Soluzioni compitino del 05/06/2008

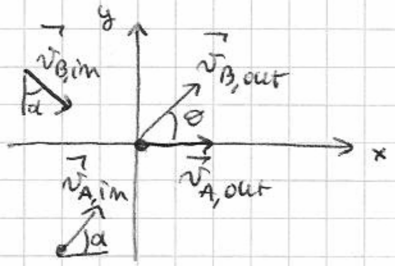
Problema 1

Urto elastico $\Rightarrow$ conserv. dell'energia cinetica.

In più, si conserva la quantità di moto $\Rightarrow$

$$\begin{cases} m_A \vec{v}_{A\,in} + m_B \vec{v}_{B\,in} = m_A \vec{v}_{A,out} + m_B \vec{v}_{B,out} \\ \frac{1}{2} m_A v_{A\,in}^2 + \frac{1}{2} m_B v_{B\,in}^2 = \frac{1}{2} m_A v_{A,out}^2 + \frac{1}{2} m_B v_{B\,out}^2 \end{cases}$$

So
$$\begin{cases} m_A = m_B = m \\ \vec{v}_{A\,in} = v\cos\alpha\,\hat{x} + v\sin\alpha\,\hat{y} \qquad \vec{v}_{A,out} = v_{A,out}\,\hat{x} \\ \vec{v}_{B\,in} = v\sin\alpha\,\hat{x} - v\cos\alpha\,\hat{y} \qquad \vec{v}_{B\,out} = v_{B,out}\cos\vartheta\,\hat{x} + v_{B,out}\sin\vartheta\,\hat{y} \end{cases}$$



$\alpha = \frac{\pi}{3}$ $\Rightarrow$

$$\begin{cases} \cancel{m}\, v\cos\alpha + \cancel{m}\, v\sin\alpha = \cancel{m}\, v_{A,out} + \cancel{m}\, v_{B,out}\cos\vartheta \\ \cancel{m}\, v\sin\alpha - \cancel{m}\, v\cos\alpha = \cancel{m}\, v_{B,out}\sin\vartheta \\ \cancel{\tfrac{1}{2}}\cancel{m}\, v^2 + \cancel{\tfrac{1}{2}}\cancel{m}\, v^2 = \cancel{\tfrac{1}{2}}\cancel{m}\, v_{A,out}^2 + \cancel{\tfrac{1}{2}}\cancel{m}\, v_{B\,out}^2 \end{cases}$$

$$\Rightarrow \begin{cases} v\cos\alpha + v\sin\alpha - v_{A,out} = v_{B\,out}\cos\vartheta \\ v\sin\alpha - v\cos\alpha = v_{B,out}\sin\vartheta \\ 2v^2 = v_{A,out}^2 + v_{B,out}^2 \end{cases}$$

Faccio il quadrato delle prime due eq. e sostituisco nella terza $\Rightarrow$

$$2v^2 = v_{A,out}^2 + v^2\cos^2\alpha + v^2\sin^2\alpha + v_{A,out}^2 + \cancel{2v^2\cos\alpha\sin\alpha}$$
$$- 2v_{A,out}\, v(\cos\alpha + \sin\alpha) + v^2\cos^2\alpha + v^2\sin^2\alpha - \cancel{2v^2\cos\alpha\sin\alpha}$$

$$\cancel{2}\cancel{v^2} = \cancel{4} v_{A,out}^{\cancel{2}} + \cancel{2v^2} - \cancel{4}\, \cancel{v_{A,out}}\, v\,(\cos\alpha + \sin\alpha)$$

$$0 = v_{A,out} - v\,(\cos\alpha + \sin\alpha)$$

$$\Rightarrow \quad v_{A,out} = v\,(\cos\alpha + \sin\alpha)$$

$$v_{B,out} \sin\theta = v\,(\sin\alpha - \cos\alpha)$$

$$v_{B,out} \cos\theta = v\cos\alpha + v\sin\alpha - v\cos\alpha - v\sin\alpha = 0$$

$$\Rightarrow \quad \theta = \frac{\pi}{2}$$

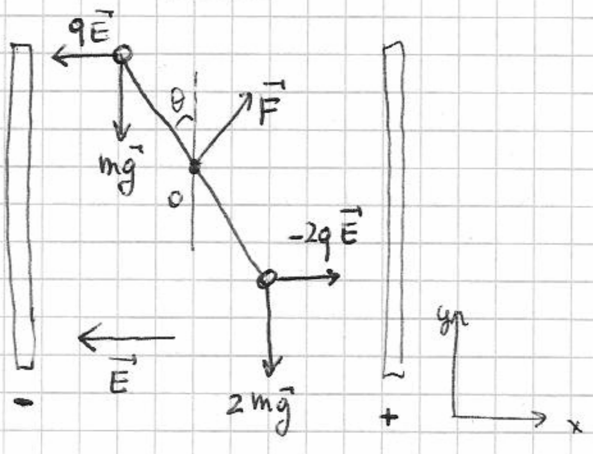
$$\Rightarrow \quad v_{B,out} = v(\sin\alpha - \cos\alpha) = v\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) = \frac{v}{2}(\sqrt{3} - 1)$$

$$\Delta E_{c,A} = \frac{1}{2} m\, v_{A,out}^2 - \frac{1}{2} m v^2 =$$

$$= \frac{1}{2} m \left(v^2 \cancel{\cos^2\alpha} + v^2 \cancel{\sin^2\alpha} + 2v^2 \cos\alpha \sin\alpha - \cancel{v^2}\right)$$

$$= m v^2 \cos\alpha \sin\alpha = m v^2 \frac{\sqrt{3}}{4} .$$

Problema 2



Risultante di tutte le forze $= 0$

$$\Downarrow$$

$$q\vec{E} + m\vec{g} + \vec{F} - 2q\vec{E} + 2m\vec{g} = 0$$

$$-qE\,\hat{x} - mg\,\hat{y} + \vec{F} + 2qE\,\hat{x} - 2mg\,\hat{y} = 0$$

$$\Rightarrow \quad \vec{F} = qE\,\hat{x} - 3mg\,\hat{y}$$

$$\Rightarrow \quad |\vec{F}| = \sqrt{(qE)^2 + 9m^2g^2}$$

In un condensatore $E = \frac{\Delta V}{d} \Rightarrow |\vec{F}| = \sqrt{\left(q\frac{\Delta V}{d}\right)^2 + 9m^2g^2}$

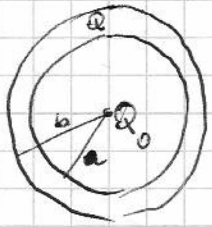
Somma di tutti i momenti rispetto a O = 0

$$\Rightarrow \quad mg\frac{l}{2}\sin\theta + qE\frac{l}{2}\cos\theta - 2mg\frac{l}{2}\sin\theta + 2qE\frac{l}{2}\cos\theta = 0$$

$$\Rightarrow \quad 3qE\cos\theta = mg\sin\theta$$

$$\Rightarrow \quad tg\,\theta_{eq} = \frac{3qE}{mg} = \frac{3q\,\Delta V}{mg\,d}$$

<u>Problema 3</u>



Applichiamo il teorema di Gauss. Come sup. su cui calcolare il flusso di E, sceglieme una sfera di raggio r

$$\Rightarrow \quad 4\pi r^2 E = \frac{q_{in}}{\varepsilon_0} \qquad \text{se} \quad r<a \qquad 4\pi r^2 E = \frac{Q_0}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{Q_0}{r^2}$$

se $a<r<b$

$$4\pi r^2 E = \frac{1}{\varepsilon_0}\left(Q_0 + \frac{Q}{\frac{4}{3}\pi(b^3-a^3)}\frac{4}{3}\pi(r^3-a^3)\right)$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\left(Q_0 + Q\frac{r^3-a^3}{b^3-a^3}\right)$$

se $r>b$

$$4\pi r^2 E = \frac{1}{\varepsilon_0}(Q_0+Q) \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{(Q_0+Q)}{r^2}$$

$\Downarrow$

campo elettrico generato da una carica posta nel centro della sfera pari a Q_0+Q.

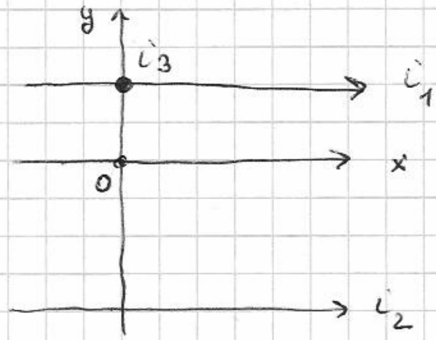
Il potenziale per $r>b$ $\quad V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q_0+Q}{r}$ e $V(\infty)=0$

$\Rightarrow$ la carica q si muove in $E(r>b)$ $\Rightarrow$ in un campo conservativo $\Rightarrow$ si conserva la sua energia meccanica $\Rightarrow$

$$\frac{1}{2} m v_0^2 + \underbrace{U_E(\infty)}_{0} = \underbrace{U_E(r=b)}_{q\,V(r=b)}$$

$$\Rightarrow \quad \cancel{\frac{1}{2}} m v_{min}^2 = q\,\frac{(Q_0+Q)}{\underset{2}{\cancel{4}}\pi\varepsilon_0\, b} \quad \Rightarrow \quad v_{min} = \sqrt{\frac{1}{2\pi\varepsilon_0}\,\frac{(Q_0+Q)q}{mb}}$$

Problema 4



$$i_1 = i_2 = i_3 = i$$

$$\vec{B}(0) = \vec{B}_1(0) + \vec{B}_2(0) + \vec{B}_3(0)$$

$$\vec{B}_1(0) = -\frac{\mu_0}{2\pi}\,\frac{i}{d}\,\hat{z}$$

$$\vec{B}_2(0) = +\frac{\mu_0}{2\pi}\,\frac{i}{2d}\,\hat{z}$$

$$\vec{B}_3(0) = \frac{\mu_0}{2\pi}\,\frac{i}{d}\,\hat{x}$$

$$\Rightarrow \quad B_z(0) = \frac{\mu_0}{2\pi}\,\frac{i}{2d} - \frac{\mu_0}{2\pi}\,\frac{i}{d} = -\frac{\mu_0}{4\pi}\,\frac{i}{d}$$

$$|\vec{B}| = \sqrt{\left|\frac{\mu_0}{2\pi}\,\frac{i}{d}\,\hat{x}\right|^2 + B_z^2} = \frac{\mu_0}{2\pi}\,\frac{i}{d}\sqrt{1+\frac{1}{4}}$$

$$= \frac{\mu_0}{4\pi}\,\frac{i}{d}\sqrt{5}$$