Prima verifica del 5/4/07

Problema 1

1.)
$$\begin{cases} x(t) = v_0 \cos\alpha \; t \\ y(t) = h_0 + v_0 \sin\alpha \; t - \frac{1}{2} g t^2 \end{cases}$$

Se $d_p = v_0 \cos\alpha \; t$ $\quad y(t) \geq h_p \implies$

$$t = \frac{d_p}{v_0 \cos\alpha} \qquad h_p \leq h_0 + \cancel{v_0} \sin\alpha \frac{d_p}{\cancel{v_0} \cos\alpha} - \frac{1}{2} g \frac{d_p^2}{v_0^2 \cos^2\alpha}$$

$$\frac{1}{2} \frac{g\, d_p^2}{v_0^2 \cos^2\alpha} \leq h_0 - h_p + d_p \operatorname{tg}\alpha$$

$$\implies v_0^2 \geq \frac{g\, d_p^2}{2\cos^2\alpha \, (h_0 - h_p + d_p \operatorname{tg}\alpha)}$$

$$\implies v_{min} = \frac{d_p}{\cos\alpha} \sqrt{\frac{g}{2(h_0 - h_p + d_p \operatorname{tg}\alpha)}}$$

2.)
$$\begin{cases} y(t) = 0 \\ x(t) = d_p + d \end{cases}$$

$$\implies h_0 + v_0 \sin\alpha \; t - \frac{1}{2} g t^2 = 0 \qquad g t^2 - 2 v_0 \sin\alpha \; t - 2h_0 = 0$$

$$\Downarrow$$

$$t = \frac{v_0 \sin\alpha \pm \sqrt{v_0^2 \sin^2\alpha + 2 h_0 g}}{g}$$

$$\Rightarrow \quad d_p + d = v_0 \cos\alpha \; \frac{v_0 \sin\alpha + \sqrt{v_0^2 \sin^2\alpha + 2h_0 g}}{g}$$

$$d = v_0 \cos\alpha \; \frac{v_0 \sin\alpha + \sqrt{v_0^2 \sin^2\alpha + 2h_0 g}}{g} - d_p$$

3.) Due modi per risolvere:

(i) $\begin{cases} v_x = v_0 \cos\alpha \\ v_y = v_0 \sin\alpha - g t \end{cases}$

$$\Rightarrow \quad v_y = v_0 \sin\alpha - v_0 \sin\alpha - \sqrt{v_0^2 \sin^2\alpha + 2h_0 g}$$
$$= -\sqrt{v_0^2 \sin^2\alpha + 2h_0 g}$$

$$\Rightarrow \quad v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 \cos^2\alpha + v_0^2 \sin^2\alpha + 2h_0 g}$$
$$= \sqrt{v_0^2 + 2h_0 g}$$

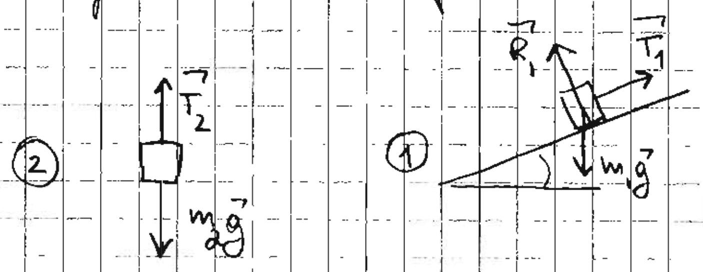
(ii) Dalla conservazione dell'energia:

$$\frac{1}{2}\cancel{m} v_0^2 + \cancel{m} g h_0 = \frac{1}{2}\cancel{m} v^2$$

$$v = \sqrt{v_0^2 + 2 g h_0}$$

Problema 2

4.) Diagrammi delle forze



Equilibrio $\Rightarrow \begin{cases} \vec{T}_2 + m_2\vec{g} = 0 \\ m_1\vec{g} + \vec{R}_1 + \vec{T}_1 = 0 \end{cases}$

$$\Rightarrow \begin{cases} T_2 = m_2 g \\ -m_1 g \cos\alpha + R_1 = 0 \\ T_1 - m_1 g \sin\alpha = 0 \end{cases}$$

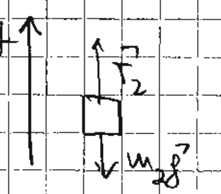
In più so che $T_1 = T_2 \Rightarrow$

$$m_2 \cancel{g} - m_1 \cancel{g} \sin\alpha = 0 \quad \Rightarrow \quad \frac{m_1}{m_2} = \frac{1}{\sin\alpha}$$

5.) Equazioni del moto sono

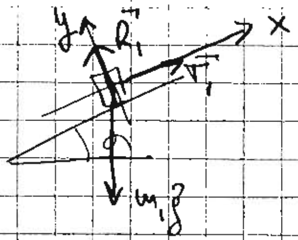
$$\begin{cases} \vec{T}_2 + m_2\vec{g} = m_2\vec{a}_2 \\ \vec{T}_1 + \vec{R}_1 + m_1\vec{g} = m_1\vec{a}_1 \end{cases}$$

Per 2 scelgo



$T_2 - m_2 g = m_2 a_{2y}$ componente lungo y

Per 1 scelgo



$$\begin{cases} T_1 - m_1 g \sin\alpha = m_1 a_1 \\ R_1 - m_1 g \cos\alpha = 0 \end{cases}$$ ← componente lungo x

Filo inestensibile $\Rightarrow$ $|a_1| = |a_2|$

In più so che se $\Delta x_1 > 0$ allora $\Delta y_2 < 0$ $\Rightarrow$

$$a_1 = -a_2$$

Filo ha massa nulla $\Rightarrow T_1 = T_2$

$$\begin{cases} T - m_2 g = m_2 a_2 \\ T - m_1 g \sin\alpha = -m_1 a_2 \end{cases}$$

Faccio la diff. tra le 2 equ $\Rightarrow$

$$\cancel{T} - m_2 g - \cancel{T} + m_1 g \sin\alpha = (m_2 + m_1)\, a_2$$

$$\Rightarrow a_2 = g \, \frac{m_1 \sin\alpha - m_2}{m_1 + m_2} =$$

$$= g \, \frac{\cancel{m_2}}{\cancel{m_2}} \, \frac{3 \sin\alpha - 1}{4}$$

6.) $$T = m_2 \, g \, \frac{3\sin\alpha - 1}{4} + m_2 g = m_2 g \, \frac{3\sin\alpha + 3}{4}$$

$$= m_2 g \, \frac{3(\sin\alpha + 1)}{4}$$

Problema 3

7\) Conserv. dell' energia

$$\frac{1}{2} m v_0^2 - \frac{G m_L m}{r_L} = - \frac{G m_L m}{(r_L + d)}$$

$$\frac{1}{2} \cancel{m} v_0^2 = G m_L \cancel{m} \left(\frac{1}{r_L} - \frac{1}{r_L + d} \right)$$

$$v_0 = \sqrt{2 G m_L \left(\frac{1}{r_L} - \frac{1}{r_L + d} \right)}$$

8\) $a = \frac{v^2}{r_L + d} \Rightarrow \cancel{m} \frac{v^2}{\cancel{r_L + d}} = \frac{G m_L \cancel{m}}{(r_L + d)^{\cancel{2}}}$

$$v = \sqrt{\frac{G m_L}{r_L + d}}$$

Problema 4

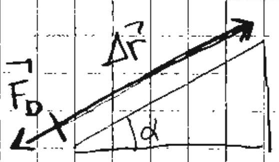
9\) Conservazione dell' energia

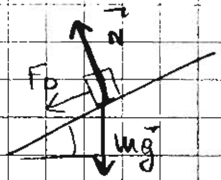
$$m g h = \frac{1}{2} m v^2 \Rightarrow v = \sqrt{2 g h}$$

10\) Teorema di cons. dell' energia in presenza di attrito

$$m g h_{max} = \frac{1}{2} m v_0^2 + \mathcal{L}_{att}$$

$$\mathcal{L}_{att} = - F_D \cdot \frac{h_{max}}{\sin \alpha}$$





$$F_D = \mu_D N = \mu_D mg\cos\alpha$$

$$\Rightarrow \mathcal{L}_{att} = -\mu_D mg \frac{\cos\alpha}{\sin\alpha} h_{max}$$

$$\cancel{m}\cancel{g}\, h_{max} = \underbrace{\frac{1}{2} m v_0^2}_{\cancel{mg}\, h} - \mu_D \cancel{mg} \frac{h_{max}}{tg\alpha}$$

$$h_{max}\left(1+\frac{\mu_D}{tg\alpha}\right) = h$$

$$h_{max} = \frac{h}{1+\frac{\mu_D}{tg\alpha}}$$