Transport properties of $\beta$-stable nuclear matter

Arianna Carbone$^1$ and Omar Benhar$^2$

$^1$ Departament d’Estructura i Constituents de la Matèria, Universitat de Barcelona, E-08028 Barcelona, Spain
$^2$ INFN, Sezione di Roma & Dipartimento di Fisica, ”Sapienza” Università di Roma, I-00185 Roma, Italy

E-mail: ariannac@ecm.ub.es; Omar.Benhar@roma1.infn.it

Abstract. The transport properties of matter in the interior of rotating neutron stars play a critical role in determining the evolution of these compact objects. In this brief report we discuss a study of the shear viscosity of stellar matter composed by neutrons, protons and electrons in equilibrium with respect to $\beta$ decay and electronic capture. The aim of this work is calculate in a fully consistent manner the equation of state and the transport properties of nuclear matter, in particular the shear viscosity coefficient, using the same dynamical model.

1. Introduction

Transport properties of neutron star matter characterize in a direct way its evolution. Especially viscosity plays an important role, because it measures the damping effect on the instability caused by the emission of gravitational waves by neutron stars. Hence, the knowledge of this quantity is an essential element to predict the stability of a rotating neutron star.

The CFS mechanism [1, 2, 3] describes this kind of instability. First pointed out by Chandrasekhar in 1970 [1, 2], gravitational radiation is emitted when a non-radial oscillation mode of the star is excited by an internal or external perturbation; this perturbation tends to dissipate in non rotating stars, while, as shown by Friedman and Schutz [3] it drives all perfect fluid rotating stars to instability. Furthermore, the $r$-modes, modes of rotating stars whose restoring force is the Coriolis force, were defined always unstable due to the emission of gravitational radiation in all perfect fluid rotating stars [4, 5].

If, on the other hand, we consider matter which does not behave as a perfect fluid, viscosity is that dissipative process which opposes the onset of the CFS instability; it appears that viscosity can damp or actually suppress totally the oscillating modes of rotating neutron stars which are driven unstable due to the emission of gravitational radiation [6].

With this work we calculate the shear viscosity of a three-component, neutrons, protons and electrons, $\beta$-stable system [7], combining the Landau-Abrikosov-Khalatnikov formalism [8, 9, 10], to obtain the transport coefficients, with the many-body Correlated Basis Function (CBF) theory [11], necessary to obtain the nucleon-nucleon scattering probability, the proton fraction and the neutron and proton effective masses.

2. Landau theory for normal Fermi liquids

The Landau theory for normal Fermi liquids [8] applies to strongly interacting non superfluid fermionic systems at very low temperatures; it is therefore appropriate for the calculation of the
transport properties of matter interior to neutron stars.

Following the Landau-Abrikosov-Khalatnikov formalism [8, 9, 10], successively expanded by Flowers and Itoh to the case of a fermionic system composed by more than one kind of quasiparticle family [12, 13], the multicomponent Boltzmann-Landau transport equation which has to be solved takes the form

\[
\frac{\partial n_\alpha}{\partial t} + \frac{\partial n_\alpha}{\partial r} \cdot \frac{\partial \epsilon_{p\alpha}}{\partial p} - \frac{\partial n_\alpha}{\partial p} \cdot \frac{\partial \epsilon_{p\alpha}}{\partial r} = \sum_\beta I_{\alpha\beta},
\]

where \(n_\alpha = n_{p\alpha}(r,t)\) denotes the distribution of quasiparticles of type \(\alpha (\alpha = n, p, e)\), carrying momentum \(p\) and energy \(\epsilon_{p\alpha}\). The term on the right hand side represents the collisions integral involving both like and unlike quasiparticles; in the low temperature limit, the collision term only takes into account processes involving quasiparticles carrying momenta close to the Fermi momenta [12, 13].

Solving the transport equation (1), the total shear viscosity results as the sum of the contributions given by all the quasiparticle families; each contribution of type \(\alpha\) reads [12, 13]

\[
\eta_\alpha = \frac{1}{5} \rho_\alpha m^{*}_\alpha v^{2}_{F\alpha} \tau_\alpha \frac{2}{\pi^2(1 - \ell_{\alpha\alpha})} C(\ell_{\alpha\alpha}) ,
\]

where \(\rho_\alpha, m^{*}_\alpha\) and \(v_{F\alpha}\) denote the density, effective mass and Fermi velocity, respectively; \(\ell_{\alpha\alpha}\) and \(C(\ell_{\alpha\alpha})\) are factors given in Ref.[13]; \(\tau_\alpha\) is the quasiparticle lifetime at temperature \(T\) and it is given by the expression [13]

\[
\tau_\alpha = \frac{4\pi^4}{m^{*2}_\alpha T^2 \sum_\beta m^{*2}_\beta \langle W_{\alpha\beta} \rangle} .
\]

In the above equation, the quantity \(W_{\alpha\beta}\) represents the scattering probability between quasiparticles of type \(\alpha\) and \(\beta\). These quantities take into account both strong and electromagnetic interaction. Nuclear interactions contributing to \(W_{nn}, W_{pp}\) and \(W_{np}=W_{pn}\) have been described within the dynamical model of Ref.[11] using the CBF effective interaction.

The effective interaction \(v_{\text{eff}}\) described in Ref.[11] is based on a truncated version of the NN potential referred to as Argonne \(v_{18}\) [14], providing an excellent fit of deuteron properties and the full Nijmegen phase shift database. It also includes the effects of interactions involving three- and many-nucleon forces, described according to the approach originally proposed in Ref.[15]. The EOS of symmetric nuclear matter and pure neutron matter obtained using the dynamical model of Ref.[11] turn out to be in fairly good agreement with the results of the state-of-the-art calculations of Ref.[16], carried out using the full Argonne \(v_{18}\) potential.

The calculations of \(W_{ep}=W_{pe}, W_{en}=W_{ne}\) and the electromagnetic part of \(W_{pp}\) have been calculated following Refs.[12, 13].

3. Shear viscosity of \(\beta\)-stable nuclear matter

The calculation of the shear viscosity of \(\beta\)-stable nuclear matter requires the proton and electron fraction, which is obtained solving the coupled equations of \(\beta\)-equilibrium and charge neutrality [7]

\[
\mu_n = \mu_p = \mu_e , \quad \rho_p = \rho_e ,
\]

where \(\mu_\alpha\) denotes the chemical potential of quasiparticles of type \(\alpha\). The proton and neutron chemical potential have been calculated using the single particle spectrum obtained from the CBF effective interaction of Ref.[11].

Figure 1 shows the baryon density-dependence of the proton fraction resulting from the numerical solution of the coupled equations (4). We clearly see that the presence of protons and
electrons is very low and it reaches a maximum value of 6% at double the value of equilibrium density of nuclear matter.

Figure 2 shows the results obtained for the temperature-independent quantities $\eta_\alpha T^2$, as well as $\eta T^2$, plotted as a function of baryon density. It appears that the proton viscosity $\eta_p$ is always very small, due to their low density and mobility. On the other hand, in spite of the fact that $\rho_p=\rho_e$, the electron viscosity $\eta_e$ turns out to be much higher, as electrons are ultra-relativistic. As expected, the dominant contribution is $\eta_n$, as the neutron fraction is larger than 90% over the whole range of baryon density. For $\rho \leq \rho_0$, $\rho_0 = 0.16 \text{ fm}^{-3}$ being the equilibrium density of symmetric nuclear matter, the total viscosity can be identified with the neutron contribution. The electron contribution becomes barely visible only at larger density.

Furthermore, for comparison, we plot in figure 2 the shear viscosity of pure neutron matter calculated using both free-space and in medium collision probabilities [11], which shows how the presence of the medium strongly suppresses quasiparticle scattering probabilities, enhancing their mean lifetime and consequently their contribution to the shear viscosity.

The difference between the shear viscosity of pure neutron matter and $\beta$-stable nuclear matter lies in the larger number of channels available in collision processes, in the second case, which in turn lowers the quasiparticle lifetime and furthermore the total shear viscosity.

To underline the importance of the composition of matter in determining the different contributions of the quasiparticle families to the total shear viscosity, we plot in figure 3 the quantities $\eta_\alpha T^2$ and $\eta T^2$ obtained using a proton fraction double the one plotted in figure 1. Note that changing the proton fraction amounts to using a different model of nuclear dynamics to determine the EOS. In this case the electron viscosity exceeds neutron viscosity at values of baryon density just above nuclear matter saturation density.

### Table 1. Explanation of data in figure 2 and figure 3.

<table>
<thead>
<tr>
<th>Description</th>
<th>Line Style</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$ of $\beta$-stable NM</td>
<td></td>
</tr>
<tr>
<td>$\eta_n$ of $\beta$-stable NM</td>
<td></td>
</tr>
<tr>
<td>$\eta_e$ of $\beta$-stable NM</td>
<td></td>
</tr>
<tr>
<td>$\eta_p$ of $\beta$-stable NM</td>
<td></td>
</tr>
<tr>
<td>$\eta$ of pure NM with screening effects</td>
<td>♦</td>
</tr>
<tr>
<td>$\eta$ of pure NM without screening effects</td>
<td>×</td>
</tr>
</tbody>
</table>
4. Conclusions

We reported the results for the shear viscosity of $\beta$-stable nuclear matter composed by neutrons, protons and electrons, obtained combining in a consistent manner the Landau-Abrikosov-Khalatnikov formalism with the CBF many-body approach. The effective strong interaction derived in the frame of this approach takes into account the presence of the medium which results in screening effects.

The role played by screening of the bare NN interaction, mainly due to short range correlations, was already pointed out in Ref.[11] for the case of pure neutron matter. In $\beta$-stable matter the inclusion of this effect turns out to be critical. The results displayed in figure 2 show that, in the absence of screening, the electron viscosity would become dominant at $\rho \geq 0.2$ fm$^{-3}$. Overall, due to the availability of a larger number of reaction channels, the viscosity of $\beta$-stable matter turns out to be lower than that of pure neutron matter. Furthermore electron viscosity would dominate at $\rho \geq \rho_0$ in the case of dynamical models predicting a larger proton fraction, typically $x_p \sim 8\%$ at nuclear matter equilibrium density.

References