2) Si supponga che la spirale sia percorsa con $r\dot{\theta} = w = \text{cost.}$

$$r\dot{\theta} = w = \left(r_0 + \frac{b}{2\pi}\theta\right)\frac{d\theta}{dt}$$

$$\Rightarrow \int_{\theta_0}^{\theta}\left(r_0 + \frac{b}{2\pi}\theta\right)d\theta = \int_0^t w\,dt$$

$$r_0\theta + \frac{b}{4\pi}\theta^2 + K_0 = wt$$

$K_0 =$ cost che dipende dalle condizioni iniziali: per $t=0$ voglio $\theta = 0$ $\Rightarrow K_0 = 0$

$$\frac{b}{4\pi}\theta^2 + r_0\theta - wt = 0$$

$$b\theta^2 + 4\pi r_0\theta - 4\pi wt = 0$$

soluzione fisica

$$\theta = \frac{-2\pi r_0 \pm \sqrt{4\pi^2 r_0^2 + 4\pi b w t}}{b}$$

$$\Rightarrow \theta = \frac{2\pi r_0}{b}\left[\sqrt{1 + \frac{bwt}{\pi r_0^2}} - 1\right]$$

$$\frac{bw}{\pi r_0^2} = \frac{\cancel{[L]}\,\cancel{[L]}\,[T]^{-1}}{\cancel{[L]}^2} \approx [T]^{-1}$$

$$\boxed{\frac{bw}{\pi r_0^2} = \omega_0} \qquad \Rightarrow \theta = \frac{2\pi r_0}{b}\left(\sqrt{1+\omega_0 t} - 1\right)$$

$$\Rightarrow r(t) = \cancel{r_0} + \frac{\cancel{b}}{\cancel{2\pi}}\,\frac{\cancel{2\pi} r_0}{\cancel{b}}\left(\sqrt{1+\omega_0 t}\ \cancel{-1}\right)$$

$$= r_0\sqrt{1+\omega_0 t}$$

$$\vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$$

$$= r_0\,\frac{1}{2}\,\frac{\omega_0}{\sqrt{1+\omega_0 t}}\,\hat{e}_r + w\,\hat{e}_\theta$$

$$= \frac{\omega_0 r_0}{2\sqrt{1+\omega_0 t}}\,\hat{e}_r + w\,\hat{e}_\theta \quad \Rightarrow$$