$$|\vec{v}| = \sqrt{\frac{\omega_0^2 r_0^2}{4(1+\omega_0 t)} + w^2} .$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{e}_r + \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})\,\hat{e}_\theta$$

$$\ddot{r} = \frac{\omega_0 r_0}{2}\left(-\frac{1}{2}\right)\frac{\omega_0}{(1+\omega_0 t)^{3/2}} = -\frac{\omega_0^2 r_0}{4(1+\omega_0 t)^{3/2}}$$

$$r\dot{\theta}^2 = (r\dot{\theta})\dot{\theta} = w\,\frac{\cancel{2}\pi r_0}{b}\,\frac{1}{\cancel{2}}\,\frac{\omega_0}{\sqrt{1+\omega_0 t}} = w\,\frac{\cancel{\pi r_0}}{\cancel{b}}\,\frac{\cancel{b}w}{\cancel{\pi} r_0^{\cancel{2}}}\,\frac{1}{\sqrt{1+\omega_0 t}}$$

$$= \frac{w^2}{r_0}\,\frac{1}{\sqrt{1+\omega_0 t}}$$

$$r^2\dot{\theta} = r(r\dot{\theta}) = w\, r_0\sqrt{1+\omega_0 t} \Rightarrow$$

$$\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta}) = \frac{w}{r}\,\frac{r_0\omega_0}{2\sqrt{1+\omega_0 t}} = \frac{w\,\omega_0}{2(1+\omega_0 t)}$$

$$\Rightarrow \vec{a} = \left(-\frac{\omega_0^2 r_0}{4(1+\omega_0 t)^{3/2}} - \frac{w^2}{r_0}\,\frac{1}{\sqrt{1+\omega_0 t}}\right)\hat{e}_r + \frac{w\,\omega_0}{2(1+\omega_0 t)}\,\hat{e}_\theta$$

$$\hat{T} = \frac{\omega_0 r_0}{2\sqrt{1+\omega_0 t}\,v}\,\hat{e}_r + \frac{w}{v}\,\hat{e}_\theta$$

$$\hat{N} = -\frac{w}{v}\,\hat{e}_r + \frac{\omega_0 r_0}{2v\sqrt{1+\omega_0 t}}\,\hat{e}_\theta$$

$$\dot{v} = \frac{1}{2}\,\frac{1}{v}\,\frac{\omega_0^2 r_0^2}{4}\,(-)\,\frac{\omega_0}{(1+\omega_0 t)^2} = -\frac{\omega_0^3 r_0^2}{8v(1+\omega_0 t)^2}$$

$$\vec{a} = \dot{v}\,\hat{T} + \frac{v^2}{\rho}\,\hat{N} = -\frac{\omega_0^3 r_0^2}{8v(1+\omega_0 t)^2}\,\frac{\omega_0 r_0}{2v\sqrt{1+\omega_0 t}}\,\hat{e}_r - \frac{\omega_0^3 r_0^2}{8v(1+\omega_0 t)^2}\,\frac{w}{v}\,\hat{e}_\theta$$

$$+ \frac{v^2}{\rho}\left(-\frac{w}{v}\,\hat{e}_r\right) + \frac{v^{2}}{\rho}\,\frac{\omega_0 r_0}{2\cancel{v}\sqrt{1+\omega_0 t}}\,\hat{e}_\theta =$$