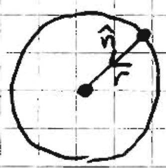
Esercizio 1

1)  $\vec{F} = m\vec{a}$

$\hat{n}$ = versore radiale diretto verso il centro del cerchio

$$\vec{F} = G\frac{m_s m}{r^2}\hat{n}$$

$$\Rightarrow \quad G\frac{m \cancel{m_s}}{r^2} = \omega^2 r \cancel{m_s}$$

$$\vec{a} = \frac{v^2}{r}\hat{n} = \omega^2 r\, \hat{n}$$

$$\omega^{\cancel{2}} = \sqrt{\frac{Gm}{r^3}}$$

$$\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} \Rightarrow \quad T = 2\pi\sqrt{\frac{r^3}{Gm}} = T_S$$

2) Dalla relazione precedente

$$\frac{T^2}{r^3} = \frac{2\pi}{Gm} = \text{costante se } m \text{ è costante} \Rightarrow$$

$$\frac{T_S^2}{r^3} = \frac{(2T_S)^2}{r_2^3} \Rightarrow \quad r_2^3 = 4r^3 \Rightarrow r_2 = \sqrt[3]{4}\, r$$

3) $$T^2 m = 2\pi\frac{r^3}{G} = \text{costante se } r \text{ è costante} \Rightarrow$$

$$T_G^2 m_G = T_S^2 m \Rightarrow \quad T_G = T_S\sqrt{\frac{m}{m_G}}$$

Esercizio 2

4) Conservazione dell'energia meccanica per m_a $\Rightarrow$

$$m_a g h = \frac{1}{2} m_a v_{in}^2 \Rightarrow v_{in} = \sqrt{2gh}$$

5) Urto elastico $\Rightarrow$ conservazione della quantità di moto e dell'energia cinetica $\Rightarrow$

$$\begin{cases} \cancel{m_a} v_{in} = m_a v_a + m_b v_b = \cancel{m_a} v_a + 3 \cancel{m_a} v_b \\ \cancel{\frac{1}{2}} \cancel{m_a} v_{in}^2 = \frac{1}{2} m_a v_a^2 + \frac{1}{2} m_b v_b^2 = \cancel{\frac{1}{2}} \cancel{m_a} v_a^2 + \cancel{\frac{1}{2}} 3 \cancel{m_a} v_b^2 \end{cases}$$

$$\begin{cases} v_{in} = v_a + 3 v_b \\ v_{in}^2 = v_a^2 + 3 v_b^2 \end{cases} \Rightarrow v_a = v_{in} - 3 v_b$$

$$\cancel{v_{in}^2} = \cancel{v_{in}^2} + 9 v_b^2 - 6 v_{in} v_b + 3 v_b^2$$

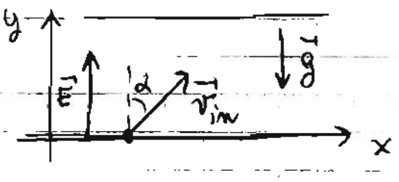
$$\Rightarrow \cancel{12} v_b^{\cancel{2}} = \cancel{6} \frac{v_{in}}{2} \cancel{v_b} \qquad v_b = \frac{v_{in}}{2}$$

$$v_a = v_{in} - \frac{3}{2} v_{in} = -\frac{v_{in}}{2}$$

6) Conservazione dell'energia meccanica per m_b

$$\frac{1}{2} \cancel{m_b} v_b^2 = \cancel{m_b} g h_b \Rightarrow h_b = \frac{v_b^2}{2g} = \frac{v_{in}^2}{8g} = \frac{\cancel{2g} h}{\cancel{8g}_4} = \frac{h}{4}$$

Esercizio 3

7) 

$$\begin{cases} x(t) = v_{in} \sin\alpha \, t \\ y(t) = v_{in} \cos\alpha \, t + \frac{1}{2} a t^2 \end{cases} \quad \begin{cases} v_x = v_{in} \sin\alpha \\ v_y = v_{in} \cos\alpha + at \end{cases}$$

8) Devo trovare a: $\vec{F} = m\vec{a}$

$$\vec{F} = q\vec{E} + m\vec{g} \Rightarrow \vec{a} = \hat{y}\left(-g + \frac{q}{m} E\right)$$

$$E = \frac{\Delta V}{d} \Rightarrow a_y = \frac{q\,\Delta V}{md} - g$$

Se $x(t) = \ell$, $y(t) = 0 \Rightarrow v_{in} \sin\alpha \, t = \ell \qquad t = \frac{\ell}{v_{in}\sin\alpha}$

$$\Rightarrow 0 = \cancel{v_{in}} \cos\alpha \frac{\ell}{\cancel{v_{in}} \sin\alpha} + \frac{1}{2} a \left(\frac{\ell}{v_{in}\sin\alpha}\right)^2$$

$$a = -2 \frac{v_{in}^2 \cos\alpha \sin\alpha}{\ell} = q\frac{\Delta V}{md} - g$$

$$\Rightarrow \Delta V = \frac{md}{q}\left(g - 2\frac{v_{in}^2 \cos\alpha \sin\alpha}{\ell}\right) \underset{\substack{\sin\alpha = \frac{\sqrt{2}}{2} \\ \| \\ \cos\alpha}}{=} \frac{md}{q\ell}\left(\ell g - v_{in}^2\right)$$

9) Un modo per risolverlo è con la conservazione dell'energia

$$\frac{1}{2} m v_{in}^2 = \frac{1}{2} m (v_{in} \sin\alpha)^2 + mg\,(d - y) - \frac{\Delta V q}{d}(d - y)$$

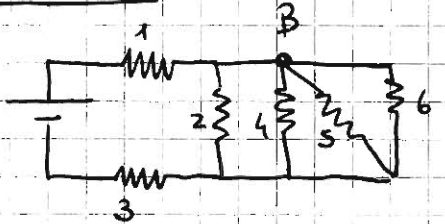
$$\frac{1}{2} \cancel{m}\, \cancel{v_{in}^2} \cos^2\alpha = (d - y)\left(mg - \frac{\Delta V q}{d}\right)$$

$$= (d - y)\left(\cancel{mg} - \cancel{mg} + 2m \frac{v_{in}^2 \cos\alpha \sin\alpha}{\ell}\right)$$

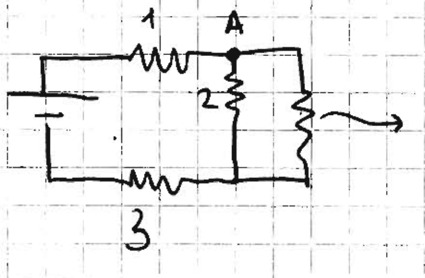
$$= (d - y) \frac{\cancel{m}\, \cancel{v_{in}^2}}{\ell}$$

$$d - y = \frac{\ell}{2}\,\frac{1}{2} \Rightarrow y = d - \frac{\ell}{4}.$$

Esercizio 4

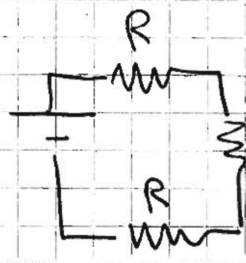


10) Circuito iniziale è equivalente a $(R_4 \parallel R_5 \parallel R_6)$:



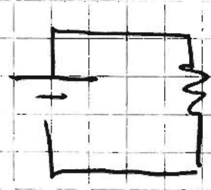
$$R_{eq} = \left(\frac{1}{R} + \frac{1}{R} + \frac{1}{R}\right)^{-1} = \frac{R}{3}$$

R_2 e R_{eq} sono in $\parallel$ $\Rightarrow$



$$R_{eq,1} = \left(\frac{1}{R} + \frac{3}{R}\right)^{-1} = \frac{R}{4}$$

R_1, $R_{eq,1}$ e R_3 sono in serie $\Rightarrow$



$$R + R + \frac{R}{4} = \frac{9}{4} R$$

$$\Rightarrow \quad \varepsilon = \frac{9}{4} R\, i \qquad i = \frac{4}{9} \frac{\varepsilon}{R}$$

11) Nel nodo A

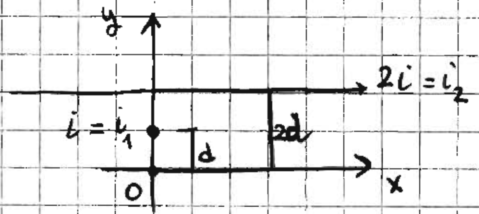
$$\begin{cases} i_2 + i_{eq} = i \\ R\, i_2 = R_{eq}\, i_{eq} \end{cases} \Rightarrow \begin{cases} i_2 + 3 i_2 = i \\ \cancel{R} i_2 = \frac{\cancel{R}}{3} i_{eq} \end{cases} \Rightarrow i_2 = \frac{i}{4}$$

12) $i_{eq} = \frac{3}{4} i$. In B

$$\begin{cases} i_{eq} = i_4 + i_5 + i_6 \\ \cancel{R} i_4 = \cancel{R} i_5 = \cancel{R} i_6 \end{cases} \Rightarrow \quad i_6 = \frac{i_{eq}}{3}$$

$$\Rightarrow i_6 = \frac{i}{4} \Rightarrow P_6 = i_6^2 R = \frac{i^2 R}{16} = \left(\frac{4}{9} \frac{\varepsilon}{R}\right)^2 \frac{R}{16} = \frac{\varepsilon^2}{81 R}.$$

Esercizio 5

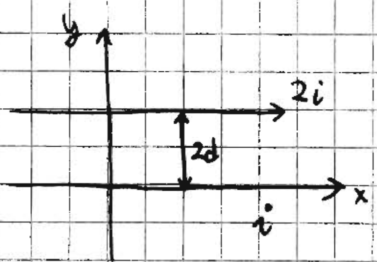


13) $\vec{B}_0 = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0}{2\pi}\frac{i}{d}\hat{x} + \frac{\mu_0}{2\pi}\frac{\cancel{2}i}{\cancel{2}d}(-\hat{z})$

$\Rightarrow B_{0,z} = -\frac{\mu_0}{2\pi}\frac{i}{d}$

14) $|\vec{B}_0| = \sqrt{\left(\frac{\mu_0}{2\pi}\frac{i}{d}\right)^2 + \left(-\frac{\mu_0}{2\pi}\frac{i}{d}\right)^2} = \frac{\mu_0}{2\pi}\frac{i}{d}\sqrt{2}$

15)



$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{\cancel{2}i^2}{\cancel{2}d} = \frac{\mu_0}{2\pi}\frac{i^2}{d}$