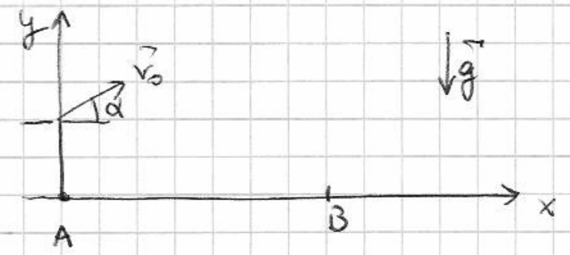
Compito del 19/06/2008

Problema 1



1. $$\begin{cases} x(t) = v_0 \cos\alpha \; t \\ y(t) = h + v_0 \sin\alpha \; t - \frac{1}{2} g t^2 \end{cases}$$

Istante in cui la pallina cade in buca è dato da

$$d = v_0 \cos\alpha \; t \Rightarrow t = \frac{d}{v_0 \cos\alpha}$$

In quest'istante $y(t) = 0 \Rightarrow$

$$0 = h + \cancel{v_0} \sin\alpha \frac{d}{\cancel{v_0} \cos\alpha} - \frac{1}{2} g \frac{d^2}{v_0^2 \cos^2\alpha}$$

$$0 = h + tg\alpha \; d - \frac{1}{2} g \frac{d^2}{v_0^2 \cos^2\alpha}$$

$$\frac{g d^2}{2 v_0^2 \cos^2\alpha} = h + tg\alpha \; d \Rightarrow v_0 = \frac{d}{\cos\alpha} \sqrt{\frac{g}{2(h + tg\alpha \; d)}}$$

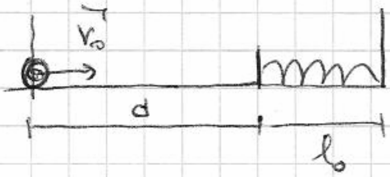
2. Conservazione dell'energia meccanica:

$$\frac{1}{2} \cancel{m} v_0^2 + \cancel{m} g h = \frac{1}{2} \cancel{m} v_B^2 \Rightarrow v_B = \sqrt{v_0^2 + 2gh}$$

3. All'altezza max. $v_y = 0 \Rightarrow v = v_x = v_0 \cos\alpha$

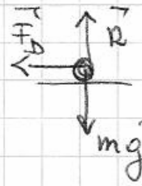
Problema 2

4.



Teorema delle forze vive $\quad \frac{1}{2} m v_{in}^2 - \frac{1}{2} m v_0^2 = L_{attrito}$

Forze sulla pallina



$R = mg$

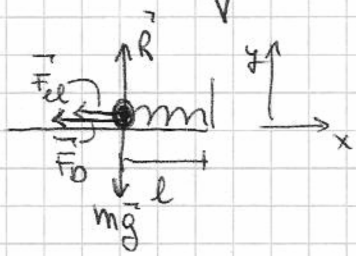
$F_D = \mu_D mg$

$\Rightarrow \quad L_{attrito} = -\mu_D mg \cdot d$

$\Rightarrow \quad \frac{1}{2} \cancel{m} v_{in}^2 - \frac{1}{2} \cancel{m} v_0^2 = -\mu_D \cancel{m} g d$

$$v_{in} = \sqrt{v_0^2 - 2\mu_D g d}$$

5.



$l = l_0 + \Delta l \qquad \Delta l = p\, l_0$

$$m\vec{a} = m\vec{g} + \vec{R} + \vec{F}_D + \vec{F}_{el} \qquad \begin{cases} ma = -F_D - F_{el} \\ 0 = R - mg \end{cases} \Rightarrow$$

$R = mg$

$ma = -\mu_D R - k(l - l_0) = -\mu_D mg - k\Delta l$

$\Rightarrow \quad a = -\mu_D g - \frac{k}{m} p l_0$

6. $\quad E_{mecc}(fin) - E_{mecc}(in) = L_{attrito}$

$$\frac{1}{2} k \Delta l_{max}^2 - \frac{1}{2} m v_{in}^2 = -\mu_D mg \Delta l_{max}$$

$$k \Delta l_{max}^2 + 2\mu_D mg \Delta l_{max} - m v_{in}^2 = 0$$

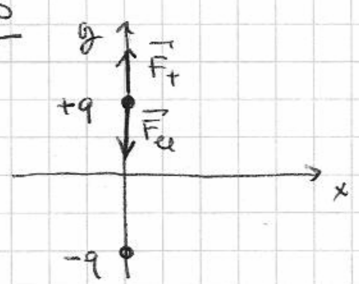
$$\Delta l_{max} = \frac{-\mu_D mg \pm \sqrt{(\mu_D mg)^2 + k m v_{in}^2}}{k} =$$

sol. non fisica

$$= \frac{-\mu_D mg + \sqrt{(\mu_D mg)^2 + k v_0^2 - 2k\mu_D g d m}}{k}.$$

Problema 3

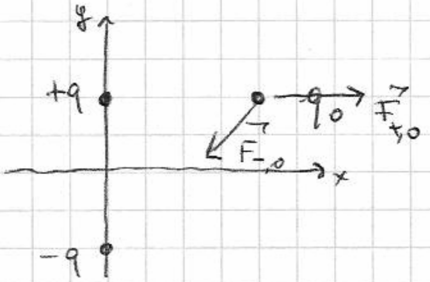
7.



$$\vec{F}_+ + \vec{F}_{el} = 0$$

$$\Rightarrow \quad F_+ = k_e \frac{qq}{d^2} = k_e \frac{q^2}{d^2}$$

9.



$$\vec{F}_0 = \vec{F}_{+,0} + \vec{F}_{-,0}$$

$$\vec{F}_{+,0} = k_e \frac{qq_0}{d^2} \hat{x}$$

$$\vec{F}_{-,0} = k_e \frac{qq_0}{2d^2} \left(-\frac{\sqrt{2}}{2} \hat{x} - \frac{\sqrt{2}}{2} \hat{y} \right)$$

$$\Rightarrow \quad \vec{F}_0 = k_e \frac{qq_0}{d^2} \left(\left(1 - \frac{\sqrt{2}}{4}\right) \hat{x} - \frac{\sqrt{2}}{4} \hat{y} \right)$$

$$m\vec{a} = \vec{F}_0 \quad \Rightarrow \quad \vec{a} = \frac{k_e}{m} \frac{qq_0}{d^2} \left(\left(1 - \frac{\sqrt{2}}{4}\right) \hat{x} - \frac{\sqrt{2}}{4} \hat{y} \right)$$

$$|\vec{a}| = \frac{k_e}{m} \frac{qq_0}{d^2} \sqrt{1 - \frac{\sqrt{2}}{2} + \frac{2}{16} + \frac{2}{16}} = \frac{k_e}{m} \frac{qq_0}{d^2} \sqrt{\frac{5}{4} - \frac{\sqrt{2}}{2}} .$$

8.

Conservazione dell'energia

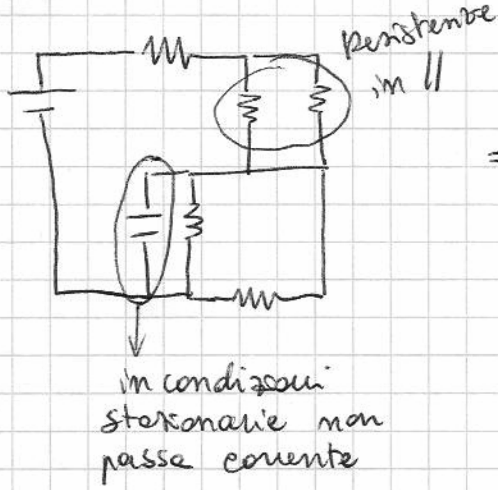
$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v^2 + k_e \frac{qq_0}{d} - k_e \frac{qq_0}{\sqrt{2} d}$$

$$v_0^2 = v^2 + 2 \frac{k_e}{m} \frac{qq_0}{d} - 2 \frac{k_e}{m} \frac{qq_0}{\sqrt{2} d}$$

$$\Rightarrow \quad v = \sqrt{v_0^2 - 2 \frac{k_e}{m} \frac{qq_0}{d} \left(1 - \frac{1}{\sqrt{2}} \right)}$$

Problema 4

10.



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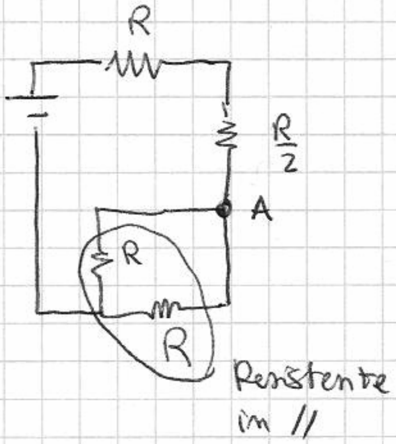

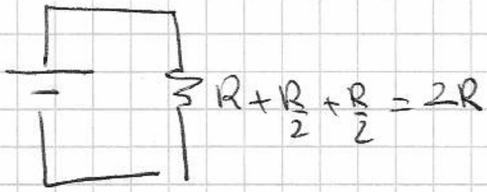

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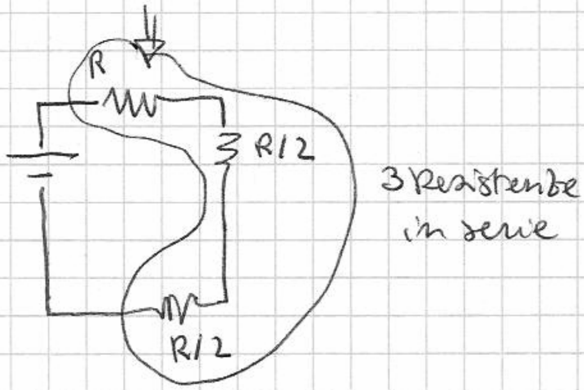


$$\Rightarrow \quad \mathcal{E} = 2R\,i \quad \Rightarrow \quad i = \frac{\mathcal{E}}{2R}$$

11. $$U = \frac{1}{2} C \Delta V^2 = \frac{1}{2} C \left(R\, i_3\right)^2$$

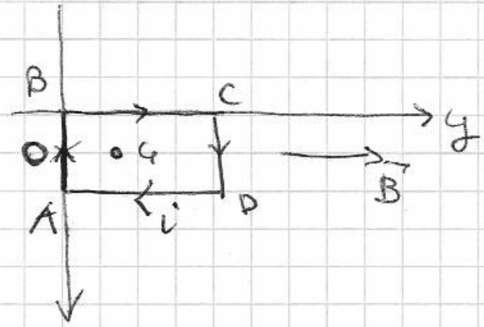
Al nodo A $\qquad i = i_3 + i_5$

$$R\, i_3 = R\, i_5 \quad \Rightarrow \quad i_3 = i_5 = \frac{i}{2}$$

$$\Rightarrow \quad U = \frac{1}{2} C \left(R \frac{i}{2}\right)^2 = \frac{1}{2} C \left(\cancel{R} \frac{\mathcal{E}}{4\cancel{R}}\right)^2 = \frac{1}{32} C \mathcal{E}^2$$

12. $$P = R_5\, i_5^2 = R\,\frac{i^2}{4} = \frac{\cancel{R}}{4}\,\frac{\mathcal{E}^2}{4R^{\cancel{2}}} = \frac{\mathcal{E}^2}{16R}$$

Problema 5



13. Forze che agiscono sulla lastra

$$\vec F_{AB} = (-\hat z)\, i a B$$
$$\vec F_{CD} = (+\hat z)\, i a B$$
$$M\vec g \, , \; \vec F_c$$

Equilibrio $\Rightarrow$ $\left.\begin{array}{l} \Sigma \vec M_O = 0 \\ \Sigma \vec F = 0 \end{array}\right\} \Rightarrow$

$$Mg \frac{\cancel{b}}{3}(-\hat x) + i a B \cancel{b} (\hat x) = 0$$

$$i a B = \frac{Mg}{3} \Rightarrow \quad B = \frac{Mg}{3 i a}$$

14. $F_{AB} = i a B = \frac{Mg}{3\cancel{ia}} \cancel{ia} = \frac{Mg}{3}$

15. $\underbrace{\vec F_{AB} + \vec F_{CD}}_{\text{si annullano}} + \vec F_c + M\vec g = 0 \Rightarrow F_c = Mg$