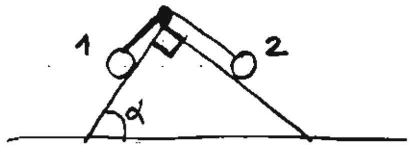
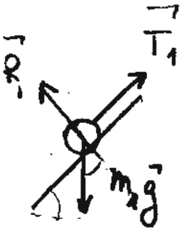
<u>Soluzioni</u>

1)



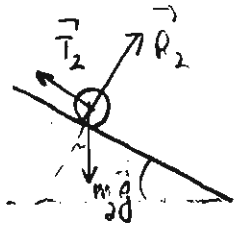
$\alpha = 60°$

(a) Forze su 1



$$m_1 \vec{a}_1 = \vec{R}_1 + \vec{T}_1 + m_1 \vec{g}$$

Forze su 2



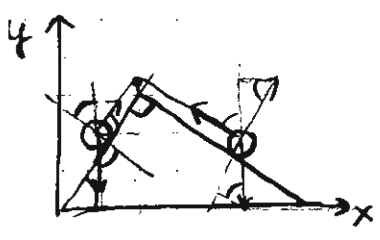
$$m_2 \vec{a}_2 = \vec{R}_2 + \vec{T}_2 + m_2 \vec{g}$$

$$\begin{cases} T_1 = m_1 g \sin\alpha \\ R_1 = m_1 g \cos\alpha \end{cases} \qquad \begin{cases} T_2 = m_2 g \sin\left(\frac{\pi}{2} - \alpha\right) = m_2 g \cos\alpha \\ R_2 = m_2 g \cos\left(\frac{\pi}{2} - \alpha\right) = m_2 g \sin\alpha \end{cases}$$

$\Rightarrow$ poichè $T_1 = T_2$ $\qquad m_1 g \sin\alpha = m_2 g \cos\alpha$

$$\boxed{\frac{m_1}{m_2} = \frac{1}{\operatorname{tg}\alpha} = 0{,}577}$$

(b) m_1 e m_2 dati $\Rightarrow$



$$\begin{cases} m_1 a_{1y} = -m_1 g + R_1 \cos\alpha + T_1 \sin\alpha \\ m_1 a_{1x} = -R_1 \sin\alpha + T_1 \cos\alpha \end{cases}$$

$$\begin{cases} m_2 a_{2y} = -m_2 g + R_2 \sin\alpha + T_2 \cos\alpha \\ m_2 a_{2x} = R_2 \cos\alpha - T_2 \sin\alpha \end{cases}$$

Eq. complicate !! Ulteriori condizioni

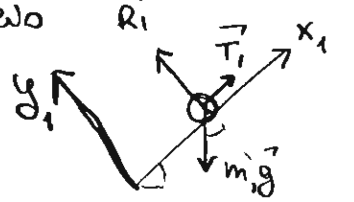
massa filo = 0 $\longrightarrow$ $T_1 = T_2$

moto sul piano $\longrightarrow$ $\dfrac{a_{1y}}{a_{1x}} = \operatorname{tg}\alpha$, $\dfrac{a_{2x}}{a_{2y}} = -\operatorname{tg}\alpha$

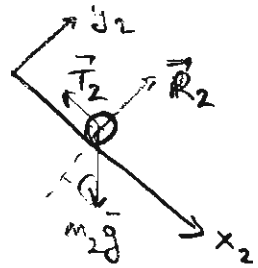
filo inest. $\longrightarrow$ $|a_1| = |a_2|$

8 incognite e 8 eqn . . . ma brutte !

Rifaccio



$$\begin{cases} m_1 a_{1,x} = -m_1 g \sin\alpha + T_1 \\ 0 = R_1 - m_1 g \cos\alpha \end{cases}$$



$$\begin{cases} m_2 a_{2x} = m_2 g \cos\alpha - T_2 \\ 0 = R_2 - m_2 g \sin\alpha \end{cases}$$

$T_1 = T_2$

$|a_{1x}| = |a_{2x}|$ $\Rightarrow$ $a_{1x} \equiv a$

$$\begin{cases} m_1 a = T - m_1 g \sin\alpha \\ m_2 a = m_2 g \cos\alpha - T \end{cases} \Rightarrow (m_1 + m_2) a = (m_2 \cos\alpha - m_1 \sin\alpha) g$$

$$\Downarrow$$

$$\begin{cases} R_1 = m_1 g \cos\alpha \\ R_2 = m_2 g \sin\alpha \end{cases} \qquad a = \frac{m_2 \cos\alpha - m_1 \sin\alpha}{m_1 + m_2}\, g$$

$$T = m_1 a + m_1 g \sin\alpha = g\,\frac{m_1 m_2 \cos\alpha - \cancel{m_1^2 \sin\alpha} + \cancel{m_1^2 \sin\alpha} + m_1 m_2 \sin\alpha}{m_1 + m_2}$$

$$= m_1 m_2\, g\, \frac{\cos\alpha + \sin\alpha}{m_1 + m_2} . \Rightarrow \begin{cases} a = 0{,}438\, m/s^2 \\ T = 8.92\, N \end{cases}$$

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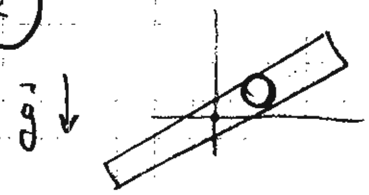
2)



m

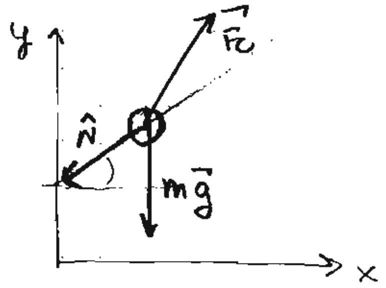
ω

$$\vec{F} = m\,\vec{a}$$

$$\vec{a} = \frac{dv}{dt}\,\hat{T} + \frac{v^2}{R}\,\hat{N}$$

$$\vec{v} = \omega R\,\hat{T} \quad \Rightarrow \quad \vec{a} = \frac{\omega^2 R^{\cancel{2}}}{\cancel{R}}\,\hat{N}$$

$$\vec{F} = m\,\omega^2 R\,\hat{N}$$



$$\vec{F_c} + m\vec{g} = m\,\omega^2 R\,\hat{N}$$

$$F_{c_x} = -\,m\,\omega^2 R\cos\theta$$

$$F_{c_y} - mg = -\,m\,\omega^2 R\sin\theta$$

$$\Rightarrow \quad F_{c_x} = -\,m\,\omega^2 R\cos\theta \qquad = -1.5\ N$$

$$F_{c_y} = mg - m\,\omega^2 R\sin\theta \qquad = 4.35\ N$$

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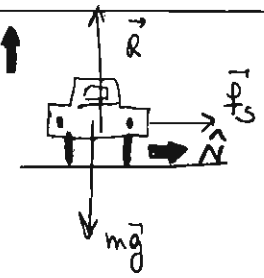
3) 

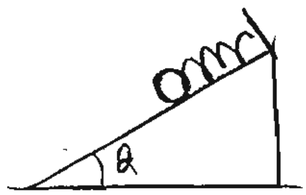
$$\vec{R} + \vec{f}_s + m\vec{g} = m\,\vec{a}$$

Moto circolare $\Rightarrow$ $\vec{a} = a_c\,\hat{N} = \dfrac{v^2}{r}\,\hat{N}$

$\Rightarrow$ Scomponendo in $\hat{N}$ e $\hat{z}$
$$\begin{cases} R - mg = 0 \\ f_s = m\dfrac{v^2}{r} \end{cases} \qquad R = mg$$

$$f_s = m\frac{v^2}{r} \leq \mu_s R = \mu_s mg \quad \Rightarrow \quad \frac{v^2}{r} \leq \mu_s g$$

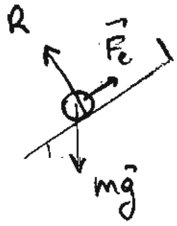
$$v \leq \sqrt{\mu_s g r} = 13.10\ m/s$$
$$= 47.2\ km/h$$

4) 

k, l_0

m

θ

(a) 

$$m\vec{g} + \vec{F}_e + \vec{R} = 0$$

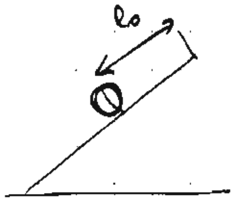
$$\Rightarrow \quad mg\sin\theta - F_e = 0$$

$$F_e = mg\sin\theta$$

$$k(l - l_0) = mg\sin\theta$$

$$x_{eq} = l_{eq} = l_0 + \frac{mg}{k}\sin\theta = 0.395\ m$$

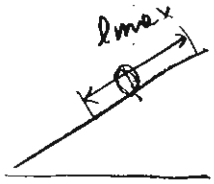
(b)



$t=0$

$$E_{in} = -mg\, l_0 \sin\theta$$

$$E_{fin} = \frac{1}{2} k (l_m - l_0)^2 - mg\, l_m \sin\theta$$



$$\Rightarrow \quad E_{in} = E_{fin}$$

$$\downarrow$$

$$-mg\, l_0 \sin\theta = \frac{1}{2} k (l_m - l_0)^2 - mg\, l_m \sin\theta$$

$$\frac{1}{2} k (l_m - l_0)^2 - mg (l_m - l_0) \sin\theta = 0$$

Soluzioni $\quad l_m = l_0$

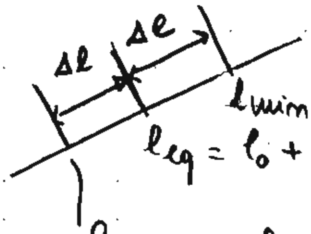
$$\frac{1}{2} k (l_m - l_0) = mg \sin\theta$$

$$l_m = l_0 + 2 \frac{mg}{k} \sin\theta$$

$\Rightarrow \quad l_0$ è l_{min}

$l_0 + 2\frac{mg}{k} \sin\theta$ è l_{max}

$\Rightarrow$



$l_{min} = l_{eq} - \Delta l$

$l_{eq} = l_0 + \frac{mg}{k} \sin\theta$

$$\Delta l = \frac{mg}{k} \sin\theta$$

$$l_{max} = l_{eq} + \Delta l = 0.590 \text{ m}$$

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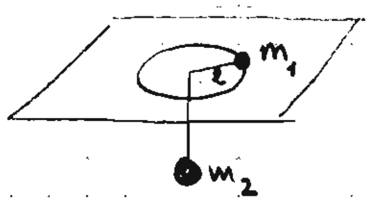
5)

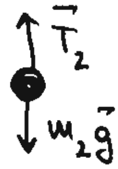


m_1

m_2

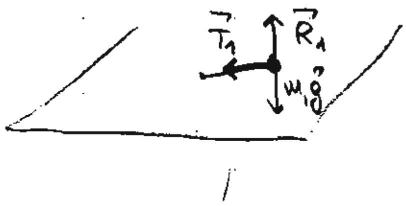
l

(a) Forze su 2



Poiché 2 non cade $\quad T_2 = m_2 g$

Forze su 1



$$\vec{T}_1 + \vec{R}_1 + m_1\vec{g} = m_1\vec{a}_1$$

Il moto è circ. unif. $\Rightarrow \quad \vec{a}_1 = \hat{N}\,\dfrac{v_1^2}{l}$

$$\Rightarrow \quad R_1 - m_1 g = 0$$

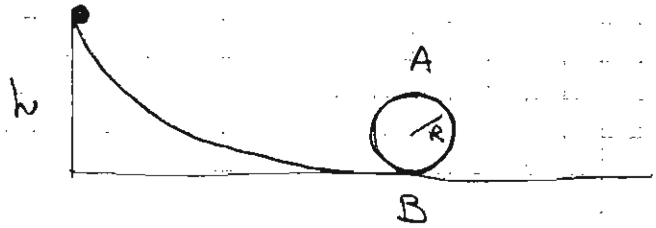
$$T_1 = m_1\frac{v_1^2}{l}$$

$$T_1 = T_2 \quad \Rightarrow \quad m_2 g = m_1\frac{v_1^2}{l}$$

$$\Rightarrow \quad v_1 = \sqrt{\frac{m_2}{m_1}\,g\,l} = 3.5 \text{ m/s}$$

(b) $\quad T = m_2 g = 9.8 \text{ N}$

6)



m

R

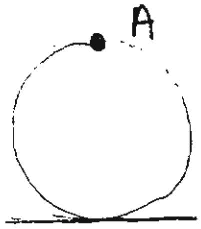
(a) Cons. dell' energia $\Rightarrow$

$$E_{in} = E_A$$

$$\cancel{m} g h = \frac{1}{2} \cancel{m} v_A^2 + \cancel{m} g 2R$$

$$v_A = \sqrt{2 g (h-2R)} = \sqrt{3gR} = 3.84 \text{ m/s}$$

(c)



$$m a_c = mg - F_q$$

$$m \frac{v^2}{R} = mg - F_q \leftarrow \text{vera sempre}$$

Per fare il giro, occorre che sulla pallina ci sia una forza opportuna per farle proseguire il moto circolare.

Tale forza è esercitata o dalla guida e peso, o al limite dal solo peso ⇒

$$F_g = mg - m\frac{v^2}{R} \geq 0$$

Dalla cons. dell' energia (vd. punto (e))

$$v = \sqrt{2g(h-2R)} \Rightarrow$$

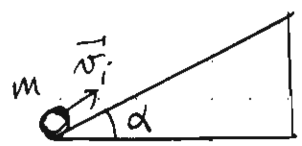
$$mg - m\frac{2g}{R}(h-2R) \geq 0$$

$$\cancel{mg} - 2mg\frac{h}{R} + \overset{5}{\cancel{4}}\,mg \geq 0$$

$$h \leq \frac{5\,mg}{2\,mg}R = \frac{5}{2}R$$

$$\Rightarrow \quad h_{min} = \frac{5}{2}R = 1.25\ m$$

7)



Teorema delle forze vive (o cons. dell' en. mec.)

$$E_{c\,fin} - E_{c\,in} = \Sigma \mathcal{L}_i$$

$$0 - \frac{1}{2}m v_i^2 = \mathcal{L}_{peso} + \mathcal{L}_{attrito}$$

$$-\frac{1}{2}m v_i^2 = -mg\,h_{max} - \mu_D N \frac{h_{max}}{\sin\alpha}$$

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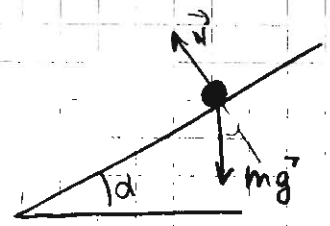
$N = mg\cos\alpha$



$$\frac{1}{2}\cancel{m}\, v_i^2 = \cancel{m}g\, h_{max} + \mu_D\, \cancel{m}g\, \frac{\cos\alpha}{\sin\alpha}\, h_{max}$$

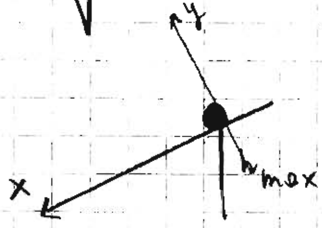
$$\Rightarrow \quad h_{max} = \frac{v_i^2}{2g\left(1+\frac{\mu_D}{tg\alpha}\right)}$$

Sempre dal TFV

$$\frac{1}{2}\cancel{m}\, v_f^2 - \frac{1}{2}\cancel{m}\, v_i^2 = -\mu_D\, \cancel{m}g\cos\alpha\, \frac{h_{max}}{\sin\alpha} \quad \times 2$$

$$\Rightarrow \quad v_f = \sqrt{v_i^2 - \cancel{4}\,\frac{\mu_D}{tg\alpha}\,\cancel{g}\,\frac{v_i^2}{\cancel{2g}\left(1+\frac{\mu_D}{tg\alpha}\right)}} = v_i\sqrt{\frac{1-\frac{\mu_D}{tg\alpha}}{1+\frac{\mu_D}{tg\alpha}}}$$

Oppure



$$\begin{cases} x(t) = \frac{1}{2}at^2 \\ v_x(t) = at \end{cases}$$

$$\Sigma\vec{F} = m\vec{a} \qquad m\vec{g} + \vec{N} + \vec{F}_D = m\vec{a} \qquad \begin{cases} mg\sin\alpha - \mu_D N = ma \\ N = mg\cos\alpha \end{cases}$$

$$\Rightarrow \quad a = g\sin\alpha\left(1-\frac{\mu_D}{tg\alpha}\right)$$

$$d = \frac{h_{max}}{\sin\alpha} = \frac{1}{2}at^2 \qquad t = \sqrt{\frac{2\, h_{max}}{a\sin\alpha}}$$

$$\Rightarrow \quad v_x = \sqrt{2a\,\frac{h_{max}}{\sin\alpha}} = \sqrt{\frac{\cancel{2}\cancel{g}\,\cancel{\sin\alpha}\,(1-\mu_D/tg\alpha)}{\cancel{\sin\alpha}}\;\frac{v_i^2}{\cancel{2g}\,(1+\mu_D/tg\alpha)}}$$

= a prima !