The inclusion of tensor terms in the nonrelativistic nuclear Energy Density Functionals

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Abstract. This contribution is intended to briefly review recent attempts to include tensor terms in effective nuclear Energy Density Functionals (EDFs). We stress that effective tensor forces should not be confused with the well-known bare tensor force. Consistently with the basic idea of EDFs, collective states are shown to provide better constraints for the tensor parameters, as compared with single-particle states. In fact, basic limitations of the description of the single-particle strength in atomic nuclei provided by EDFs, are highlighted in the last part of this contribution.

1. Introduction

Modern EDFs constitute the basic tool for microscopic nuclear structure calculations if a systematic exploration of wide regions of the nuclear chart is envisaged, and if also excited states at relatively large excitation energies are to be considered. There exist several kinds, and many parameterizations, for nuclear EDFs. In the nonrelativistic description, one often starts from effective Hamiltonians based either on zero-range Skyrme interactions or finite-range Gogny interactions. Recently, local functionals derived from the Skyrme Hamiltonians are generalized by including systematically all terms consistent with symmetries (up to a given order in the derivative operators). Relativistic Mean Field (RMF), or covariant, functionals have also been devoted much attention: in this case the EDF is derived from a Lagrangian that includes nucleons treated as Dirac particles and effective mesons (plus the associated couplings).

The basic features of EDFs for nuclei are reviewed, e.g., in Ref. [1]. They are, as a rule, fitted by using empirical properties of uniform nuclear matter together with the masses and charge radii of selected reference nuclei. They can describe in a reasonable way the global trends of the ground-state properties of nuclei (binding energies, radii, and deformations). They can be used to describe superheavy nuclei, drip-line isotopes, or even exotic forms of nuclear matter like that found inside specific layers of neutron stars (at least, at densities that are not much larger, or smaller, than usual terrestrial nuclear densities).

Starting from an EDF, the ground-state properties are obtained by minimizing the total energy. The scheme is in principle based on the Density Functional Theory (DFT) originally introduced by Hohenberg and Kohn. In practice, if one starts from an Hamiltonian $H_{\text{eff}} = T + V_{\text{eff}}$ (where $T$ is the kinetic energy and $V_{\text{eff}}$ is the effective force) and calculates the total energy as

$$E[\rho] = \langle \Phi | H_{\text{eff}} | \Phi \rangle,$$  

(1)
where $|\Phi\rangle$ is a general Slater determinant, formally the minimization of the total energy amounts to solving the Hartree-Fock (HF) equations. However, if the interaction is adequately designed and the parameters are well fitted, the model, although formally is a mean field model, can effectively include a large class of many-body correlations.

The time-dependent extension of the static DFT describe nuclear oscillations around the ground-state, namely collective states like the low-lying density modes or the giant resonances. Starting from an effective Hamiltonian $H_{\text{eff}}$, time-dependent DFT looks the same as time-dependent HF (in close analogy with the discussion of the previous paragraph). It is well known from textbooks that the small amplitude limit of time-dependent HF is the so-called Random Phase Approximation (RPA) theory. Self-consistent RPA on top of Skyrme-HF, or Gogny-HF, or RMF calculations for the nuclear ground-state, have been performed for several years.

In the last five years or so, much interest has concerned the relevance of effective tensor terms within, e.g., the Skyrme Hamiltonian. In fact, only central terms (and one or two spin-orbit terms) are included in most of the Skyrme parameter sets which have been widely used so far in the literature. Many groups have recently performed HF calculations by including tensor terms on top of the usual Skyrme central terms (see Refs. [2, 3, 4, 5, 6, 7, 8, 9]). In Ref. [10], Skyrme results the for single-particle HF gaps are compared with results obtained within the framework of HF with the Gogny force (including the tensor force introduced in [11]), as well as with relativistic HF (RHF) results [12, 13, 14]. A similar yet less complete comparison had been performed in Ref. [15]. It has to be stressed that in the relativistic case the tensor interaction is associated with pion and $\rho$-meson exchange and cannot be accommodated in the Hartree, or RMF, description.

As far as RPA calculations are concerned, tensor terms have been introduced in the Skyrme framework, both for normal states [16, 17] and charge-exchange states [18, 19, 20, 21]. We will argue below that these calculations seem to lead to more clear ways to constrain the tensor force parameters, as compared with those in which HF single-particle states are considered. Gogny-RPA calculations including tensor correlations have been recently reported in Ref. [22].

The outline of the present contribution is the following. After recalling the basic features of tensor terms added on top of Skyrme zero-range forces in Sec. 2, we discuss the main results concerning the properties of single-particle states and collective vibrations in Secs. 3 and 4, respectively. Some intrinsic limitations of the DFT description of single-particle states are eventually discussed in Sec. 5, before presenting some short conclusion in Sec. 6.

### 2. Tensor force: formalism

The zero-range form of the effective nucleon-nucleon force has been proposed originally by T.H.R. Skyrme [23, 24]. We do not repeat here many well-known features of the models based on the use of the Skyrme interaction. Since its first application in HF calculations until now, it has been proven to be quite useful for the understanding of many nuclear properties. It should be never forgotten, however, that this force is meant as an effective force for HF or RPA calculations.

The tensor terms that can be added on top of the central ones have been written already in the original papers, and read

$$V^T = \frac{T}{2} \left\{ \left[ (\sigma_1 \cdot k') (\sigma_2 \cdot k') - \frac{1}{3} (\sigma_1 \cdot \sigma_2) k^2 \right] \delta(r_1 - r_2) ight. $$

$$+ \delta(r_1 - r_2) \left[ (\sigma_1 \cdot k)(\sigma_2 \cdot k) - \frac{1}{3} (\sigma_1 \cdot \sigma_2) k^2 \right] \right\}$$

$$+ U_2 \left\{ (\sigma_1 \cdot k') \delta(r_1 - r_2) (\sigma_2 \cdot k) + (\sigma_2 \cdot k') \delta(r_1 - r_2) (\sigma_1 \cdot k) - \frac{2}{3} (\sigma_1 \cdot \sigma_2) k' \cdot \delta(r_1 - r_2) k \right\} .$$

(2)
In the above expression, the operator $k$ is defined as $(\nabla_1 - \nabla_2) / 2i$, whereas $k'$ is its complex conjugate and it reads $-(\nabla'_1 - \nabla'_2) / 2i$. Operators without (with) prime symbols are meant to act at right (left). The coupling constants $T$ and $U$ denote the strengths of the triplet-even and triplet-odd tensor interactions, respectively.

These terms provide additional contributions to both the energy density $\mathcal{H}$ (the total energy is defined as $\int d^3r \mathcal{H}(\mathbf{r})$), and to the spin-orbit part of the HF average potential. In fact, these additional contributions have the same structure as other contributions coming from the exchange part of the central Skyrme terms. We will keep this distinction clear in what follows. However, from the viewpoint of pure DFT they cannot be distinguished.

The central exchange and tensor contributions to the energy density $\mathcal{H}$ are given by

$$\mathcal{H} = \frac{1}{2} \alpha (J_n^2 + J_p^2) + \beta J_n J_p,$$

where $J_n$ and $J_p$ are the so-called spin-orbit densities for neutrons and protons, respectively. Their definition in spherical systems is (see, for instance, Ref. [25])

$$J_q(r) = \frac{1}{4\pi r^3} \sum_i (2j_i + 1) \left[ j_i(j_i + 1) - l_i(l_i + 1) - \frac{3}{4} \right] R_i^2.$$

Here (and in what follows) $q=0(1)$ labels neutrons (protons). The sum over $i \equiv n, l, j$ runs over all occupied states having the given $q$, and $R_i$ are the radial wavefunctions. The HF spin-orbit potential is given by

$$U_{s.o.}^{(q)} = W_0 \frac{2}{2r} \left( \frac{d\rho_q}{dr} + \frac{d\rho'_q}{dr} \right) + (\alpha \frac{J_q}{r} + \beta \frac{J'_q}{r}).$$

The first term comes from the Skyrme two-body spin-orbit interaction and depends on the usual densities $\rho_q$, whereas the second term includes both the central exchange and the tensor contributions. These central exchange and tensor contributions enter the above expressions as

$$\alpha = \alpha_C + \alpha_T,$$

$$\beta = \beta_C + \beta_T,$$

with

$$\alpha_C = \frac{1}{8}(t_1 - t_2) - \frac{1}{8}(t_1x_1 - t_2x_2),$$

$$\beta_C = -\frac{1}{8}(t_1x_1 + t_2x_2),$$

$$\alpha_T = \frac{5}{12}U,$$

$$\beta_T = \frac{5}{24}(T + U).$$

The first thing to be noted is that $J_q$ is almost negligible in the $l \cdot s$ nuclei, that is, in those systems in which both spin-orbit partners ($j_\lessgtr = l - 1/2$ and $j_\gtrsim = l + 1/2$) are either occupied or unoccupied. In these cases, the tensor force gives no contribution either to the total energy or to the spin-orbit potential. These contributions can be appreciated by looking at the trends of those quantities along isotopic, or isotonic, chains when $J_q$ is varying according to the positive (negative) contributions provided by the occupation of the $j_\gtrsim (j_\lessgtr)$ orbitals, while the usual densities $\rho_q$ vary more smoothly.

This fact explains while the first attempt [26] to introduce the tensor force on top of the old SIII parameter set [27] was considered unsatisfactory. The results for the single-particle states
were of the same quality, or even worse, than the results obtained with the interaction SIII, since few magic nuclei had been studied.

As mentioned in the Introduction, the issue of the tensor force has been considered again, after a long time interval, since its role has been shown to be relevant in the context of the shell-model calculations for exotic nuclei [28]. However, another general word of caution is worth to be mentioned here. In Ref. [28] the bare tensor force due to pion and $\rho$-meson exchange has been employed. While this force is certainly not small, most of its effects, when fitting EDFs, are absorbed into the central parameters. In this respect, we do not expect to necessarily find consistency between the strength of the bare tensor force, and that of the effective tensor force added on top of Skyrme sets. This latter has to be adjusted by looking at some kind of experimental data.

\[
S_{12} \equiv 3(\sigma_1 \cdot \mathbf{r}_{12})(\sigma_2 \cdot \mathbf{r}_{12}) - \sigma_1 \cdot \sigma_2
\]

\[
\langle S_{12} \rangle = +2 \quad \quad \langle S_{12} \rangle = -1
\]

**Figure 1.** Schematic picture of states having either (a) parallel spins and spatial wavefunction concentrated in space or (b) parallel spins and wavefunctions more spread out in space. See the text for an explanation why case (a) and (b) should respectively correspond to \((j_>, j_<)\) and to \((j_>, j_>)\) (or \((j_<, j_<)\)).

### 3. Effect of the tensor force on single-particle states

As mentioned in the previous Section, in Ref. [28] the proton-neutron tensor force has been advocated as one of the driving mechanisms of the shell structure evolution in exotic nuclei. This force has been shown to produce a strong attraction between neutron and protons with \(j_>\) and \(j_<\) (for instance, either attraction or repulsion has been shown to take place, respectively, between \(f_{7/2}\) neutrons and \(d_{3/2}\) or \(d_{5/2}\) protons). The intuitive picture for such a mechanism is depicted in the left part (a) of figure 1. The two states \(j_>\) and \(j_<\) have spatial angular momenta that are opposite to the spins depicted in the figure. In other words, the semiclassical picture is that of two orbits with large relative momentum, and due to the uncertainty principle the
wavefunction is spatially confined (as represented schematically by the curve that surrounds the spins). The configuration is deuteron-like and the tensor is known to act attractively in this case. The complementary picture (spatial angular momenta parallel to the spins depicted, small relative momenta of the orbits, and spread wavefunctions) holds for the case (b). In that case, therefore, a repulsive effect is expected.

Although we have stressed that the effective tensor force produced by some kind of fit performed within the Skyrme framework does not necessarily have the strength of the bare tensor force, we expect to see qualitatively the same effects.

**Figure 2.** Energy differences between the $1h_{11/2}$ and $1g_{7/2}$ single-proton states along the $Z = 50$ isotopes. The calculations are performed without (crosses) and with (circles) the tensor terms in the spin-orbit potential (5), on top of the set SLy5 [29]. The experimental data (joined by the line) are from Ref. [30].

One of the first studies of this kind is the one of Ref. [4]. The motivation of this study has been the remark that even modern Skyrme sets are unable to produce the correct trend of the single-particle main peaks along isotopic or isotonic chains. From figure 2, it is evident that the calculation with the force SLy5 [29] is not able to reproduce the experimental data, while the insertion of the tensor terms provides a clear improvement. The tensor interaction employed here has $(\alpha_T, \beta_T) = (170, 100)$ MeV fm$^5$.

The interpretation of this result follows the line of the above argument. In the $Z = 50$ core, only the proton $g_{9/2}$ orbital dominates the proton spin-orbit density $J_p$. Consequently, with a negative value of $\alpha_T$, the spin-orbit potential (5) is enlarged in absolute value (note that $W_0$ is positive and the radial derivatives of the densities are negative), the values of the proton spin-orbit splittings are increased, and the energy difference $\Delta \epsilon \equiv \epsilon(h_{11/2}) - \epsilon(g_{7/2})$ is reduced with respect to the case without tensor. However, the term in $\alpha$ does not give any isospin dependence to the spin-orbit potential for a fixed proton number, at variance with the term in
From $N - Z = 6$ to $14$, the $g_{7/2}$ neutron orbit is gradually filled and $J_n$ is reduced. Then, the positive value of $\beta_T$ enlarges in absolute value the spin-orbit potential and increases the spin-orbit splitting, so that the energy difference $\Delta \epsilon$ becomes smaller. From $N - Z = 14$ to $20$, the $s_{1/2}$ and $d_{3/2}$ neutron orbits are occupied, and in this region the spin-orbit density is not so much changed as the $s_{1/2}$ orbital does not provide any contribution. Instead, for $N - Z = 20$ to $32$, the $h_{11/2}$ orbital is gradually filled. This gives a positive contribution to the spin-orbit potential and the spin-orbit splitting becomes smaller. $\Delta \epsilon$ consequently increases, and this effect is well pronounced in our theoretical results. The magnitude of $\beta$ determines the slope of the isospin dependence, so that a larger $\beta$ would give a steeper slope. In any event, it is clear that a positive value of $\beta$ provides an effect of attraction between $j_\geq$ and $j_\leq$ states, and repulsion otherwise, just as in the case of Ref. [28].

It must be added that in more systematic calculations that have been done later, the situation looks more complicated. The negative value of $\alpha$ has been questioned in Ref. [6]. More generally, in that work it has been shown that it is difficult to accommodate in the framework of Skyrme Hamiltonians plus tensor terms, the systematics of single-particle levels in nuclei. This remains true even if one does not add the tensor terms in a perturbative way on top of existing Skyrme sets, but also if one fits on an equal footing central and tensor parameters. The issue remains contradictory even if one restricts to states around the single-particle gap, and tries to learn from the comparison of Skyrme plus tensor, Gogny plus tensor and RHF including pion plus $\rho$-meson exchange as in Ref. [10]. We will argue below (see Sec. 5) that single-particle states in nuclei are too much affected by correlations that cannot be included in the DFT framework (for instance, particle-vibration coupling). Therefore, although they can give some first hint about the relevance of tensor terms, they cannot be used as a stringent constraint.

![Figure 3](image-url)  
**Figure 3.** Schematic view of the effect of the tensor force on the mean-field responses to different operators. The levels displayed on the left side evolve to those depicted on the right side due to the effect of the tensor force. The arrows show the transitions that are excited mainly by either non spin-flip (the single arrow in the left part) or spin-flip (the two arrows in the right part) external fields.
4. Effect of the tensor force on collective states

Collective states like the giant resonances are more in the framework of (time-dependent) DFT. One should aim at studying the effects of the tensor force on their properties, and identifying constraints on the tensor force parameters accordingly. This program has been carried out in Refs. [16, 17, 18, 19, 20, 21] in the Skyrme case, and will be reviewed in the present Section.

Naively, one could imagine that the tensor force will affect more the spin-flip rather than the non spin-flip states. At the mean-field level, this can be seen schematically in figure 3. Also, we expect that the residual RPA tensor force will be more relevant for the spin-flip particle-hole (ph) transitions than for the non spin-flip ones. In fact, our systematic investigation of $2^+$ and $3^-$ low-lying density modes, and high-lying giant resonances [16, 17] has concluded that (i) the impact of the tensor force on giant resonances is indeed small, (ii) although the low-lying levels are more affected because they are more sensitive to the detail of the shell structure, they cannot provide a clear and unique signature of how one could fix the value of the tensor force parameters. At the same time, one of the conclusions of Ref. [17] has been that some of the Skyrme forces that belong to the family $T_{IJ}$ introduced in Ref. [6], namely the sets $T_{36}, T_{44}, T_{45}$ and $T_{46}$ give the best results for the $2^+$ and $3^-$ states in $^{40}$Ca and $^{208}$Pb. The same is true for SGII plus tensor terms (added perturbatively). The values for energies and electromagnetic reduced transition probabilities to the ground-state turn out to be accurate at the level of $\approx 10$-20%.

In Refs. [16, 17], the spin-flip M1 excitations have been studied as well. Although in principle they are more sensitive to the introduction of the tensor force, many of the Skyrme sets produce larger values of the transition strength for the isoscalar rather than the isovector M1 operator. This isoscalar dominance is not in agreement with the experimental findings.

Therefore, we have found that spin-flip charge-exchange transitions (where by definition the isoscalar component is absent) are better candidates to study the impact of the tensor force. The first implementation of charge-exchange RPA on top of HF that includes the tensor force consistently, can be found in Ref. [18]. The model has been applied to the case of the Gamow-Teller (GT) response which is associated with the operator

$$\hat{O}_{GT\pm} = \sum_i t_{\pm} \sigma_i = \sqrt{4\pi} \sum_i t_{\pm} [Y_0 \otimes \sigma_i^1]_1.$$  \hspace{1cm} (12)

The main outcomes of this work are: (i) the peak of the GT$_-$ response in $^{90}$Zr and $^{208}$Pb is lowered by about 2 MeV when tensor correlations are included, and (ii) at the same time, about 10% of the GT strength is pushed in the high-energy region between 30 and 60 MeV by the tensor correlations. The first conclusion shows the relevance of the GT resonance if one is interested in fixing the tensor force parameters by considering sizeable effects induced by this force. The second conclusion is interesting, and consistent with the previous finding [31] that a large shift of strength towards the higher excitation energy region, far from the GT peak, can be obtained in the full model space of 1 particle-1 hole (1p-1h) plus 2 particle-2 hole (2p-2h) excitations also due to the effect of the tensor force. We find that a precursor of this effect takes place in the more limited 1p-1h space.

In Ref. [19] it has been shown that the downward shift of the GT$_-$ peak is associated with an upward shift of the spin-quadrupole SQ strength associated with the operator

$$\hat{O}_{SQ\pm} = \sum_i t_{\pm} r_i^2 [Y_2 \otimes \sigma_i^1]_1.$$  \hspace{1cm} (13)

So, the effect induced by the tensor force on the GT resonance can be seen as a “deuteron-like” coupling between modes having $L=0$ and $L=2$. Because of the lack of detailed experimental information, however, the position of GT and SQ peaks are not two observables that can constrain the two parameters $T$ and $U$ that characterize a zero-range tensor force.
It has been found in Ref. [20] that a more powerful way to constrain these parameters is provided by the spin-dipole (SD) strength. This latter is associated with the operator

\[ \hat{O}_\pm^\lambda = \sum_i r_i \left[ Y_1^i \otimes \sigma_1^i \right]_\lambda. \]  

(14)

In the case of \(^{208}\)Pb, experimentally it has been possible to disentangle the strength associated with the three total angular momentum components \(\lambda\) equal to 0\(^-\), 1\(^-\) and 2\(^-\) [32]. At the same time, from theory it has been found that the residual tensor force introduced in the charge-exchange RPA has a \textit{unique} multipole dependence. Because of this, only specific values for the tensor force parameters \(T\) and \(U\) can reproduce the experimentally observed sequence of peaks associated with the different values of \(\lambda\).

This dependence can be understood as follows. The diagonal matrix elements of the residual RPA interaction, in the isovector channel, are given after proper antisymmetrization by

\[ V_{ph,AS}^{(\lambda)} = \left( -\frac{T}{2} + \frac{U}{2} \right) a_\lambda \langle \tau_1 \cdot \tau_2 \rangle, \]  

(15)

where

\[ a_\lambda = -\frac{5}{12} \left\{ \begin{array}{c} 1 \\ -1/6 \\ 1/50 \end{array} \right\} \left| \langle \rho | \hat{O}_\lambda^\rho | h \rangle \right|^2 \text{ for } \lambda = \left\{ \begin{array}{c} 0^- \\ 1^- \\ 2^- \end{array} \right\}. \]  

(16)

So, we can see that diagonal p-h matrix elements in the 0\(^-\) case are the largest, and that in the 1\(^-\) case they are somewhat smaller. The effect on the 2\(^-\) states is expected to be very small. For interactions in which the coupling constant \(T\) is positive and \(U\) is negative (or positive but small), the tensor correlations are strongly repulsive for 0\(^-\) and weakly repulsive for 2\(^-\), whereas for 1\(^-\) they are attractive. In this way, one can explain the experimental findings.

The constraints set by the Gamow-Teller and spin-dipole resonances are put together in Ref. [21]. One of the results is illustrated in figure 4. If one wishes to add a tensor force on top of SLy5, the acceptable area for the parameters is roughly triangle-shaped. This area is defined by demanding that the difference \(\delta E\) between theoretical and experimental energies is smaller than 2.5 MeV. \(U\) can range from -300 to -50 MeV fm\(^5\), and the minimum value for \(T\) is 450 MeV fm\(^5\). The maximum value of \(T\) depend on \(U\) (the absolute maximum being close to 650 MeV fm\(^5\)). In the figure, we also show that some of the T\(IJ\) forces display quite reasonable results.

In conclusion, we have shown that it is possible to select Skyrme forces that, after the inclusion of tensor correlations, have a satisfactory behaviour when compared towards experiment in the case of quite different collective modes. T44, T45 and T46 are reasonably accurate for low-lying modes while T43 has passed several tests associated with charge-exchange spin-flip modes. These sets have quite similar values for the parameters.

We believe that the procedures we have undertaken can be useful if one wishes to improve the fine tuning of an effective tensor force. Another issue that has been discussed in Ref. [33] is the stability of functionals with the tensor terms added. We do not discuss this topic here.

5. Limits of the DFT description for the single-particle strength

It is well-known that mean field models do not account well for the single-particle energies, in particular close to the Fermi level. This fact can be considered to be consistent with the idea that these models are an implementation of DFT for atomic nuclei, and single-particle states may not be within the DFT framework.

The issue has been recently addressed in Ref. [34]. In this paper, results for the single-particle states obtained from a recent implementation of the DFT are compared with those coming from the particle-vibration coupling (PVC) approach. PVC is supposed to take care of most of the
Figure 4. (Color online) The region of $T$ and $U$ values constrained by the criterion $\delta E \leq 2.5$ MeV for the GT and total SD centroid energies in $^{90}\text{Zr}$ and $^{208}\text{Pb}$, and for the SD $1^-$ centroid energy in $^{208}\text{Pb}$. The points correspond to some of the forces $T_{IJ}$ already described in the main text (however, the central part of them is different from that of SLy5, so their position in this plot is only indicative).

dynamical correlations missing in the static DFT models, in keeping with the fact that collective vibrations are the most important low-lying states of spherical nuclei.

Experimentally, the single-particle strength displays also a non-negligible fragmentation, and theory should account for it. The fragmentation lies definitely outside the framework of DFT. This is perhaps the main motivation to try to generalize this framework, in which the basic variable is the static density, to a many-body theory where dynamical correlations are included. A simple version of this is the PVC model (also called “dynamical” shell model). There are of course other motivations: for instance, the damping width of single-particle and collective states (like the nuclear giant resonances) can be described only by means of theories beyond mean field or present-day DFT.

In the review paper [35], several phenomenological PVC calculations performed in the 80’s are compared and discussed. Qualitatively, they all demonstrate the relevance of the PVC to produce the single-particle strength fragmentation, as well as the increase of the level density (that is, the enhancement of the effective mass $m^*$) close to the Fermi energy which is empirically observed. However, none of them is consistent and it is hard to extract quantitative conclusions due to the many approximations involved. Recently, microscopic PVC calculations have become available, based on the consistent use of either Skyrme interactions [36], or RMF Lagrangians [37, 38].

The purpose of this Section is to make a short summary of the PVC formalism, and to show a few results aimed to demonstrate that single-particle states are affected enough by PVC, so that
accurate fitting of the tensor forces based on their properties is not probably a useful strategy.

5.1. Basic elements of PVC theory
Using standard many-body techniques, one can introduce the energy-dependent part $\Sigma(\omega)$ of the self-energy. It enters the well-known Dyson equation,

$$
G(\omega) = G_0(\omega) + G_0(\omega)\Sigma(\omega)G(\omega),
$$

(17)

that allows determining the Green’s function $G$ in terms of the unperturbed one $G_0$. In the present context, $G_0$ is the mean-field (i.e., Hartree-Fock) Green’s function. The $\Sigma(\omega)$ term is associated with the coupling to (mainly density) vibrations.

In the so-called diagonal approximation, as defined in Ref. [39], the quantities appearing in the Dyson equation (17) are written in the basis in which the mean-field is diagonal (that is, the HF basis), and only the diagonal matrix elements of $\Sigma(\omega)$ are considered. If the indices $i$, $p$ and $h$ label, respectively, a generic single-nucleon state, a particle state above the Fermi energy and a hole state below the Fermi energy, the diagonal matrix element $\Sigma_i$ reads

$$
\Sigma_i(\omega) = \frac{1}{2j_i + 1} \left( \sum_{nL, p > F} \frac{|\langle i|V||p, nL\rangle|^2}{\omega - \varepsilon_p - \omega_{nL} + i\eta} + \sum_{nL, h < F} \frac{|\langle i|V||h, nL\rangle|^2}{\omega - \varepsilon_h + \omega_{nL} - i\eta} \right). \tag{18}
$$

The energies $\omega_{nL}$ are the energies of the vibrational states (phonons) with angular momentum $L$, labeled by the index $n$. A (small) imaginary part $\eta$ is added in the denominators (it is set at 0.05 MeV in the calculations presented below). The numerators contain the squared modulus of a reduced matrix element called PVC vertex (see Ref. [36] for details).

The solution of the Dyson equation in the diagonal approximation is equivalent to a matrix diagonalization [37, 39]. In several cases, the results obtained by means of full diagonalization do not differ significantly from those obtained by means of perturbation theory (see, e.g., [40]). In this latter case, dressed single-particle energies are calculated by means of the equation

$$
\tilde{\varepsilon}_i = \varepsilon_i + \Sigma_i(\omega)|_{\omega=\varepsilon_i}. \tag{19}
$$

5.2. Results for a typical nucleus: $^{132}$Sn
We show results obtained by starting from the Hartree-Fock (HF) solution associated with the Skyrme parameter set SLy5 [29]. With the same parameter set we perform fully self-consistent RPA calculations in order to obtain the vibrations. The tensor force already employed in Ref. [4] is included in HF but not in the self-consistent RPA. The PVC vertex includes the central momentum-independent part of the Skyrme force. The self-energy is treated in the perturbative approximation, that is, the dressed single-particle energies are obtained by means of Eq. (19).

Results for $^{132}$Sn are displayed in figure 5. The different columns, from left to right, refer to calculations at the level of HF without the tensor force, HF including the tensor interaction of Ref. [4] and HF plus PVC, respectively. The main message of this figure is that correlations associated with PVC may produce shifts of the single-particle levels that are comparable, or larger, with respect to the shifts produced by the tensor force.

We have chosen to display the result for this nucleus, since some of its properties have been already discussed in Sec. 3, despite the fact that this calculation neglects the momentum-dependent part of the Skyrme force in the PVC vertex. This approximation is not quantitatively satisfactory, as it has been shown in Ref. [36] to overestimate the shifts associated with PVC. However, the qualitative conclusion that these shifts cannot be neglected when compared with shifts induced by the tensor correlations, remains true.
Figure 5. Results obtained for the neutron levels of $^{132}$Sn at various levels of approximation: HF with SLy5, HF with SLy5 plus tensor, HF plus PVC (from left to right). The last column at right displays experimental levels.

6. Conclusion
Recently, much attention has been devoted to the inclusion of an effective tensor force in the mean field or DFT framework. After a few years, it is timely to review the main conclusions, at least those that can be drawn by considering tensor terms added to the Skyrme effective interactions.

We have found that the most effective way to constrain the parameters of the tensor force, is the study of collective modes. In particular, charge-exchange spin-flip resonances are clearly affected by the tensor correlations. Since recent experimental data have been provided on the position of the peaks associated with the different components of the spin-dipole resonance, this represents one of the strongest tests for effective Hamiltonians that include the tensor terms. We have shown that these observables, together with the properties of the Gamow-Teller resonance, allow fixing the tensor force parameters to a large extent.

Non charge-exchange modes are also affected by tensor correlations although the effects are more subtle and less clear to disentangle from other effects. This becomes even more clear if one turns the attention towards single-particle properties. We have shown indeed that dynamical correlations associated with particle-vibration coupling can produce similar, or larger, shifts of the single-particle energy levels when compared with tensor correlations.

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