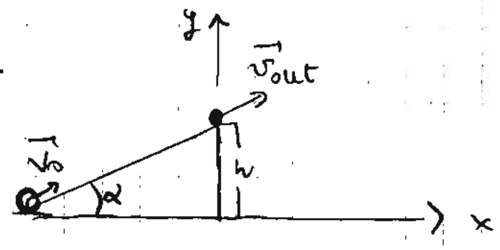
Esame dell' 8/6/07

Pb. 1



1) Dalla conservazione dell' energia meccanica

$$\frac{1}{2} \cancel{m} v_0^2 = \frac{1}{2} \cancel{m} v_{out}^2 + \cancel{m} g h \quad \Rightarrow \quad v_{out} = \sqrt{v_0^2 - 2gh}$$

2) Ad $h_{max} \Rightarrow v_y = 0 \Rightarrow \vec{v} = \vec{v}_x \hat{x}$. Lungo x, il moto è rettilineo uniforme con

$$v_x = v_{out} \cos\alpha = v_{out} \cos(\pi/6)$$

3)
$$x(t) = v_x t \qquad\qquad v_x = v_{out} \cos(\pi/6)$$
$$y(t) = h + v_y t - \frac{1}{2} g t^2 \qquad\qquad v_y = v_{out} \sin(\pi/6)$$

$$y(t) = 0 \Rightarrow h + v_y t - \frac{1}{2} g t^2 = 0$$

$$g t^2 - 2 v_y t - 2h = 0 \qquad t = \frac{v_y \pm \sqrt{v_y^2 + 2hg}}{g}$$

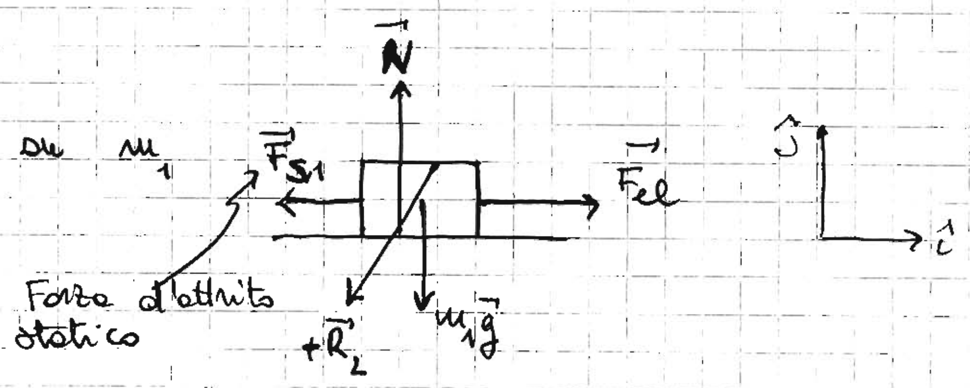
La soluzione corretta è quella positiva $\Rightarrow$

$$d_x = v_x \frac{v_y}{g} \left(1 + \sqrt{1 + \frac{2gh}{v_y^2}} \right)$$

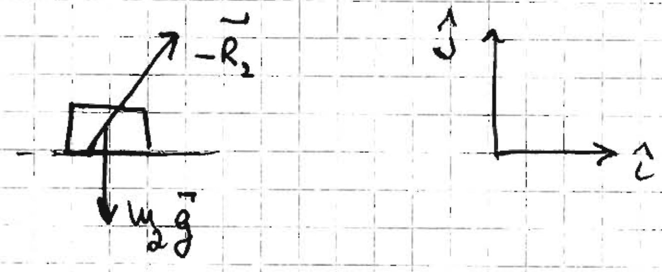
$$= \frac{v_{out}^2}{g} \cos(\pi/6) \sin(\pi/6) \left(1 + \sqrt{1 + \frac{2gh}{v_{out}^2 \sin^2(\pi/6)}} \right)$$

Pb. 2

4) Forze su m_1



Forze su m_2



Equilibrio $\Rightarrow$

$$\begin{cases} 0 = m_1\vec{g} + \vec{F}_{el} + \vec{N} + \vec{R}_2 + \vec{F}_{S,1} \\ 0 = m_2\vec{g} - \vec{R}_2 \end{cases}$$

$$\Rightarrow \begin{cases} 0 = -m_1 g + N - R_{2y} \\ 0 = -F_{S,1} + F_{el} - R_{2x} \\ 0 = -m_2 g + R_{2y} \\ 0 = +R_{2x} \end{cases} \Rightarrow \begin{cases} R_{2y} = m_2 g \\ R_{2x} = 0 \end{cases} \qquad \begin{cases} N = m_1 g + m_2 g \\ F_{el} = F_{S1} \end{cases}$$

Poichè $F_{S,1} \leq \mu_S N = \mu_S (m_1+m_2) g$

$$F_{el} \leq \mu_S (m_1+m_2) g$$

$$\shortparallel$$

$$k d \leq \mu_S (m_1+m_2) g \Rightarrow \mu_S \geq \frac{k d}{(m_1+m_2) g}$$

5) 6) Con lo stesso diagramma delle forze

$$\begin{cases} m_1 \vec{a}_1 = m_1 \vec{g} + \vec{F}_{el} + \vec{N} + \vec{R}_2 + \vec{F}_{d,1} \\ m_2 \vec{a}_2 = m_2 \vec{g} - \vec{R}_2 \end{cases}$$

forza d'attrito dinamico

$$\begin{cases} 0 = -m_1 g + N - R_{2y} \\ m_1 a_1 = F_{el} - R_{2x} - \mu_d N \\ 0 = -m_2 g + R_{2y} \\ m_2 a_2 = + R_{2x} \end{cases} \qquad R_{2x} \equiv \vec{F}_{d,2} = \mu_d R_{2y}$$

forza d'attrito dinamico su 2

$$R_{2y} = m_2 g \qquad m_2 a_2 = + \mu_d R_{2y} = + \mu_d m_2 g$$

$$\Rightarrow \quad a_2 = \mu_d g$$

Per 1:

$$\begin{cases} 0 = -m_1 g + N - m_2 g \\ m_1 a_1 = kd - \mu_d m_2 g - \mu_d N \end{cases}$$

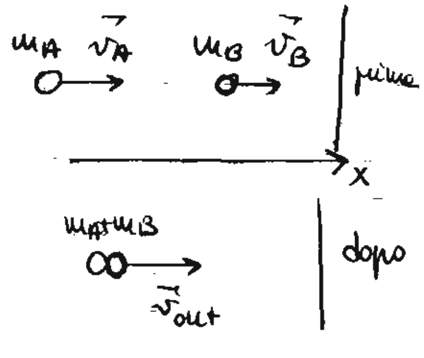
$$\begin{cases} N = (m_1 + m_2) g \\ m_1 a_1 = kd - \mu_d (2 m_2 + m_1) g \end{cases}$$

$$\Rightarrow \quad a_1 = \frac{kd - \mu_d (2 m_2 + m_1) g}{m_1}$$

Pb. 3

7) $$m_A \vec{v}_A + m_B \vec{v}_B = (m_A + m_B)\, \vec{v}_{out}$$

$$\Rightarrow \quad v_{out} = \frac{m_A v_A + m_B v_B}{m_A + m_B}$$



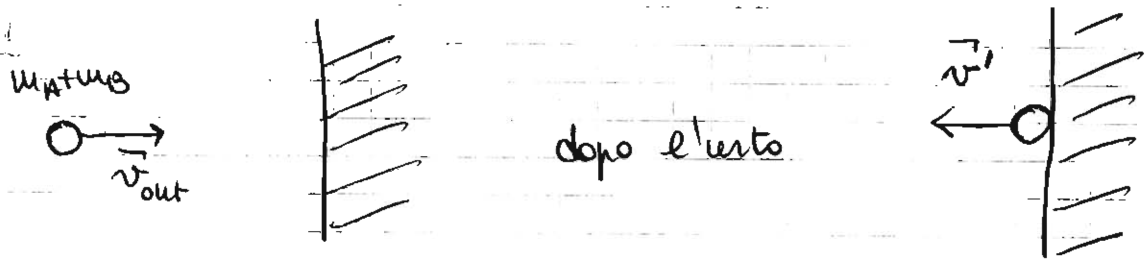
8) $$E_{diss} = E_{cin}\,(\text{prima dell'urto}) - E_{cin}\,(\text{dopo l'urto})$$

$$= \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 - \frac{1}{2} \cancel{(m_A+m_B)} \frac{(m_A v_A + m_B v_B)^2}{(m_A+m_B)^{\cancel{2}}}$$

$$= \frac{1}{2} \frac{1}{m_A+m_B} \Big[\cancel{m_A^2 v_A^2} + m_A m_B v_A^2 + m_A m_B v_B^2 + \cancel{m_B^2 v_B^2}$$
$$- \cancel{m_A^2 v_A^2} - \cancel{m_B^2 v_B^2} - 2\, m_A m_B v_A v_B \Big]$$

$$= \frac{1}{2} \frac{m_A m_B}{m_A + m_B} \left[v_A - v_B \right]^2$$

9)



Troviamo v' : $$\frac{1}{2}(m_A+m_B)\, v_{out}^2 \underset{\uparrow}{=} \frac{1}{2}(m_A+m_B)\, v'^2$$

urto elastico

$$\Rightarrow \quad v' = v_{out}$$

Per la scelta della direzione e verso, si utilizza il teorema dell'impulso. L'unica forza impulsiva che agisce su $m_A + m_B$ è $\vec{R}$ (reazione del muro, $\perp$ al muro, perché se urto elastico $\Rightarrow$ non ci sono forze di attrito dissipative)

$$\Rightarrow \quad \vec{I}_{\vec{R}} = \int_0^{\Delta t = \text{tempo dell'urto}} \vec{R}\, dt \quad \text{deve essere } \perp \text{ al muro}$$

$$\Rightarrow \quad \text{poiché} \quad \vec{I} = \underbrace{\vec{Q}_{fin}}_{\text{quantità di moto delle palline A+B}} - \vec{Q}_{in}$$

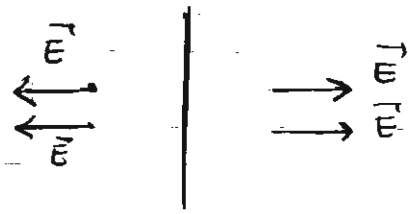
$$I_{/\!/} = 0 = m_{A+B}\, v'_{/\!/} - 0 \quad \Rightarrow \quad v'_{/\!/} = 0 \quad \Rightarrow \quad \vec{v}\,' \perp \text{muro}$$

inoltre
$$I_{\perp} = I_R = (m_A + m_B)\, v' - (m_A + m_B)(-v')$$

$$= 2\,(m_A + m_B)\, v' = 2\,(m_A + m_B)\, v_{out}$$

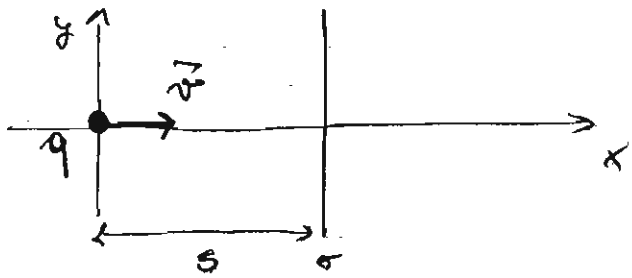
Pb. 4

10) Il campo elettrico di un piano infinito con densità sup. $+\sigma$ è uniforme, $\perp$ al piano (vd. figura) e di modulo $E = \dfrac{\sigma}{2\varepsilon_0}$



(lo si può eventualmente dimostrare facilmente col teorema di Gauss)

⇒



$$\vec{F} = q\vec{E} = -q\,\frac{\sigma}{2\varepsilon_0}\,\hat{x} \quad \Rightarrow \quad \vec{a} = \frac{\vec{F}}{m} = -\frac{q}{m}\,\frac{\sigma}{2\varepsilon_0}\,\hat{x}$$

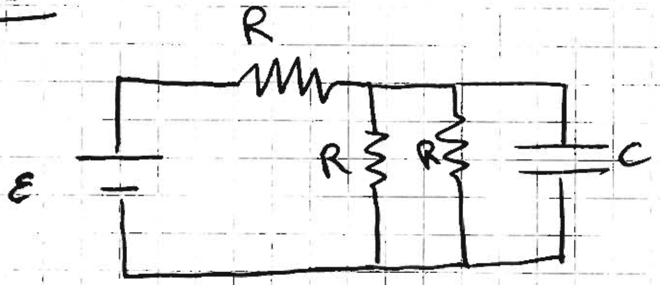
11) $v_x = v_0 - at$ $v_x = 0 \Rightarrow t = \frac{v_0}{a}$

$x(t) = v_0 t - \frac{1}{2} a t^2$

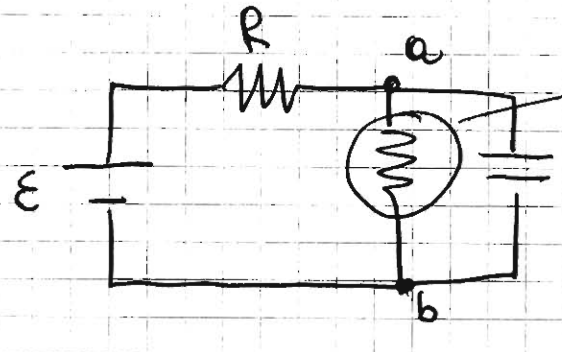
$\Rightarrow x = \frac{v_0^2}{2a} \Rightarrow d = s - x = s - \frac{v_0^2 \, m \varepsilon_0}{\sigma q}$

Pb. 5

12)



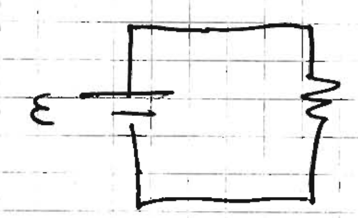
↓ equivalente a



$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} \Rightarrow$

$R_{eq} = \frac{R}{2}$

In condizioni stazionarie, il cond. è carico e quindi non ci passa corrente $\Rightarrow$ i passa solo dalle resistenze $\Rightarrow$



$R + \frac{R}{2} = \frac{3}{2} R$

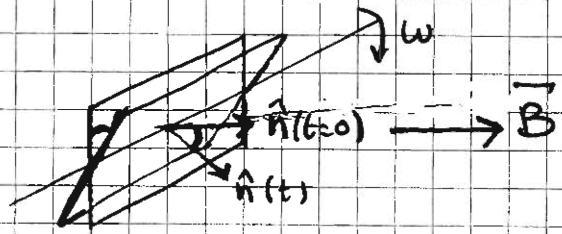
$\Rightarrow \frac{3}{2} R i = \varepsilon$ $i = \frac{2}{3} \frac{\varepsilon}{R}$

13) $\Delta V_{ab} = i \frac{R}{2} = \frac{\cancel{2}}{3} \frac{\varepsilon}{\cancel{R}} \frac{\cancel{R}}{\cancel{2}} = \frac{\varepsilon}{3}$

Poiché $\frac{Q}{\Delta V_{ab}} = C \Rightarrow Q = C \cdot \frac{\varepsilon}{3}$

Pb. 6

14)



$$\phi_B(t) = B(a \cdot b)\cos(\omega t)$$

$$\Rightarrow \quad \frac{d\phi_B}{dt} = -B\,ab\,\omega \sin \omega t \quad \Rightarrow$$

$$\mathcal{E} = B\,ab\,\omega \sin \omega t$$

15) $$P_{max} = \frac{\mathcal{E}_{max}^2}{R} = \frac{(B a b \omega)^2}{R}$$