$$d_{max} = \frac{v_0^3}{g(2gh+2v_0^2)}\sqrt{2gh+v_0^2} + \sqrt{\frac{2gh+v_0^2}{2gh+2v_0^2}}\,\frac{v_0}{\sqrt{g}}\sqrt{\frac{v_0^4}{g}\frac{1}{2hg+2v_0^2}+2h}$$

$$= \frac{v_0^3}{g(2gh+2v_0^2)}\sqrt{2gh+v_0^2} + \sqrt{\frac{2gh+v_0^2}{2gh+2v_0^2}}\,\frac{v_0}{\sqrt{g}}\sqrt{\frac{v_0^4+2hg(2hg+2v_0^2)}{g(2hg+2v_0^2)}}$$

$$= \frac{v_0^3}{g}\frac{\sqrt{2gh+v_0^2}}{2gh+2v_0^2} + \frac{v_0}{\sqrt{g}}\sqrt{\frac{2gh+v_0^2}{2gh+2v_0^2}}\,\frac{2gh+v_0^2}{\sqrt{g(2hg+2v_0^2)}}$$

$$= \frac{v_0}{g}\frac{\sqrt{2gh+v_0^2}}{2gh+2v_0^2}\left[v_0^2+2gh+v_0^2\right]$$

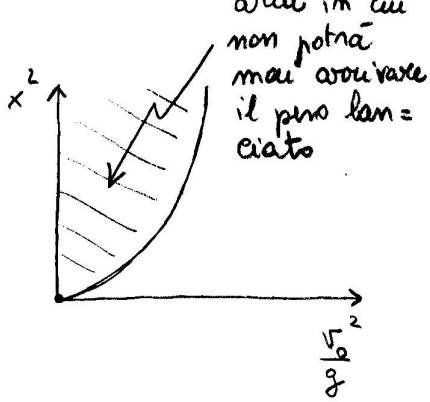
$$= \frac{v_0}{g}\sqrt{2gh+v_0^2}$$

Check

- $h = 0$ $\quad d_{max} = \frac{v_0^2}{g}$ (come doveva essere)
- $h \gg \frac{v_0^2}{g}$ $\quad d_{max} \simeq \frac{v_0}{g}\sqrt{2gh} = \sqrt{\frac{2h}{g}}\,v_0$

sia $x \equiv d_{max} \Rightarrow$

$$x^2 = \frac{v_0^2}{g^2}(2gh+v_0^2) = \frac{v_0^4}{g^2} + 2h\frac{v_0^2}{g}$$



[record $\simeq$ 23 m
$v_0 \simeq$ 14 m/s