \leq 6.5\)

\(5.4 \leq s \leq 8.0\)

\(5.4 \leq s \leq 5.6\)

\(\theta \text{ (Degrees)}\)
Summary and Outlook

▶ We have constructed a relativistic, fully spin dependent model, which describes the high energy nucleon-nucleon scattering amplitudes.
▶ While we describe the data well, we believe we can improve upon the fit.
▶ Utilize the amplitudes to describe final state interactions in the $D(e, e'p)n$ calculation.
▶ When complete, we will provide the amplitudes via a program or code for the community.
Electromagnetic Effects

- In order to describe proton-proton scattering we also need to take into account electromagnetic interactions.

\[ \Gamma = F_1 \gamma^\mu - \frac{F_2}{2m} i\sigma^{\mu\nu} q_\nu \]

\[
F_1 = \frac{G_E - G_M t/4m^2}{1 - t/4m^2} \quad F_2 = \frac{G_M - G_E}{1 - t/4m^2}
\]

\[ G_E = G_M / 2.79 = (1 - t/.71)^{-2} \]

- To approximate higher order effects, we also allow for a helicity dependent phase for the dominant amplitudes.

\[
T_{++;++}^{EM} \approx T_{+-;+-}^{EM} \approx e^{i\phi_1(t)} T_{++;++}^{EM} \bigg|_{1 \text{photon}}
\]

\[
T_{++;++}^{EM} \approx A e^{i\phi_2(t)} T_{++;++}^{EM} \bigg|_{1 \text{photon}}
\]
\[ X^2 = 4.9 \]
\[(X_{SIGpp}, X_{SIGpn}, X_{DSGppH}, X_{DSGppL}, X_{DSGpnA}, X_{PppH}, X_{Ppp}, X_{Ppn}, X_{AZXpp}, X_{AZXpn}, X_{AZZpp}, X_{AZZpn}, X_{DTpp}, X_{DTpn}, X_{AXXpp}) \]
\[(189.85449675079789, 25.075946302709312, 1648.5956834235128, 30534.265011369982, 3107.4561391554071, 704.17520994637914, 11369.075182146697, 704.17520994637914, 4394.0279028217074, 100.8000053578187, 387.06625135748175, 52.317494816433403, 6880.520955580645, 215.03225731892957, 1073.3736097413077, 199.71357425625374, 1068.2075172285674, 1060.4130983805419) \]
\[(192, 72, 758, 3447, 745, 250, 3136, 250, 568, 81, 188, 37, 1587, 117, 608, 89, 281, 276) \]
\[(0.98882550391040569, 0.34827703198207377, 2.1749283422473784, 8.858214392625658, 4.1710820659804124, 2.8167008397855167, 3.6253428514498394, 2.2625062801800899, 7.7359646176438508, 1.2444445105903543, 2.0588630391355411, 1.4139863463900919, 4.3355519568876151, 1.8378825411874322, 1.7654171212850456, 2.243972744452289, 3.8014502392475711, 3.8420764434077603) \]