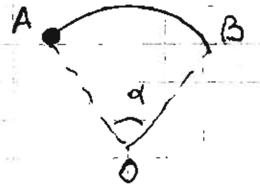
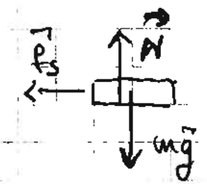
Compito del 5/9/2007

Pb. 1

1. 

$$\omega = \frac{v_0}{R} = \frac{\alpha\,(Rad)}{t}$$

$$\Rightarrow \quad t = \frac{\alpha\,(Rad)\,R}{v_0} = \frac{\alpha(°)\,\pi R}{180\,v_0}$$

2. 

Da $\vec{F} = m\vec{a}$

$$\vec{N} + m\vec{g} + \vec{f}_s = m\vec{a}$$

$$\Rightarrow \begin{cases} N - mg = 0 \\ f_s = m\dfrac{v_0^2}{R} \end{cases} \qquad \begin{array}{l} N = mg \\ m\dfrac{v_0^2}{R} = f_s \le \mu_s N = \mu_s mg \end{array}$$

$$\Rightarrow \quad \cancel{m}\frac{v_0^2}{R} \le \cancel{m}\mu_s g \qquad v_0 \le \sqrt{Rg\mu_s}$$

Pb. 2

3. Cons. dell' en. meccanica

$$\frac{1}{2} m v_0^2 + m g\, l_0 \sin\alpha = \frac{1}{2} k \underbrace{(l - l_0)}_{\Delta l}{}^2 + m g\, l \sin\alpha$$

$$\cancel{\tfrac{1}{2}} k\,\Delta l^2 + 2 m g\,\Delta l \sin\alpha - \cancel{\tfrac{1}{2}} m v_0^2 = 0$$

$$\Delta l = \frac{-mg\sin\alpha \pm \sqrt{(mg\sin\alpha)^2 + mkv_0^2}}{k}$$

sol. fisica

$$\Delta l = -\frac{mg\sin\alpha}{k} + \sqrt{\left(\frac{mg\sin\alpha}{k}\right)^2 + \frac{m v_0^2}{k}}$$

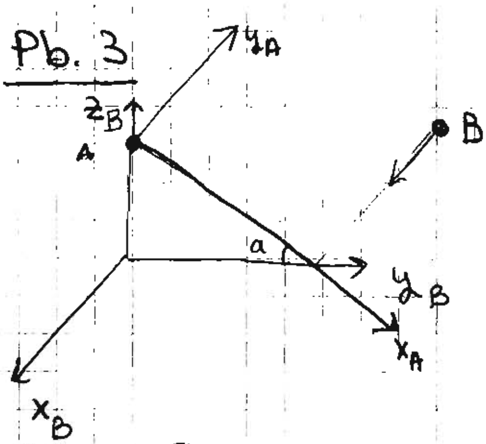
4. Sempre cons. dell' energia ($\Delta l = l_0 \Rightarrow l_{molla} = 2 l_0$)

$$\frac{1}{2} m v_0^2 + m g\, l_0 \sin\alpha = \frac{1}{2} m v^2 + \frac{1}{2} k \left(2 l_0 - l_0\right)^2 + m g\, 2 l_0 \sin\alpha$$

$$\frac{1}{2} m v_0^2 - m g\, l_0 \sin\alpha - \frac{1}{2} k\, l_0^2 = \frac{1}{2} m v^2$$

$$v = \sqrt{v_0^2 - 2 g\, l_0 \sin\alpha - \frac{k}{m} l_0^2}$$

Pb. 3



5.
$$\begin{cases} x_A(t) = \frac{1}{2} g \sin\alpha\, t^2 \\ y_A(t) = 0 \end{cases} \qquad \begin{cases} x_B(t) = -4h + v_B t \\ y_B(t) = h / \mathrm{tg}\alpha \\ z_B(t) = 0 \end{cases}$$

Tempo che impiega A ad arrivare a terra:

$$\frac{h}{\sin\alpha} = \frac{1}{2} g \sin\alpha\, \bar{t}^2 \Rightarrow \bar{t} = \sqrt{\frac{2h}{g}}\, \frac{1}{\sin\alpha}$$

In questo intervallo di tempo $x_B(\bar{t}) = 0 \Rightarrow$

$$4h = v_B\, \bar{t} \qquad v_B = \frac{4h}{\bar{t}} = \frac{4h \sin\alpha \sqrt{g}}{\sqrt{2h}}$$

$$\alpha = 30^\circ \Rightarrow \sin\alpha = \frac{1}{2} \Rightarrow v_B = \sqrt{2hg}$$

6. Urto anelastico $\Rightarrow$

$$m_A \vec{v}_A + m_B \vec{v}_B = (m_A + m_B)\, \vec{v}_{fin}$$

$$\vec{v}_B = \sqrt{2gh}\ \hat{x}_B$$

$v_A \rightarrow$ cons. energia $\frac{1}{2} m_A v_A^2 = m_A g h \Rightarrow v_A = \sqrt{2gh}$

$$\vec{v}_A = \sqrt{2gh}\ \hat{y}_B$$

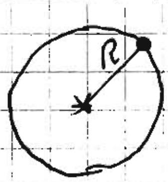
$$\Rightarrow \quad \vec{v}_{fin} = \sqrt{2gh} \left(\frac{m_A}{m_A+m_B}\, \hat{y}_B + \frac{m_B}{m_A+m_B}\, \hat{x}_B \right)$$

$$\Rightarrow \quad |\vec{v}_{fin}| = \sqrt{2gh}\ \frac{\sqrt{m_A^2 + m_B^2}}{m_A + m_B}$$

7. $$tg\,\beta = \frac{v_{fin,y}}{v_{fin,x}} = \frac{m_A}{m_B}$$

$$\Rightarrow \quad \beta = tg^{-1}\left(\frac{m_A}{m_B}\right)$$

Pb. 4

8. 

$$m \frac{v^2}{R} = \cancel{m}\, \omega^2 R = \frac{G M_S \cancel{m}}{R^2}$$

$$\omega = \sqrt{\frac{G M_S}{R^3}} = \frac{2\pi}{T}$$

$$\Rightarrow \quad T = 2\pi \sqrt{\frac{R^3}{G M_S}}$$

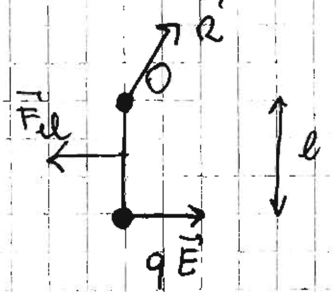
9. Cons. dell' energia

$$E_c - \frac{G M_S m}{R} = 0 \leftarrow \begin{cases} E_{c_{fin}} = 0 \\ E_{pot,\,fin} = 0 \end{cases}$$

$$E_c = \frac{G M_S m}{R}$$

<u>Pb. 5</u>

10.



<u>Forze sulla sbarretta</u>

<u>Equilibrio</u> $\Rightarrow$ $\sum \vec F_{ext} = 0$

$$\vec M_{o,\,ext} = 0$$

$$\Downarrow$$

$$F_{el}\,\frac{\cancel{\ell}}{2} = q E \cancel{\ell}$$

$$F_{el} = k\Delta\ell$$

$$E = \frac{\Delta V}{d} \quad \Rightarrow \quad \frac{k\Delta\ell}{2} = q\,\frac{\Delta V}{d}$$

$$\Delta V = \frac{k\,\Delta\ell\, d}{2q}$$

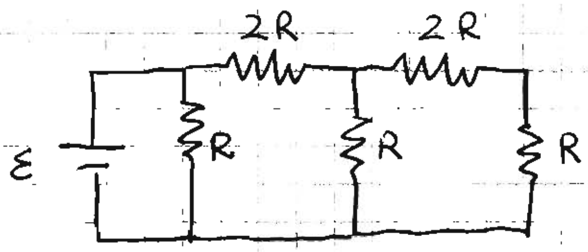
11. Punto più vicino $\Rightarrow$ sbarretta orizzontale. Dal teorema delle forze vive:

$$\frac{1}{2} m v^2 = \mathcal{L}_{elettrica} = q\,\frac{\Delta V}{d}\cdot \ell$$

$$\Rightarrow \quad v = \sqrt{2\,\frac{\Delta V}{m}\,\frac{\ell}{d}\,q}$$

Pb. 6

12.



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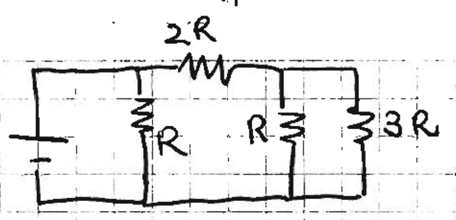
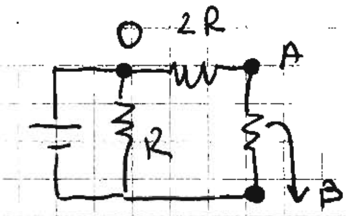


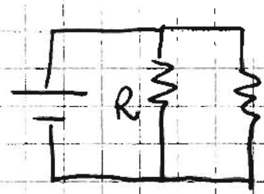
$=$



$$\frac{1}{R_{tot}} = \frac{1}{R} + \frac{1}{3R} = \frac{4}{3R}$$

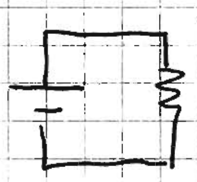
$$\Rightarrow \quad R_{tot} = \frac{3}{4}R$$

$=$



$$2R + \frac{3}{4}R = \frac{11}{4}R$$

$\parallel$



$$\frac{1}{R_{tot}} = \frac{1}{R} + \frac{4}{11R} = \frac{15}{11R}$$

$$\Rightarrow \quad \mathcal{E} = R_{tot}\, i \quad \Rightarrow \quad i = \frac{15\,\mathcal{E}}{11\,R}$$

13. $P = \dfrac{\Delta V_{AB}^{2}}{R}$

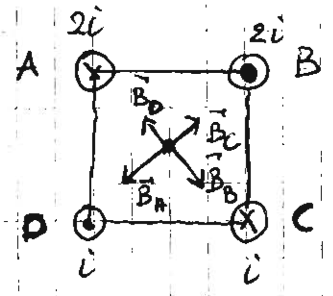
$$\Delta V_{AB} = \frac{3}{4}R\, i_{AB} \quad \rightarrow \text{Nodo } O: \quad i_{AB}\,\frac{11}{4}R = i_1 R_1 = \mathcal{E}$$

$$\Rightarrow \quad i_{AB} = \frac{4\,\mathcal{E}}{11\,R} \quad \Rightarrow \quad \Delta V_{AB} = \frac{3}{\cancel{4}}\cancel{R}\,\frac{\cancel{4}}{11}\,\frac{\mathcal{E}}{\cancel{R}} = \frac{3}{11}\,\mathcal{E}$$

$$\Rightarrow \quad P = \frac{9}{121}\,\frac{\mathcal{E}^2}{R}$$

Pb. 17

14.



$$B_A = \frac{\mu_0}{2\pi}\,\frac{2i}{\frac{\sqrt{2}}{2}a}$$

$$B_B = \frac{\mu_0}{2\pi}\,\frac{2i}{\frac{\sqrt{2}}{2}a} = B_A$$

$$B_C = \frac{\mu_0}{2\pi}\,\frac{i}{\frac{\sqrt{2}}{2}a} = \frac{B_A}{2}$$

$$B_D = \frac{\mu_0}{2\pi}\,\frac{i}{\frac{\sqrt{2}}{2}a} = B_C$$

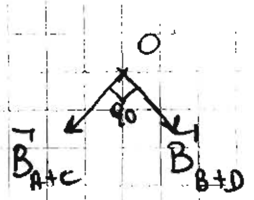
direzioni e versi vd. figura

$$|\vec{B}_A + \vec{B}_C| = \frac{\mu_0}{2\pi}\,\frac{i}{\frac{\sqrt{2}}{2}a}$$

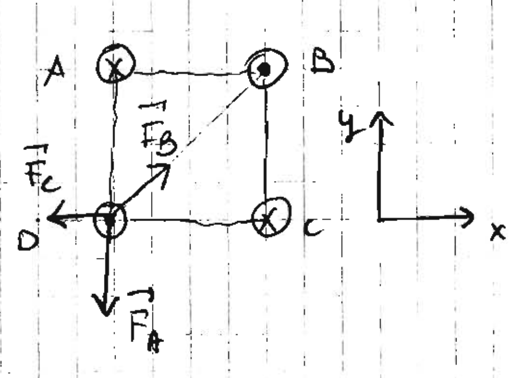
$$|\vec{B}_B + \vec{B}_D| = \frac{\mu_0}{2\pi}\,\frac{i}{\frac{\sqrt{2}}{2}a}$$

$$\Rightarrow \quad B_{tot} = \frac{\mu_0}{2\pi}\,\frac{i}{\frac{\cancel{\sqrt{2}}}{2}a}\,\cancel{\sqrt{2}}$$

$$= \frac{\mu_0}{4\pi}\,\frac{4i}{a}$$



15.



$$\frac{\vec{F}_A}{\ell} = -\hat{y}\,\frac{\mu_0}{2\pi}\,\frac{2i^2}{a}$$

$$\frac{\vec{F}_C}{\ell} = -\hat{x}\,\frac{\mu_0}{2\pi}\,\frac{i^2}{a}$$

$$\frac{\vec{F}_B}{\ell} = \frac{\mu_0}{2\pi}\,\frac{2i^2}{\sqrt{2}a}\left(\frac{\sqrt{2}}{2}\hat{x} + \frac{\sqrt{2}}{2}\hat{y}\right)$$

$$\Rightarrow \frac{\vec{F}_{tot}}{\ell} = \frac{\mu_0}{2\pi}\frac{i^2}{a}\left(\underbrace{\cancel{\hat{x}} + \hat{y}}_{\frac{\vec{F}_B}{\ell}} \underbrace{- 2\hat{y}}_{\frac{\vec{F}_A}{\ell}} \underbrace{\cancel{-\hat{x}}}_{\frac{\vec{F}_C}{\ell}}\right)$$

$$= \frac{\mu_0}{2\pi}\frac{i^2}{a}(-\hat{y})$$

$$\left|\frac{\vec{F}_{tot}}{\ell}\right| = \frac{\mu_0}{4\pi}\frac{2i^2}{a}$$