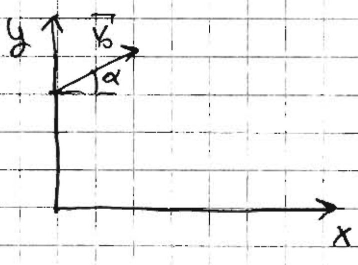
Soluzioni del compito del 28/06/2007

Pb. 1



1) $\begin{cases} x(t) = v_0 \cos\alpha \, t \\ y(t) = h + v_0 \sin\alpha \, t - \frac{1}{2} g t^2 \end{cases}$

$$\begin{cases} 0 = h + v_0 \sin\alpha \, t - \frac{1}{2} g t^2 \\ d = v_0 \cos\alpha \, t \end{cases} \Rightarrow t = \frac{d}{v_0 \cos\alpha}$$

$$0 = h + \cancel{v_0} \sin\alpha \frac{d}{\cancel{v_0} \cos\alpha} - \frac{1}{2} g \frac{d^2}{v_0^2 \cos^2\alpha}$$

$$\frac{1}{2} \frac{g d^2}{v_0^2 \cos^2\alpha} = h + d \, tg\alpha$$

$$v_0 = \sqrt{\frac{g}{2(h + d \, tg\alpha)}} \frac{d}{\cos\alpha}$$

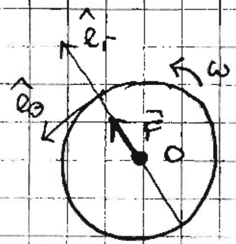
2) Dalla cons. dell' energia

$$\frac{1}{2} \cancel{m} v_0^2 + \cancel{m} g h = \frac{1}{2} \cancel{m} (v_0 \cos\alpha)^2 + \cancel{m} g h_{max}$$

$$\Rightarrow h_{max} = h + \frac{v_0^2 \sin^2\alpha}{2g}$$

all'altezza max. la velocità lungo y è zero
ma quella lungo x è cost.

3) $v = v_x = v_0 \cos\alpha$

Pb. 2  ω, v_0, m

4) Il moto di m è la composizione di due moti: traslazione lungo il diametro e rotazione intorno ad O

$$\Rightarrow \vec{v} = \vec{v}_{tr} + \vec{\omega} \times \vec{r}$$

$$\vec{v}_{tr} = v_0 \hat{e}_r$$

$$\vec{\omega} \times \vec{r} = \omega r(t) \hat{e}_\theta \quad e \quad r(t) = v_0 t$$

$$\Rightarrow |\vec{v}| = |v_0 \hat{e}_r + v_0 \omega t \hat{e}_\theta|$$

$$= \sqrt{v_0^2 + v_0^2 \omega^2 t^2} =$$

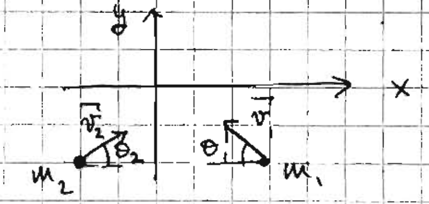
$$= v_0 \sqrt{1 + \omega^2 t^2}$$

5) Da $\vec{F} = m\vec{a}$

$$F_r = m a_r = -m \omega^2 r(t)$$

$$\Rightarrow |F_r| = m \omega^2 v_0 t$$

Pb. 3



m_1

$m_2 = 2m_1$

$|\vec v_1| = |\vec v_2| = v_{in}$

$\theta_1 = 60°, \ \theta_2 = 30°$

6) $$(m_1+m_2)\,\vec v_0 = m_1 \vec v_1 + m_2 \vec v_2$$

$$\Rightarrow \underbrace{(m_1+m_2)}_{3\cancel{m_1}} v_{0x} = -m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2$$

$$= -\cancel{m_1}\, v_{in}\,\frac{1}{2} + \cancel{2}\cancel{m_1}\, v_{in}\,\frac{\sqrt3}{\cancel{2}}$$

$$\underbrace{(m_1+m_2)}_{3\cancel{m_1}} v_{0y} = m_1 v_1 \sin\theta_1 + m_2 v_2 \sin\theta_2$$

$$= \cancel{m_1}\, v_{in}\,\frac{\sqrt3}{2} + \cancel{2}\cancel{m_1}\, v_{in}\,\frac{1}{\cancel{2}}$$

$$|\vec v_0| = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{\left(\frac{\sqrt3}{3} v_{in} - \frac{v_{in}}{6}\right)^2 + \left(\frac{v_{in}}{3} + v_{in}\frac{\sqrt3}{6}\right)^2}$$

$$= v_{in}\sqrt{\frac{1}{3} + \frac{1}{36} - \cancel{\frac{\sqrt3}{9}} + \frac{1}{9} + \frac{3}{36} + \cancel{\frac{\sqrt3}{9}}} =$$

$$= v_{in}\,\frac{\sqrt5}{3}$$

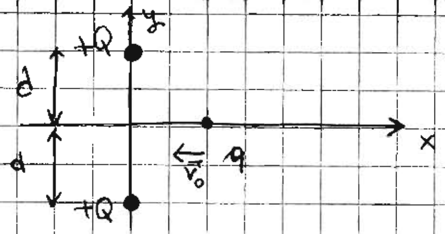
7) $$\mathrm{tg}\,\alpha = \frac{v_{0y}}{v_{0x}} = \frac{\cancel{v_{in}}\left(\frac{\sqrt3}{6} + \frac{1}{3}\right)}{\cancel{v_{in}}\left(\frac{\sqrt3}{3} - \frac{1}{6}\right)} = \frac{\sqrt3 + 2}{2\sqrt3 - 1}$$

8) $$\Delta E = \left|\frac{1}{2}\, 3m\, v_0^2 - \frac{1}{2} m_1 v_{in}^2 - \frac{1}{2} m_2 v_{in}^2\right|$$

$$= \left|\frac{1}{2}\, 3m_1 v_{in}^2 \frac{5}{9} - \frac{1}{2}\, 3m_1 v_{in}^2\right| = \frac{\cancel{3}}{\cancel{2}} m_1 v_{in}^2 \frac{\cancel{4}^{\,2}}{\cancel{9}_{\,3}}$$

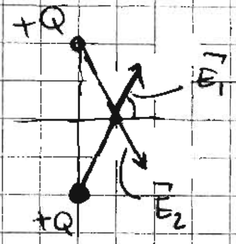
$$= \frac{2}{3} m_1 v_{in}^2$$

Pb. 4



$Q,\ d,\ m,\ v_0,\ q$

g) $\vec{a} = \dfrac{\vec{F}_e}{m} = \dfrac{q}{m}\vec{E}\left(\dfrac{d}{2}, 0\right)$



$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{k_e Q}{\left(d^2 + \frac{d^2}{4}\right)}\left(\cos\theta\,\hat{x} + \sin\theta\,\hat{y}\right)$$

$$+ k_e \frac{Q}{\left(d^2 + \frac{d^2}{4}\right)}\left(\cos\theta\,\hat{x} - \sin\theta\,\hat{y}\right)$$

$$= \frac{k_e Q}{\frac{4d^2 + d^2}{4}} \cdot 2\cos\theta\,\hat{x} =$$

$$= \cancel{8}^{4}\frac{k_e Q}{5d^2}\,\frac{\cancel{d}}{\cancel{2}}\,\frac{1}{\sqrt{d^2 + \frac{d^2}{4}}}\,\hat{x}$$

$$= 8\,\frac{k_e Q}{5\sqrt{5}\,d^2}\,\hat{x}$$

$$\Rightarrow \quad \vec{a} = -\frac{|q|}{m}\,\frac{8}{5\sqrt{5}}\,\frac{k_e Q}{d^2}\,\hat{x}$$

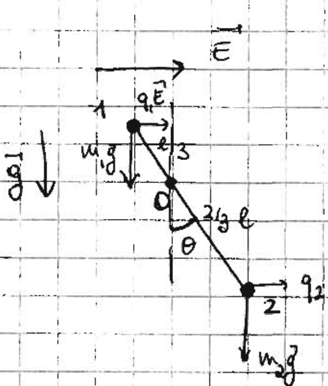
$$|\vec{a}| = \frac{|q|}{m}\,\frac{8}{5\sqrt{5}}\,\frac{k_e Q}{d^2}$$

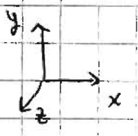
10) Cons. dell' energia

$$\frac{1}{2} m v_0^2 + |q| k_e \frac{Q}{\sqrt{2}\, d} \cdot 2 = \frac{1}{2} m v^2 - |q| k_e \frac{Q}{d} \cdot 2$$

$$v^{\cancel{2}} = \sqrt{v_0^2 + 4 \frac{Q |q| k_e}{m \cdot d} \left(1 - \frac{1}{\sqrt{2}}\right)}$$

Pb. 5





m_1, m_2, q_1, q_2

$\theta = 30°$

11) Eq. $\Rightarrow \sum$ momenti di tutte le forze ext. $\Big|_{\text{rispetto ad O}} = 0$

$$m_1 g \frac{\cancel{\ell}}{\cancel{3}} \sin\theta - q_1 E \frac{\cancel{\ell}}{\cancel{3}} \cos\theta - m_2 g \frac{2}{\cancel{3}} \cancel{\ell} \sin\theta + q_2 E \frac{2}{\cancel{3}} \cancel{\ell} \cos\theta = 0$$

$$(2q_2 - q_1) \cos\theta \, E = (2m_2 - m_1)\, g \sin\theta$$

$$E = \frac{2m_2 - m_1}{2q_2 - q_1} \, g \, \mathrm{tg}\,\theta$$

$$\Delta V = -E d \Rightarrow \Delta V = -\frac{2m_2 - m_1}{2q_2 - q_1} \, d \, g \, \mathrm{tg}\,\theta$$

12) Dalla conservazione dell' energia ($\nexists \vec{F}_{el}$, perché $\Delta V = 0 \Rightarrow \vec{E} = 0$)

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + m_1 g \frac{\ell}{3} - m_2 g \frac{2\ell}{3} =$$

$$= m_1 g \frac{\ell}{3} \cos\theta - m_2 g \frac{2}{3} \ell \cos\theta$$

Poiché il sistema ruota intorno ad O,

$$v_1 = \omega \frac{\ell}{3}$$

$$v_2 = \omega \frac{2\ell}{3} \quad \Rightarrow \quad v_2 = 2 v_1 \quad \Rightarrow$$

$$\frac{1}{2}(m_1 + 4 m_2) v_1^2 = m_1 g \frac{\ell}{3} (\cos\theta - 1) + m_2 g \frac{2}{3} \ell (1 - \cos\theta)$$

$$= \frac{\ell}{3} (1 - \cos\theta) g (2 m_2 - m_1)$$

$$v_1 = \sqrt{\frac{2}{3} \ell (1 - \cos\theta) g \frac{(2 m_2 - m_1)}{m_1 + 4 m_2}}$$

13) $\vec{E} = \frac{\Delta V}{2} \frac{1}{d} \hat{x} \Rightarrow$

$$\Delta U_{el} = -q_1 \int_{P_{1in}}^{P_{1fin}} \vec{E} \cdot d\vec{s}_1 - q_2 \int_{P_{2in}}^{P_{2fin}} \vec{E} \cdot d\vec{s}_2$$

$$= -q_1 \frac{\Delta V}{2d} \frac{\ell}{3} \sin\theta + q_2 \frac{\Delta V}{2d} \frac{2\ell}{3} \sin\theta$$

$$= \frac{\Delta V}{2d} \frac{\ell}{3} \sin\theta (2 q_2 - q_1)$$

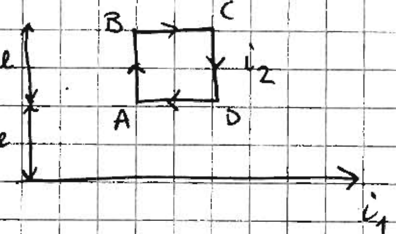
$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + m_1 g \frac{\ell}{3} (1 - \cos\theta) +$$

$$+ m_2 g \frac{2}{3} \ell (\cos\theta - 1) + \frac{\Delta V}{6d} \ell \sin\theta (2 q_2 - q_1) = 0$$

$$\frac{1}{2}(m_1 + 4 m_2) v_1^2 = \frac{\ell}{3} (1 - \cos\theta) g (2 m_2 - m_1) - \frac{\Delta V}{6d} \ell \sin\theta (2q_2 - q_1)$$

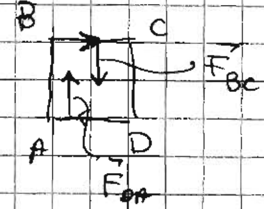
$$v_1 = \sqrt{\frac{2}{3} \ell (1 - \cos\theta) g \frac{2 m_2 - m_1}{m_1 + 4 m_2} - \frac{\Delta V}{3d} \ell \sin\theta \frac{2 q_2 - q_1}{m_1 + 4 m_2}}$$

Pb. 6



14) Sappiamo che $\vec{F}_{AB} + \vec{F}_{CD} = 0$

$$\Rightarrow \vec{F}_{tot} = \left(\vec{F}_{BC} + \vec{F}_{DA}\right) \cdot N$$



$$|\vec{F}_{BC}| = \overbrace{\frac{\mu_0}{2\pi} \frac{i_1}{2\cancel{\ell}}}^{\vec{B}} \, i_2 \cancel{\ell}$$

$$\uparrow$$

$$\vec{F} = i \, \vec{\ell} \times \vec{B}$$

$$F_{BC} = \frac{\mu_0 \, i_1 i_2}{4\pi}$$

$$|\vec{F}_{DA}| = \frac{\mu_0}{2\pi} \frac{i_1}{\cancel{\ell}} \, i_2 \cancel{\ell} \qquad F_{DA} = \frac{\mu_0 \, i_1 i_2}{2\pi}$$

$$F_{tot} = \frac{\mu_0 \, i_1 i_2}{2\pi} \left(1 - \frac{1}{2}\right) \cdot N = \frac{\mu_0 \, i_1 i_2 \, N}{4\pi}$$

15) $$R_{bobina} = N \, R_{spira} = N \underbrace{\rho}_{\text{resistività}} \frac{L_{tot}}{\text{Sezione}} = N \rho \frac{4\ell}{\pi \frac{\phi^2}{4}} = \frac{16 N \rho \ell}{\pi \phi^2}$$

$$\Rightarrow P = i_2^2 R_{bobina} = \frac{16 N \rho \ell i_2^2}{\pi \phi^2}$$