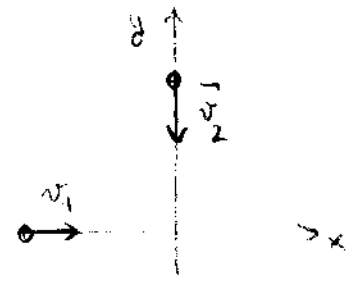
Esercizi su sistemi: urti, corpo rigido e fluidi

Problema 1

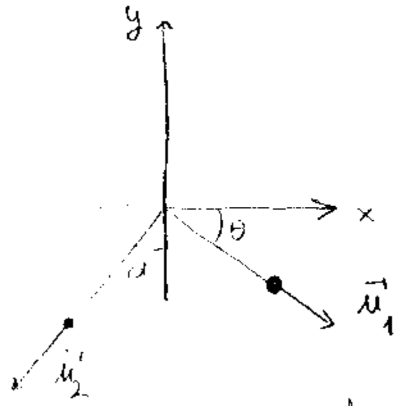
a)

+

b)



prima



dopo

Cons. della q. di moto

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{u}_1 + m_2 \vec{u}_2$$

Urto elastico $\Rightarrow$ cons. dell'energia cinetica

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$$

So che $\quad m_1 = m_2 = m$

$$\vec{v}_1 = v_1 \hat{x} \qquad\qquad \vec{u}_1 = u_1 \cos\theta \, \hat{x} - u_1 \sin\theta \, \hat{y}$$

$$\vec{v}_2 = -v_2 \hat{y} \qquad\qquad \vec{u}_2 = -u_2 \sin\alpha \, \hat{x} - u_2 \cos\alpha \, \hat{y}$$

$\Rightarrow$ riscrivo le eq. come:

$$\begin{cases} \cancel{m} v_1 \hat{x} - \cancel{m} v_2 \hat{y} = \cancel{m} (u_1 \cos\theta \, \hat{x} - u_1 \sin\theta \, \hat{y}) \\ \qquad + \cancel{m} (-u_2 \sin\alpha \, \hat{x} - u_2 \cos\alpha \, \hat{y}) \\ \cancel{\frac{1}{2}} \cancel{m} v_1^2 + \cancel{\frac{1}{2}} \cancel{m} v_2^2 = \cancel{\frac{1}{2}} \cancel{m} u_1^2 + \cancel{\frac{1}{2}} \cancel{m} u_2^2 \end{cases} \Rightarrow$$

$$\begin{cases} v_1 = u_1 \cos\theta - u_2 \sin\alpha \\ -v_2 = -u_1 \sin\theta - u_2 \cos\alpha \end{cases} \qquad v_1^2 + v_2^2 = u_1^2 + u_2^2$$

$$\begin{cases} u_2 \sin\alpha = u_1 \cos\theta - v_1 \\ u_2 \cos\alpha = v_2 - u_1 \sin\theta \end{cases} \rightarrow u_2^2 = u_1^2 + v_1^2 - 2 v_1 u_1 \cos\theta + v_2^2 - 2 v_2 u_1 \sin\theta$$

$$\cancel{v_1^2} + \cancel{v_2^2} = u_1^2 + u_1^2 + \cancel{v_1^2} + \cancel{v_2^2} - 2 v_1 u_1 \cos\theta - 2 v_2 u_1 \sin\theta$$

$$2 u_1 \left(u_1 - v_1 \cos\theta - v_2 \sin\theta \right) = 0$$

$$u_1 = v_1 \cos\theta + v_2 \sin\theta$$

$$tg\,\alpha = \frac{v_1 \cos^2\theta + v_2 \sin\theta\cos\theta - v_1}{v_2 - v_1 \cos\theta \sin\theta - v_2 \sin^2\theta} = \frac{-v_1 \sin^2\theta + v_2 \sin\theta\cos\theta}{v_2 \cos^2\theta - v_1 \cos\theta \sin\theta}$$

$$= tg\,\theta \, \frac{\cancel{v_2 \cos\theta - v_1 \sin\theta}}{\cancel{v_2 \cos\theta - v_1 \sin\theta}} = tg\,\theta \Rightarrow \alpha = \theta$$

Altro modo:

$$v_1^2 + v_2^2 = u_1^2 + u_2^2$$

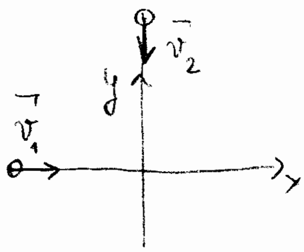
$$\vec{v}_1 + \vec{v}_2 = \vec{u}_1 + \vec{u}_2 \Rightarrow$$

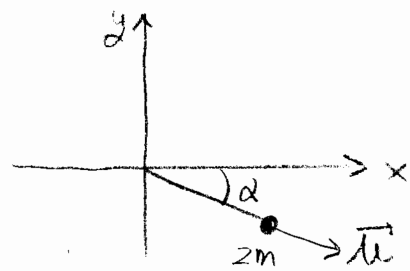
$$v_1^2 + v_2^2 + \underbrace{2 v_1 v_2 \cos 90}_{=0} = \underbrace{u_1^2 + u_2^2}_{v_1^2 + v_2^2} + 2 u_1 u_2 \cos\theta_{12}$$

$$\Rightarrow 2 u_1 u_2 \cos\theta_{12} = 0$$

$$\theta_{12} = 90 = \alpha + 90 - \theta \Rightarrow \alpha = \theta$$

c) + d)





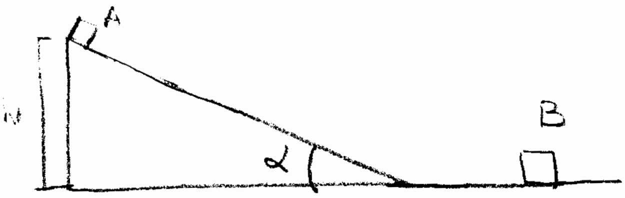
Cons. delle q. di moto $(\vec{u}_1 = \vec{u}_2 = \vec{u})$

$$m v_1 \hat{x} - m v_2 \hat{y} = 2m u (\cos\alpha \, \hat{x} - \sin\alpha \, \hat{y})$$

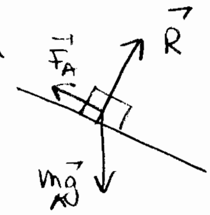
$$\Rightarrow \quad \cancel{m} v_1 = 2 \cancel{m} u \cos\alpha \qquad \cancel{m} v_2 = 2 \cancel{m} u \sin\alpha \qquad tg\,\alpha = \frac{v_2}{v_1}$$

$$4 u^2 = v_1^2 + v_2^2 \Rightarrow u = \frac{\sqrt{v_1^2 + v_2^2}}{2}$$

Problema 2



Forze su A



a) TFV $\Rightarrow$ $E_{cin,A}(f) - E_{cin,A}(i) = \mathcal{L}_{peso} + \mathcal{L}_R + \mathcal{L}_{Fatt}$

$\mathcal{L}_{peso} = m_A g h$

$\mathcal{L}_R = 0$

ipotenusa triangolo $\Rightarrow$ $l = \dfrac{h}{\sin\alpha}$

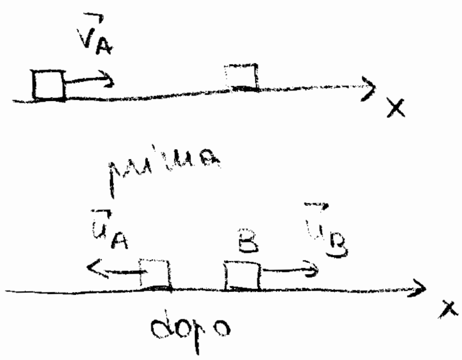
$\mathcal{L}_{Fatt} = -\mu_D R \, l = -\mu_D m_A g \cos\alpha \dfrac{h}{\sin\alpha}$

$R = m_A g \cos\alpha$

$$\Rightarrow \quad \frac{1}{2} \cancel{m_A} v_A^2 - 0 = \cancel{m_A} g h - \cancel{m_A} \mu_D g \cos\alpha \frac{h}{\sin\alpha}$$

$$v_A = \sqrt{2 g h \left(1 - \frac{\mu_D}{\operatorname{tg}\alpha}\right)}$$

b)



C. della q. d. moto
C. dell' en. cinetica

$$\begin{cases} m_A v_A = -m_A u_A + m_B u_B \\ \cancel{\frac{1}{2}} m_A v_A^2 = \cancel{\frac{1}{2}} m_A u_A^2 + \cancel{\frac{1}{2}} m_B u_B^2 \end{cases}$$

$$\Rightarrow \quad u_A = \frac{m_B u_B - m_A v_A}{m_A} \quad \Rightarrow$$

$$m_A v_A^2 = \frac{m_A}{m_A^2}\left(m_B^2 u_B^2 + m_A^2 v_A^2 - 2 m_A m_B u_B v_A\right) + m_B u_B^2$$

$$\cancel{m_A v_A^2} = \cancel{m_A}\frac{m_B^2}{m_A^{\cancel{2}}} u_B^2 + \cancel{v_A^2 m_A} - 2 m_B u_B v_A + m_B u_B^2$$

$$\cancel{m_B}\left(1 + \frac{m_B}{m_A}\right) u_B^{\cancel{2}} - 2 \cancel{m_B} v_A \cancel{u_B} = 0$$

$$u_B = \frac{2 v_A m_A}{m_A + m_B}$$

c) $$u_A = \left[m_B \left(\frac{2 v_A m_A}{m_A + m_B}\right) - m_A v_A\right] \frac{1}{m_A}$$

$$= \frac{\cancel{2} m_A m_B v_A - m_A^2 v_A - \cancel{m_A m_B v_A}}{m_A (m_A + m_B)}$$

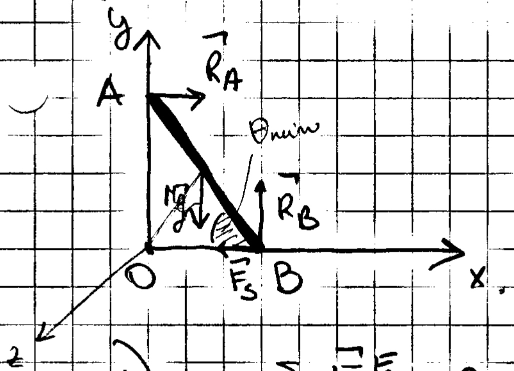
$$= \frac{\cancel{m_A} v_A}{\cancel{m_A}} \frac{m_B - m_A}{m_A + m_B}$$

TFV $$\cancel{+}\frac{1}{2}\cancel{m_A} u_A^2 = \cancel{+}\cancel{m_A} g h' + \mu_D \cancel{m_A} g \cos\alpha \frac{h'}{\sin\alpha}$$

$$h' = \frac{u_A^2}{2g}\frac{1}{\left(1 + \frac{\mu_D}{\mathrm{tg}\alpha}\right)} = \frac{(m_B - m_A)^2}{(m_A + m_B)^2}\frac{2gh}{\cancel{2g}}\frac{\left(1 - \frac{\mu_D}{\mathrm{tg}\alpha}\right)}{\left(1 + \frac{\mu_D}{\mathrm{tg}\alpha}\right)}$$

$$= h \frac{(m_B - m_A)^2}{(m_A + m_B)^2}\frac{\mathrm{tg}\alpha - \mu_D}{\mathrm{tg}\alpha + \mu_D}$$

(Esercizio della scala) Problema 3



θ_{min} t.c. scala non cade?

1) $\sum \vec{F}^E = 0 \longrightarrow \vec{R}_A + \vec{R}_B + M\vec{g} + \vec{F}_S = 0$

2) $\sum \vec{M}_O^E = 0$

$$\begin{cases} R_A - F_S = 0 \\ -Mg + R_B = 0 \end{cases} \longrightarrow \begin{cases} F_S = R_A \\ R_B = Mg \end{cases}$$

$$-R_A \cancel{l} \sin\theta + R_B \cancel{l} \cos\theta - Mg \frac{\cancel{l}}{2} \sin\left(\frac{\pi}{2} - \theta\right) = 0$$

$$-\frac{1}{2} Mg \cos\theta + R_B \cos\theta - R_A \sin\theta = 0$$

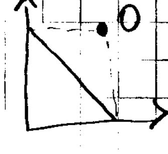
$$\cancel{-\frac{1}{2} Mg \cos\theta} + \underset{\wedge}{{}_{1/2}} Mg \cos\theta - F_S \sin\theta = 0$$

$$F_S = \frac{1}{2} Mg \frac{1}{tg\theta} \leq \mu_S R_B = \mu_S Mg$$

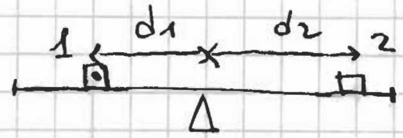
$$\Rightarrow \frac{1}{2} \frac{1}{tg\theta} \leq \mu_S \qquad tg\theta \geq \frac{1}{2\mu_S}$$

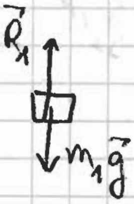
$$\Rightarrow tg\,\theta_{min} = \frac{1}{2\mu_S}$$

Assai più facile se per O si sceglieva



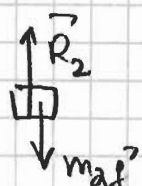
Problema 4



Forze su 1: 

1 in eq ⇒ $\vec R_1 + m_1\vec g = 0$

$\vec R_1 = -m_1\vec g$

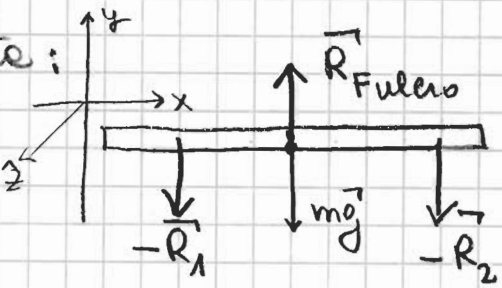
Analogamente per 2: 

2 in eq ⇒ $\vec R_2 + m_2\vec g = 0$

$\vec R_2 = -m_2\vec g$

Se $\vec R_1$ è esercitata dall'asta sul bambino 1, per il 3° principio, il bambino 1 esercita sull'asta la forza $-\vec R_1$. Analogamente per 2 ⇒

Forze sull'asta:



$-\vec R_1 = m_1\vec g$

$-\vec R_2 = m_2\vec g$

Asta in eq ⇒ $\vec F_E = 0$, $\vec M_{E,O} = 0$

$m_1\vec g + m_2\vec g + m\vec g + \vec R_F = 0$

$\Downarrow$

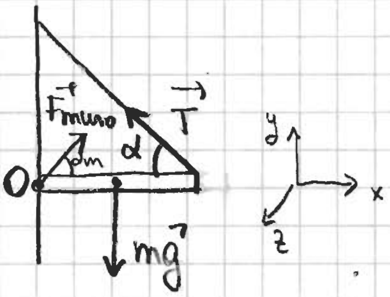
non ho informazioni su d_2! ⇐ $R_F = (m_1 + m_2 + m)\, g$

Scelgo O nel centro di massa ⇒ $\vec M_{mg} = 0$, $\vec M_{R_F} = 0$

⇒ $m_1 \cancel{g}\, d_1 \hat z - m_2 \cancel{g}\, d_2 \hat z = 0$

⇒ $d_2 = \dfrac{m_1}{m_2} d_1$

Problema 5

a) 

Eq $\Rightarrow$ $\quad \vec{F}_{muro} + \vec{T} + m\vec{g} = 0 \qquad \vec{F}_E = 0$

$$-mg\frac{\cancel{\ell}}{2}\hat{z} + \hat{z}T\cancel{\ell}\sin\alpha = 0 \qquad \vec{M}_E = 0$$

$$\Rightarrow \quad T = \frac{mg}{2\sin\alpha}$$

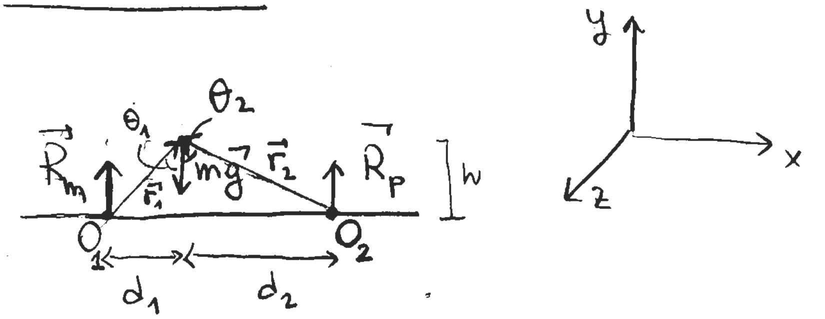
b) $\begin{cases} F_{m_x} - \dfrac{mg}{2\sin\alpha}\cos\alpha = 0 \\ F_{m_y} + \dfrac{mg}{2\cancel{\sin\alpha}}\cancel{\sin\alpha} - mg = 0 \end{cases}$ $\qquad F_{m_x} = \dfrac{mg}{2\sin\alpha}\cos\alpha \qquad F_{m_y} = \dfrac{mg}{2}$

$$\Rightarrow \quad F_m = \sqrt{F_{m_x}^2 + F_{m_y}^2} = \frac{mg}{2\sin\alpha} = T$$

$$\operatorname{tg}\alpha_m = \frac{F_{m_y}}{F_{m_x}} = \frac{mg}{2}\,\frac{2\sin\alpha}{mg\cos\alpha} = \operatorname{tg}\alpha \Rightarrow$$

$$\alpha_m = \alpha$$

Problema 6



(a) Calcolo $\vec{M}_{O,E} = 0$ e per polo O scelgo O_2

$$\Rightarrow \quad R_m(d_1+d_2)(-\hat{z}) + mg \underbrace{r_2 \sin\theta_2}_{d_2} \hat{z} = 0$$

$$R_m(d_1+d_2) = mg\, d_2$$

$$R_m = mg \frac{d_2}{d_1+d_2}$$

(b) Calcolo $\vec{M}_{O,E} = 0$ con polo in O_1

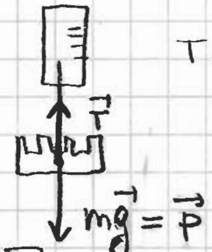
$$R_p(d_1+d_2)\hat{z} - mg \underbrace{r_1 \sin\theta_1}_{d_1} \hat{z} = 0$$

$$R_p = mg \frac{d_1}{d_1+d_2}$$

Problema 7

$T =$ quello che misura la bilancia

Nell'aria



$T = P = mg$

Nell'acqua

$$T = P_a = -F_{arch} + mg =$$

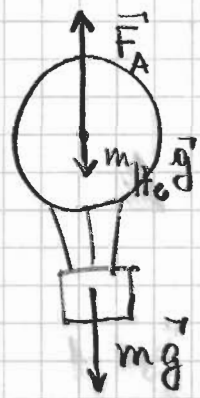
$$= -\rho_{H_2O} V g + \rho_{corona} V g$$

$$\frac{P_a}{mg} = \frac{P_a}{P} = \frac{\rho_{corona} - \rho_{H_2O}}{\rho_{corona}}$$

$$\rho_c \frac{P_a}{P} = \rho_c - \rho_{H_2O}$$

$$\rho_c = \frac{\rho_{H_2O}}{1 - \frac{P_a}{P}} = \rho_{H_2O} \frac{mg}{(mg - P_a)}$$

Problema 8



$$F_A = (m_{He} + m)\, g$$

$$\downarrow$$

$$F_A = V \rho_a g$$

$$\Rightarrow \rho_a V \cancel{g} = \rho_{He} V \cancel{g} + m \cancel{g}$$

$$V = \frac{m}{\rho_a - \rho_{He}} = 721\ m^3$$

Nota ad h maggiori ρ_{aria} diminuisce $\Rightarrow V$ ancora + grande