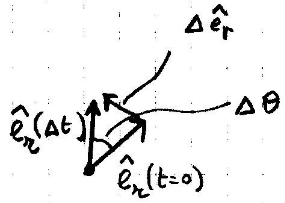
■ Verifica da $\vec{r} = r\,\hat{e}_r$

$$\dot{\vec{r}} = \dot{r}\,\hat{e}_r + r\,\frac{d\hat{e}_r}{dt}$$



$$\frac{d\hat{e}_r}{dt} = \lim_{\Delta t\to 0}\frac{\Delta\hat{e}_r}{\Delta t} = \lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t}\,\hat{e}_\theta = \dot{\theta}\,\hat{e}_\theta$$

$$\Rightarrow \dot{\vec{r}} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta \qquad ■$$

__Acc__ : $\dot{\vec{v}} = \vec{a} = \ddot{x}\,\hat{i} + \ddot{y}\,\hat{j} = \ddot{\vec{r}}$

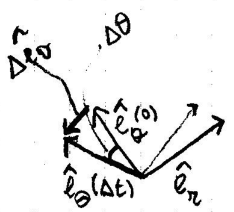
$$= \hat{i}\left(\ddot{r}\cos\theta - 2\dot{r}\dot{\theta}\sin\theta - \cancel{\dot{r}\dot{\theta}\sin\theta} - r\ddot{\theta}\sin\theta - r\dot{\theta}^2\cos\theta\right)$$
$$+ \hat{j}\left(\ddot{r}\sin\theta + 2\dot{r}\dot{\theta}\cos\theta + \cancel{\dot{r}\dot{\theta}\cos\theta} + r\ddot{\theta}\cos\theta - r\dot{\theta}^2\sin\theta\right)$$

$$= \ddot{r}\left(\hat{i}\cos\theta + \hat{j}\sin\theta\right)$$
$$+ 2\dot{r}\dot{\theta}\left(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}\right)$$
$$+ r\ddot{\theta}\left(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}\right)$$
$$- r\dot{\theta}^2\left(\cos\theta\,\hat{i} + \hat{j}\sin\theta\right)$$

$$= \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta \qquad (5)$$

■ Verifica da $\dot{\vec{r}} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$

$$\ddot{\vec{r}} = \ddot{r}\,\hat{e}_r + \dot{r}\,\frac{d\hat{e}_r}{dt} + \dot{r}\dot{\theta}\,\hat{e}_\theta + r\ddot{\theta}\,\hat{e}_\theta + r\dot{\theta}\,\frac{d\hat{e}_\theta}{dt}$$



$$\frac{de_\theta}{dt} = \lim_{\Delta t\to 0}\frac{\Delta e_\theta}{\Delta t} = \lim_{\Delta t\to 0} -\hat{e}_r\,\frac{\Delta\theta}{\Delta t} =$$
$$= -\hat{e}_r\,\dot{\theta}$$

$$\ddot{\vec{r}} = \ddot{r}\,\hat{e}_r + 2\dot{r}\dot{\theta}\,\hat{e}_\theta + r\ddot{\theta}\,\hat{e}_\theta - r\dot{\theta}^2\,\hat{e}_r$$

$$= \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta \qquad ■$$