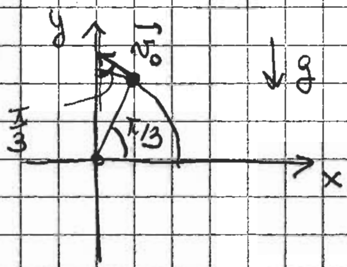
Compito dell' 11/09/08

Problema 1



All'istante iniziale

$$\vec{v}_0 = \omega L \left(- \sin\frac{\pi}{3}\,\hat{x} + \cos\frac{\pi}{3}\,\hat{y} \right)$$

$$= \frac{\omega L}{2} \left(-\sqrt{3}\,\hat{x} + \hat{y} \right)$$

$$\Rightarrow \quad x(t) = L \cos\frac{\pi}{3} + v_{0x} t = \frac{L}{2} - \frac{\sqrt{3}}{2}\omega L t$$

$$y(t) = L \sin\frac{\pi}{3} + v_{0y} t - \frac{1}{2} g t^2 = \frac{\sqrt{3}}{2} L + \frac{\omega L}{2} t - \frac{1}{2} g t^2$$

$\Rightarrow$ 1. y_{max} si ha nell'istante t in cui $v_y = 0$

$$v_y(t) = v_{0y} - g t = \frac{\omega L}{2} - g t = 0 \quad \Rightarrow \quad t = \frac{\omega L}{2g}$$

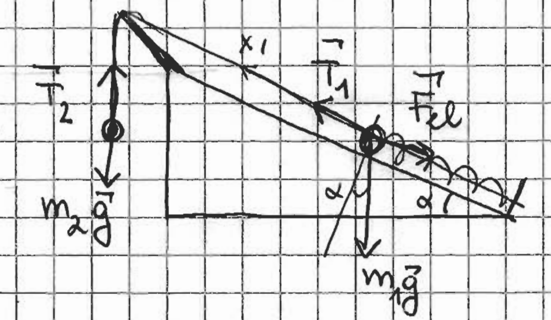
$$y_{max} = \frac{\sqrt{3}}{2} L + \frac{\omega L}{2} \cdot \frac{\omega L}{2g} - \frac{1}{2} \cancel{g} \frac{(\omega L)^2}{4 g^{\cancel{2}}}$$

$$= \frac{\sqrt{3}}{2} L + \frac{\omega^2 L^2}{4g} - \frac{\omega^2 L^2}{8g} = \frac{\sqrt{3}}{2} L + \frac{\omega^2 L^2}{8g}$$

2. t tale che $x = 0 \Rightarrow \frac{\cancel{L}}{\cancel{2}} - \frac{\sqrt{3}}{\cancel{2}} \omega \cancel{L} t = 0 \qquad t = \frac{1}{\omega\sqrt{3}}$

3. $x = -\cancel{L} = \frac{\cancel{L}}{2} - \frac{\sqrt{3}}{2} \omega \cancel{L} t \qquad \frac{3}{\cancel{2}} = \frac{\sqrt{3}}{\cancel{2}} \omega t \qquad t = \frac{\sqrt{3}}{\omega}$

Problema 2

4. 

$\alpha = \frac{\pi}{6}$

Equilibrio $\Rightarrow \begin{cases} m_2\vec{g} + \vec{T}_2 = 0 \\ \vec{T}_1 + \vec{F}_{el} + m_1\vec{g} = 0 \end{cases}$

Filo inestensibile di massa nulla $\Rightarrow |\vec{T}_1| = |\vec{T}_2| = T$

$\Rightarrow \quad m_2 g = T$

Per 1 proietto su x_1 $\Rightarrow \quad T - F_{el} - m_1 g \sin\alpha = 0$

$\Rightarrow \quad F_{el} = (m_2 - m_1 \sin\alpha)\, g = mg\,(1-\sin\alpha)$

$F_{el} = k\,\Delta l_{eq} = mg\,(1-\sin\alpha) \Rightarrow$

$$\Delta l_{eq} = \frac{mg}{k}\Big(1-\underbrace{\sin\frac{\pi}{6}}_{\frac{1}{2}}\Big) = \frac{mg}{2k}$$

5. Applichiamo la cons. dell'energia meccanica, scegliendo come zero per l'en. pot. gravitazionale quella della configurazione a $t=0$ $\Rightarrow$

$$0 = \frac{1}{2}\cancel{m_1}v_1^2 + \frac{1}{2}\cancel{m_2}v_2^2 + \frac{1}{2}k\,\Delta l_{eq}^2 + \cancel{m_1}g\,\Delta l_{eq}\sin\alpha - \cancel{m_2}g\,\Delta l_{eq}$$

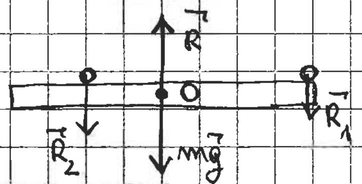
Osserviamo che $v_1 = v_2$ e ricordiamo che $m_1 = m_2$ $\Rightarrow$

$$\cancel{m}v^2 = \cancel{m}g\underbrace{\Delta l_{eq}}_{\frac{mg}{2k}}(1-\underbrace{\sin\alpha}_{1/2}) - \frac{1}{2}\frac{k}{m}\Delta l_{eq}^2 \Rightarrow v = \sqrt{\frac{mg^2}{8k}}$$

6. Allung. max $\Rightarrow v_1 = v_2 = 0 \Rightarrow 0 = \frac{1}{2}k\,\Delta l_{max}^2 + mg\,\Delta l_{max}(\sin\alpha - 1)$

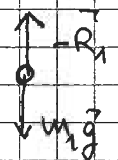
$\Rightarrow \Delta l_{max} = mg/k$

Problema 3

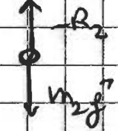


Forze sull'asta

Forze su 1



; Forze su 2



Equilibrio $\Rightarrow$ $m_1 g = R_1$

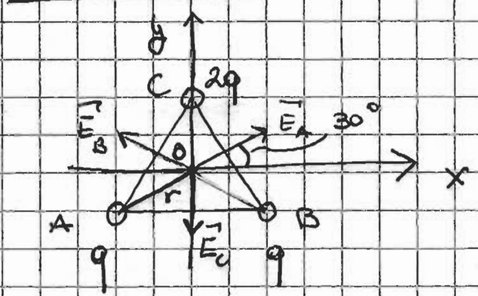
$m_2 g = R_2$

$$\begin{cases} \sum \vec{F}_i = 0 & \vec{R}_1 + \vec{R}_2 + \vec{R} + m\vec{g} = 0 \Rightarrow R = mg + R_1 + R_2 \\ \sum \vec{M}_{0,i} = 0 & R_1 \dfrac{l}{2} - R_2 d = 0 \end{cases}$$

7. $\Rightarrow \quad m_1 \cancel{g} \dfrac{l}{2} - m_2 \cancel{g} d = 0 \qquad m_2 = m_1 \dfrac{l}{2d}$

8. $R = mg + m_1 g + m_2 g = \left(m + m_1 + m_1 \dfrac{l}{2d} \right) g$

Problema 4



$$r = \frac{2}{3}\sqrt{\ell^2 - \frac{\ell^2}{4}} = \ell\frac{\sqrt{3}}{\cancel{2}}\cdot\frac{\cancel{2}}{3} = \frac{\ell}{\sqrt{3}}$$

= distanza da O di una qualunque delle tre cariche

9. $$\vec{E}(O) = \vec{E}_A + \vec{E}_B + \vec{E}_C$$

$$\vec{E}_A = k_e\frac{q}{r^2}\left(\cos 30°\,\hat{x} + \sin 30°\,\hat{y}\right) = k\frac{q}{\frac{\ell^2}{3}}\left(\frac{\sqrt{3}}{2}\hat{x} + \frac{1}{2}\hat{y}\right)$$

$$\vec{E}_B = k_e\frac{q}{r^2}\left(-\cos 30°\,\hat{x} + \sin 30°\,\hat{y}\right) = k\frac{q}{\frac{\ell^2}{3}}\left(-\frac{\sqrt{3}}{2}x + \frac{1}{2}\hat{y}\right)$$

$$\vec{E}_C = -k_e\frac{2q}{r^2}\hat{y} = -k_e\frac{2q}{\frac{\ell^2}{3}}\hat{y}$$

$$\Rightarrow \vec{E}(O) = k_e\frac{q}{\ell^2}\cdot 3\left(\frac{1}{2}\hat{y} + \frac{1}{2}\hat{y} - 2\hat{y}\right) = -k_e\frac{3q}{\ell^2}\hat{y}$$

$$\Rightarrow |\vec{E}(O)| = 3k_e\frac{q}{\ell^2}$$

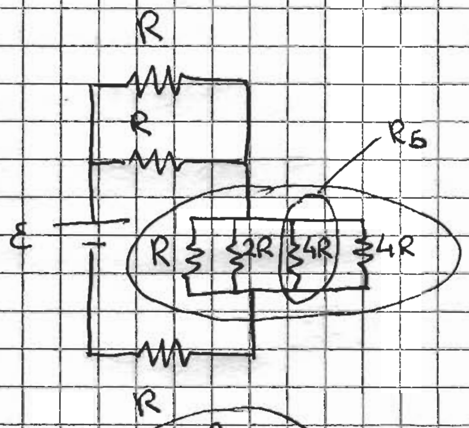
10. Conservazione dell'energia

$$\frac{1}{2}m v_0^2 = \underbrace{k_e\frac{qq_0}{\frac{\ell}{\sqrt{3}}}}_{q_0 V_A} + \underbrace{k_e\frac{qq_0}{\frac{\ell}{\sqrt{3}}}}_{q_0 V_B} + \underbrace{k_e\frac{2qq_0}{\frac{\ell}{\sqrt{3}}}}_{q_0 V_C} = 4\sqrt{3}\,k_e\frac{qq_0}{\ell}$$

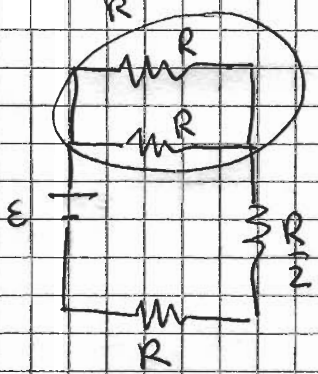
$$\Rightarrow v_0 = \sqrt{8\sqrt{3}\,\frac{k_e}{m}\frac{qq_0}{\ell}}\,.$$

Problema 5

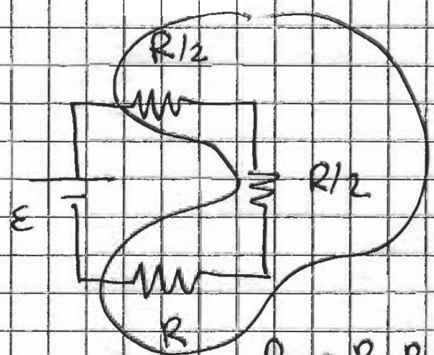
In condizioni stazionarie dal ramo contenente il condensatore non passa corrente $\Rightarrow$



$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{4R} + \frac{1}{4R} = \frac{2}{R}$$



$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R} \Rightarrow$$



$$R_{eq} = \frac{R}{2} + \frac{R}{2} + R = 2R$$

11. $\Rightarrow \quad 2R\, i = \varepsilon \qquad i = \frac{\varepsilon}{2R}$

12. $\Delta V = \frac{R}{2}\, i = \frac{\varepsilon}{4} \qquad E = \frac{1}{2} C \Delta V^2 = \frac{1}{2} C \frac{\varepsilon^2}{16} = \frac{C\varepsilon^2}{32}$

13. $R_3 i_3 = R_4 i_4 = R_5 i_5 = R_6 i_6 \quad ; \quad i = i_3 + i_4 + i_5 + i_6$

$\Downarrow$

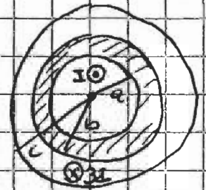
$\cancel{R}\, i_3 = 2\cancel{R}\, i_4 = 4\cancel{R}\, i_5 = 4\cancel{R}\, i_6 \Rightarrow i = 4 i_5 + 2 i_5 + i_5 + i_5$

$= 8 i_5$

$\Rightarrow i_5 = \frac{i}{8} = \frac{\varepsilon}{16R}$

$$P = R_5 i_5^2 = \cancel{4}\cancel{R} \frac{\varepsilon^2}{4^{\cancel{4}3} R^{\cancel{2}}} = \frac{\varepsilon^2}{64R}$$

Problema 6



14. $B(r=3c)$ Applichiamo il teorema d'Ampère usando come percorso per calcolare la circuitazione di B una circonferenza di raggio $3c$. Sfruttando poi la simmetria cilindrica del problema, si ha

$$\cancel{2}\pi(3c)\,B = \mu_0(-I+3I) = \cancel{2}\mu_0 I$$

$$B(r=3c) = \frac{\mu_0}{\pi}\frac{I}{3c}$$

15. Applichiamo nuovamente il teorema di Ampère usando come percorso una circonferenza di raggio $= \frac{a+b}{2}$.
A questa distanza dal centro del cilindro ci troviamo nella zona del dielettrico $\Rightarrow$

$$\cancel{2}\pi\frac{a+b}{\cancel{2}}B = \mu_0 I$$ unica corrente concatenata

$$\Rightarrow\quad B\left(r=\frac{a+b}{2}\right) = \frac{\mu_0}{\pi}\frac{I}{a+b}$$