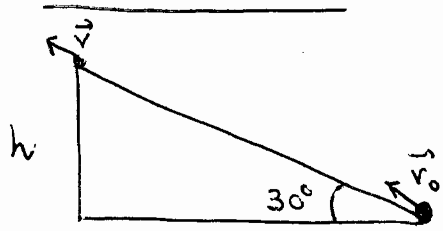
Compito del 13/01/2009

Problema 1



1. Conservazione dell' energia meccanica ⇒

$$\frac{1}{2} k \ell^2 = mgh + \frac{1}{2} m v^2$$

$$\Rightarrow \quad v = \sqrt{\frac{k}{m} \ell^2 - 2gh}$$

2. All'altezza max $v_y = 0$, mentre $v_x = \text{cost} = v \cos 30°$

Sempre sfruttando la conserv. dell' energia meccanica

$$\frac{1}{2} m v_x^2 + mg h_{max} = \frac{1}{2} m v^2 + mgh$$

$$\frac{1}{2} \cancel{m} \left(v \cdot \frac{\sqrt{3}}{2}\right)^2 + \cancel{m} g h_{max} = \frac{1}{2} \cancel{m} v^2 + \cancel{m} g h$$

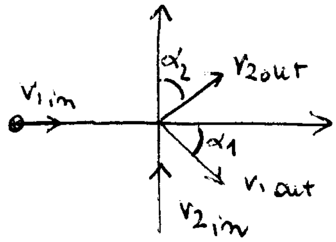
$$h_{max} = h + \frac{1}{2} \frac{v^2}{g} \left(1 - \frac{3}{4}\right) = h + \frac{1}{8} \frac{v^2}{g} =$$

$$= h + \frac{1}{8} \frac{k \ell^2}{mg} - \frac{1}{4} h = \frac{3}{4} h + \frac{1}{8} \frac{k \ell^2}{mg}$$

3. Conserv. dell' energia meccanica durante tutto il moto

$$\Rightarrow \quad \cancel{\frac{1}{2}} k \ell^2 = \cancel{\frac{1}{2}} m v_f^2 \quad \Rightarrow \quad v_f = \sqrt{\frac{k \ell^2}{m}}$$

Problema 2



4.
5.

$$\left.\begin{aligned} \cancel{m}\, v_{1in} &= \cancel{m}\, v_{1out}\cos\alpha_1 + \cancel{m}\, v_{2out}\sin\alpha_2 \\ \cancel{m}\, v_{2in} &= -\cancel{m}\, v_{1out}\sin\alpha_1 + \cancel{m}\, v_{2out}\cos\alpha_2 \end{aligned}\right\}\ \text{Cons. q.d. moto}$$

$$\cancel{\tfrac{1}{2}}\cancel{m}\, v_{1in}^2 + \cancel{\tfrac{1}{2}}\cancel{m}\, v_{2in}^2 = \cancel{\tfrac{1}{2}}\cancel{m}\, v_{1out}^2 + \cancel{\tfrac{1}{2}}\cancel{m}\, v_{2out}^2 \qquad \text{Cons. energia cin.}$$

$$\begin{cases} v_{1out}\cos\alpha_1 = v_{1in} - v_{2out}\sin\alpha_2 \\ v_{1out}\sin\alpha_1 = -v_{2in} + v_{2out}\cos\alpha_2 \end{cases}$$

$$\begin{aligned} v_{1out}^2 = v_{1in}^2 &- 2 v_{1in} v_{2out}\sin\alpha_2 \\ &+ v_{2out}^2 \sin^2\alpha_2 \\ &+ v_{2in}^2 - 2 v_{2in} v_{2out}\cos\alpha_2 \\ &+ v_{2out}^2\cos^2\alpha_2 \end{aligned}$$

$$\Rightarrow \cancel{v_{1out}^2} = \underbrace{v_{1in}^2 + v_{2in}^2}_{\cancel{v_{1out}^2} + v_{2out}^2} + v_{2out}^2 - 2 v_{1in} v_{2out}\sin\alpha_2 - 2 v_{2in} v_{2out}\cos\alpha_2$$

$$\Rightarrow \cancel{2}\, v_{2out}^{\cancel{2}} - \cancel{2}\, \cancel{v_{2out}} \left(v_{1in}\sin\alpha_2 + v_{2in}\cos\alpha_2 \right) = 0$$

$$v_{2out} = v_{1in}\sin\alpha_2 + v_{2in}\cos\alpha_2$$

$$\operatorname{tg}\alpha_1 = \frac{-v_{2in} + v_{2out}\cos\alpha_2}{v_{1in} - v_{2out}\sin\alpha_2} =$$

$$= \frac{-v_{2in} + v_{1in}\cos\alpha_2\sin\alpha_2 + v_{2in}\cos^2\alpha_2}{v_{1in} - v_{1in}\sin^2\alpha_2 - v_{2in}\sin\alpha_2\cos\alpha_2} =$$

$$= \frac{-v_{2in}\sin^2\alpha_2 + v_{1in}\cos\alpha_2\sin\alpha_2}{v_{1in}\cos^2\alpha_2 - v_{2in}\sin\alpha_2\cos\alpha_2} = \operatorname{tg}\alpha_2$$

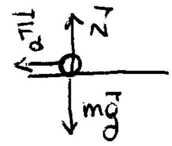
$$\Rightarrow \alpha_1 = \alpha_2$$

$$\Rightarrow \quad v_{1out} \cancel{\cos\alpha_2} = v_{1in} - v_{1in} \sin^2\alpha_2 - v_{2in} \sin\alpha_2 \cos\alpha_2$$

$$= v_{1in} \cos^2\alpha_2 - v_{2in} \sin\alpha_2 \cancel{\cos\alpha_2}$$

$$v_{1out} = v_{1in} \cos\alpha_2 - v_{2in} \sin\alpha_2$$

6. $$\begin{cases} v(t) = v_{2out} - \mu_d g t \\ s(t) = v_{2out} t - \frac{1}{2} \mu_d g t^2 \end{cases}$$



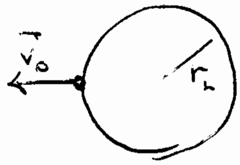
$$N = mg$$

$$F_d = \mu_d N = \mu_d mg$$

$$\Rightarrow \cancel{m} a = \mu_d \cancel{m} g$$

$$t = \frac{v_{2out}}{\mu_d g} \quad \Rightarrow \quad s(t) = \frac{v_{2out}^2}{2\mu_d g}$$

Problema 3



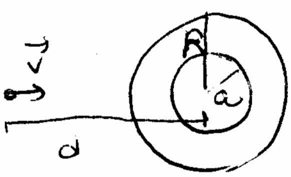
7. Cons. dell' energia meccanica (dist. max $\Rightarrow$ $v=0$)

$$\frac{1}{2}\cancel{m}v_0^2 - \frac{G m_L \cancel{m}}{r_L} = -\frac{G m_L \cancel{m}}{d}$$

$$\frac{G m_L}{d} = \frac{G m_L}{r_L} - \frac{1}{2}v_0^2 \Rightarrow d = \frac{G m_L}{\frac{G m_L}{r_L} - \frac{1}{2}v_0^2}$$

8. $$\cancel{m}\, a = \frac{G m_L \cancel{m}}{d^2} \Rightarrow a = \frac{G m_L}{d^2}$$

Problema 4



$$E(r>a) = k_e \frac{Q}{r^2}$$

$$\frac{7}{8}R^3$$

$$Q = \frac{4}{3}\pi\left(R^3 - \frac{R^3}{8}\right)\rho = \frac{7}{6}\pi R^3 \rho$$

9. $$m\,a = k_e \frac{qQ}{(2R)^2} = k_e\, q \frac{\frac{7}{6}\pi R^{\cancel{3}}\rho}{4\cancel{R^2}} = \frac{7}{24}\pi k_e\, q\rho R$$

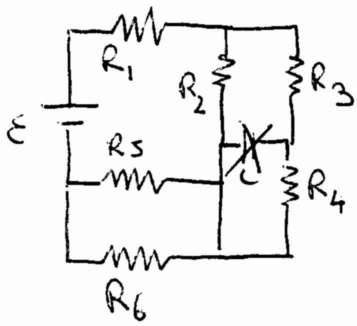
$$\Rightarrow a = \frac{7}{24}\pi k_e \frac{q\rho R}{m}$$

10. Cons. dell' energia meccanica

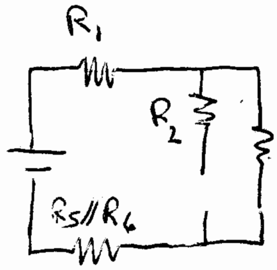
$$\frac{1}{2}mv^2 + k_e \frac{qQ}{4R} = k_e \frac{qQ}{d} \qquad d = \frac{k_e q Q}{\frac{1}{2}mv^2 + \frac{k_e q Q}{4R}}$$

$$d = \frac{k_e\, q\, \frac{7}{6}\pi R^3 \rho}{\frac{1}{2}mv^2 + \frac{7}{24}\pi q R \rho}$$

Problema 5

11. 

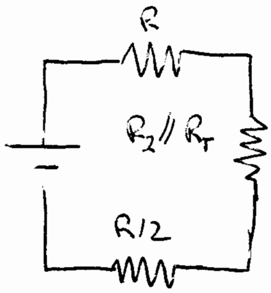
$\longrightarrow$



R_3 serie $R_4 \Rightarrow R_T = 2R$

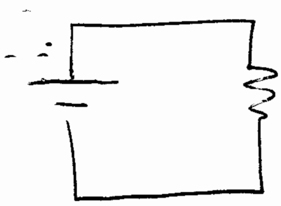
$$\frac{1}{R_T} = \frac{1}{R} + \frac{1}{R} = \frac{R}{2}$$

$\longrightarrow$



$$\frac{1}{R_T} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R} \Rightarrow R_T = \frac{2}{3}R$$

$\longrightarrow$



$$R + \frac{R}{2} + \frac{2}{3}R = \frac{6+3+4}{6}R = \frac{13}{6}R$$

$$\Rightarrow i = \frac{\varepsilon}{\frac{13}{6}R} = \frac{6}{13}\frac{\varepsilon}{R}$$

12. $\Delta V = i_4 R_4$ $\qquad i_2 \cancel{R_2} = i_4\, 2\cancel{R}$ ← R_3 serie R_4 $\qquad i_2 = 2i_4$

$$i_2 + i_4 = i \qquad 2i_4 + i_4 = i \Rightarrow i_4 = \frac{i}{3}$$

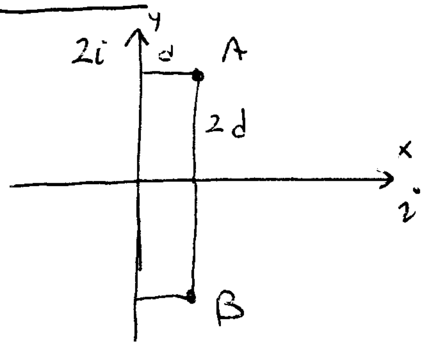
$$\Rightarrow C = \frac{Q}{\Delta V} \qquad Q = C\,\Delta V = C R \frac{i}{3} = C \cancel{R}\,\frac{2}{13}\frac{\varepsilon}{\cancel{R}}$$

$$= \frac{2}{13}\varepsilon C$$

13. $i_5 \cancel{R_5} = i_6 \cancel{R_6} \Rightarrow 2i_6 = i \Rightarrow i_6 = \frac{i}{2}$

$$P = R\left(\frac{i}{2}\right)^2 = R\frac{i^2}{4} = \frac{1}{4}R\left(\frac{6}{13}\frac{\varepsilon}{R}\right)^2 = \frac{9}{169}\frac{\varepsilon^2}{R}$$

Problema 6



14. $\vec{B}(A) = \vec{B}_1(A) + \vec{B}_2(A)$

$$\vec{B}_1(A) = \frac{\mu_0}{2\pi}\,\frac{i}{2d}\,\hat{z}$$

$$\vec{B}_2(A) = \frac{\mu_0}{2\pi}\,\frac{2i}{d}\,(-\hat{z})$$

$$\Rightarrow \vec{B}(A) = \frac{\mu_0}{2\pi}\,\frac{i}{d}\left(\frac{1}{2} - 2\right)\hat{z}$$

$$= -3\,\frac{\mu_0}{4\pi}\,\frac{i}{d}\,\hat{z}$$

15. $\vec{B}(B) = \vec{B}_1(B) + \vec{B}_2(B)$

$$\vec{B}_1(B) = \frac{\mu_0}{2\pi}\,\frac{i}{2d}(-\hat{z})$$

$$\vec{B}_2(B) = \frac{\mu_0}{2\pi}\,\frac{2i}{d}\,(-\hat{z})$$

$$\vec{B}(B) = \frac{\mu_0}{2\pi}\,(-\hat{z})\,\frac{i}{d}\left(\frac{1}{2} + 2\right) =$$

$$= \frac{\mu_0}{4\pi}\,5\,\frac{i}{d}\,(-\hat{z})$$