

$$\cancel{\frac{v_o^2}{g}} - \cancel{\frac{4}{g} v_o^2 h} \sin^2 \alpha = \cancel{\frac{2}{g} h^2} \sin^2 \alpha - \cancel{\frac{2}{g} h} \frac{v_o^2}{g} \sin^2 \alpha$$

$$\frac{v_o^2}{g} = \sin^2 \alpha \left(2h - 2 \frac{v_o^2}{g} h + 4 \frac{v_o^2}{g} h \right) = \sin^2 \alpha \left(2h + 2 \frac{v_o^2}{g} h \right)$$

$$\Rightarrow \sin^2 \alpha = \frac{\frac{v_o^2}{g}}{2h + \frac{2v_o^2}{g} h} = \frac{1}{2} \frac{1}{\left(\frac{gh}{v_o^2} + 1 \right)}$$

Controlli semplici:

$$h=0 \quad \sin^2 \alpha = \frac{1}{2} \quad \Rightarrow \quad \alpha = 45^\circ \quad (\text{come doveva essere})$$

$$h \gg \frac{v_o^2}{g} \quad \sin^2 \alpha \approx \frac{1}{2 \frac{gh}{v_o^2}} \ll 1 \quad \Rightarrow \quad \alpha \approx 0$$

Calcolare la gittata massima

$$d_{\max} = d \left(\sin^2 \alpha = \frac{1}{2} \frac{1}{\left(\frac{gh}{v_o^2} + 1 \right)} \right)$$

$$\begin{cases} \sin \alpha = \frac{v_o}{\sqrt{2gh + 2v_o^2}} \\ \cos \alpha = \sqrt{1 - \frac{v_o^2}{2gh + 2v_o^2}} \end{cases}$$

$$d_{\max} = \frac{v_o^2}{g} \frac{v_o}{\sqrt{2gh + 2v_o^2}} \sqrt{1 - \frac{v_o^2}{2gh + 2v_o^2}}$$

$$+ \sqrt{1 - \frac{v_o^2}{2gh + 2v_o^2}} \sqrt{\frac{v_o^4}{g^2} \frac{v_o^2}{2gh + 2v_o^2} + \frac{2h v_o^2}{g}}$$