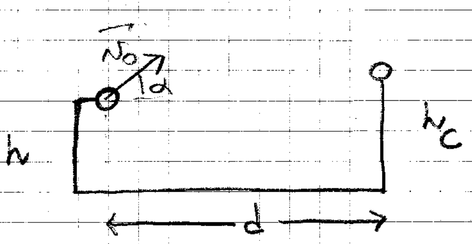
Soluzioni

1)




$h = 2\ m$

$h_c = 3.05\ m$

$d = 10\ m$

$\alpha = 40°$

$$\begin{cases} x(t) = v_0 \cos\alpha\ t \\ y(t) = h + v_0 \sin\alpha\ t - \frac{1}{2} g t^2 \end{cases}$$

$\exists\ t$ tale che $\quad d = v_0 \cos\alpha\ t$

$$h_c = h + v_0 \sin\alpha\ t - \frac{1}{2} g t^2$$

$$\Rightarrow \quad t = \frac{d}{v_0 \cos\alpha} \qquad h_c = h + \cancel{v_0} \sin\alpha \frac{d}{\cancel{v_0}\cos\alpha} - \frac{1}{2} g \frac{d^2}{v_0^2 \cos^2\alpha}$$

$$(h_c - h) = d\, tg\alpha - \frac{g d^2}{2 v_0^2 \cos^2\alpha}$$

$$\frac{g d^2}{2 v_0^2 \cos^2\alpha} = d\, tg\alpha + h - h_c$$

$$\frac{2 v_0^2 \cos^2\alpha}{g d^2} = \frac{1}{d\, tg\alpha + h - h_c}$$

$$v_0^2 = \left( \frac{g d^2}{d\, tg\alpha + h - h_c} \right) \frac{1}{2\cos^2\alpha}$$

$$v_0 = \frac{d}{\cos\alpha} \sqrt{\frac{g}{2(d\, tg\alpha + h - h_c)}} = 10.67\ m/s$$

2)
$$\begin{cases} x(t) = v_0 \cos\alpha \, t \\ y(t) = v_0 \sin\alpha \, t - \frac{1}{2} g t^2 \end{cases}$$

$\alpha$ t.c. se $\begin{cases} d = v_0 t \cos\alpha \\ h \leq v_0 t \sin\alpha - \frac{1}{2} g t^2 \end{cases} \Rightarrow$

$$t = \frac{d}{v_0 \cos\alpha} \qquad h \leq d \, tg\alpha - \frac{1}{2} g \frac{d^2}{v_0^2 \cos^2\alpha}$$

$$h \leq d \, tg\alpha - \frac{1}{2} \cancel{g} \frac{d^{\cancel{2}}}{\frac{10}{9} \cancel{g} \cancel{d}} \frac{1}{\cos^2\alpha}$$

$$\frac{1}{\cos^2\alpha} = 1 + tg^2\alpha \Rightarrow$$

$$h \leq d \, tg\alpha - \frac{9}{20} d - \frac{9}{20} d \, tg^2\alpha$$

$d = 10h \Rightarrow$

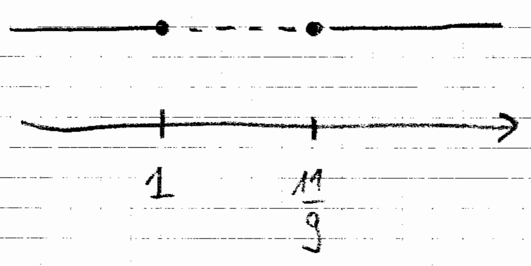
$$\frac{9}{2\cancel{0}} \cancel{10} h \, tg^2\alpha - 10h \, tg\alpha + \frac{9}{2\cancel{0}} \cancel{10} h + h \leq 0$$

$$\frac{9}{2} h \, tg^2\alpha - 10h \, tg\alpha + \frac{11}{2} h \leq 0$$

$$tg\alpha = \frac{5h \pm \sqrt{25h^2 - \frac{99}{4} h^2}}{\frac{9}{2} h} = \frac{5\cancel{h} \pm \frac{\cancel{h}}{2}}{\frac{9}{2} \cancel{h}} =$$

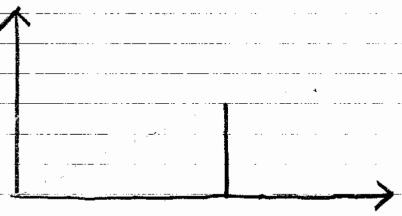
$= \nearrow \frac{11}{9}$

$\searrow 1$




$\Rightarrow$

$1 \le tg\alpha \le 11/9 \longrightarrow$



$\Rightarrow$ è $\frac{11}{9}$ la soluzione corretta!

Ora deve valere che:

$$v_0 t \cos\alpha \ge d + \overline{BC}$$

$$0 = v_0 \sin\alpha \, t - \frac{1}{2} g t^2 \Rightarrow$$

$$t = \frac{2 v_0 \sin\alpha}{g} \Rightarrow$$

$$\frac{2 v_0^2 \sin\alpha \cos\alpha}{g} \ge d + \overline{BC}$$

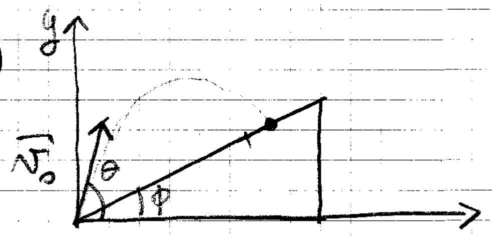
$$2 \, \frac{10}{9} \cancel{g} d \, \frac{\sin\alpha \cos\alpha}{\cancel{g}} - d \ge \overline{BC}$$

$$\overline{BC} \le \frac{20}{9} d \overbrace{\frac{1}{\sqrt{1+tg^2\alpha}}}^{\cos\alpha} \overbrace{\frac{tg\alpha}{\sqrt{1+tg^2\alpha}}}^{\sin\alpha} - d$$

$$= \frac{20}{9} d \, \frac{\frac{11}{9}}{1+\frac{121}{81}} - d = \frac{220}{\cancel{81}} \, \frac{d}{\frac{202}{\cancel{81}}} - d = \frac{110}{101} d - d$$

$$= \frac{9}{101} d = 445 \, m$$

3)




(a) $x(t) = v_0 \cos\theta \; t$

$y(t) = v_0 \sin\theta \; t - \frac{1}{2} g t^2$

$\exists\, t$ tale che $\frac{y}{x} = tg\,\phi \Rightarrow$

$$\frac{v_0 \sin\theta \,\cancel{t} - \frac{1}{2} g t^{\cancel{2}}}{v_0 \cos\theta \,\cancel{t}} = tg\,\phi$$

$$tg\,\theta - \frac{g t}{2 v_0 \cos\theta} = tg\,\phi \qquad tg\,\alpha - tg\,\beta = \frac{\sin(\alpha-\beta)}{\cos\alpha \cos\beta}$$

$$t = (tg\,\theta - tg\,\phi)\,\frac{2 v_0 \cos\theta}{g} = \frac{2 v_0 \cancel{\cos\theta}}{g}\,\frac{\sin(\theta-\phi)}{\cancel{\cos\theta}\cos\phi}$$

$$\Rightarrow t = 2 v_0 \frac{\sin(\theta-\phi)}{g \cos\phi}$$

$$d^2 = x^2 + y^2 = x^2 + x^2\, tg^2\phi = \frac{x^2}{\cos^2\phi} \Rightarrow$$

$$d^2 = v_0^2 \cos^2\theta \,\frac{4 v_0^2 \sin^2(\theta-\phi)}{g^2 \cos^2\phi}\,\frac{1}{\cos^2\phi}$$

$$d = \frac{2 v_0^2 \cos\theta \sin(\theta-\phi)}{g \cos^2\phi} \quad //$$

(b) $\frac{d\, f(\theta)}{d\theta} = 0 \Rightarrow$

$$\frac{2 v_0^2}{g \cos^2\phi}\left[(-\sin\theta)\sin(\theta-\phi) + \cos\theta \cos(\theta-\phi)\right] = 0$$

$$\cos\theta\,(\cos\theta\cos\phi + \sin\theta\sin\phi)$$
$$-\sin\theta\,(\sin\theta\cos\phi - \cos\theta\sin\phi) = 0$$

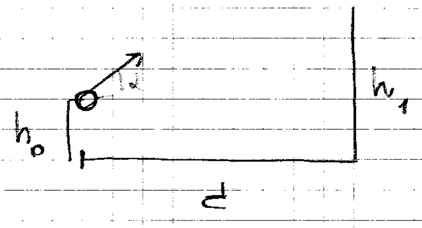
$$\cos^2\theta\cos\phi + \cos\theta\sin\theta\sin\phi$$
$$-\sin^2\theta\cos\phi + \cos\theta\sin\theta\sin\phi = 0$$

$$\cos 2\theta\cos\phi + \sin 2\theta\sin\phi = 0$$

$$\cos(2\theta - \phi) = 0 \Rightarrow$$

$$2\theta - \varphi = \frac{\pi}{2} \Rightarrow \boxed{\theta = \frac{\pi}{4} + \frac{\varphi}{2}}$$

4)




$h_0 = 1$ m

$h_1 = 21$ m

$d = 130$ m

$\alpha = 35°$

(a) $x(t) = v_0 t \cos\alpha$

$y(t) = h_0 + v_0 t \sin\alpha - \frac{1}{2} g t^2$

$$t = \frac{d}{v_0 \cos\alpha}$$

$$h_1 = h_0 + \cancel{v_0} \sin\alpha \frac{d}{\cancel{v_0}\cos\alpha} - \frac{1}{2} g \frac{d^2}{v_0^2 \cos^2\alpha}$$

$$-h_1 + h_0 + d \operatorname{tg}\alpha = \frac{1}{2} \frac{g d^2}{v_0^2 \cos^2\alpha}$$

$$\frac{2 v_0^2 \cos^2\alpha}{g d^2} = \frac{1}{h_0 - h_1 + d \operatorname{tg}\alpha}$$

$$v_0 = \sqrt{\frac{g d^2}{2\cos^2\alpha (h_0 - h_1 + d \operatorname{tg}\alpha)}} = 41.68 \text{ m/s}$$

(b) $t = \dfrac{d}{v_0 \cos\alpha} = 3.81$ s

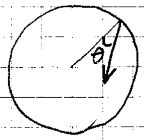
(c) $v_x = v_0 \cos\alpha = 34.14$ m/s

$v_y = v_0 \sin\alpha - g \dfrac{d}{v_0 \cos\alpha} = -13.47$ m/s

$\Rightarrow \vec{v} = (34.14\ \hat{\imath} - 13.5\ \hat{\jmath})$ m/s

$v = 36.70$ m/s $= \sqrt{v_x^2 + v_y^2}$

5)



$R = 2.5$ m/s

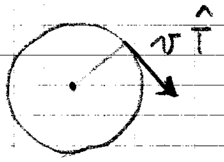
$a = 15$ m/s$^2$

$\theta = 30°$

(a) $a_\perp = a\cos\theta = 13$ m/s$^2$

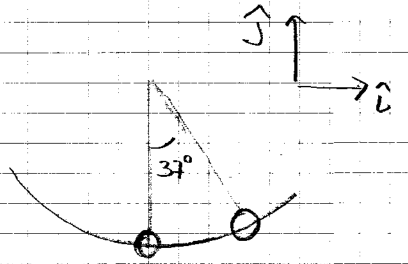
(b) $a_\parallel = a\sin\theta = 7.5$ m/s$^2$

(c) $\vec{v} = v\hat{T}$



$$a_\perp = \frac{v^2}{R} \Rightarrow v = \sqrt{a_\perp R} = 5.7 \text{ m/s}$$

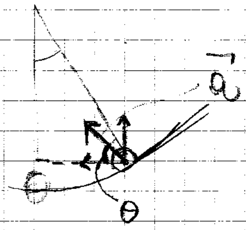
6)



$\vec{a} = (-22.5\,\hat{\imath} + 20.2\,\hat{\jmath})$ m/s$^2$

$R = 1.5$ m

(a) Disegnamo il vettore $\vec{a}$



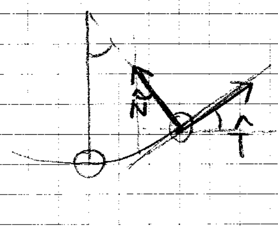
Calcolo $\theta$:

$$\tan\theta = \frac{a_y}{a_x} = \frac{20.2}{22.5} = 0.898$$

$$\Rightarrow \theta = 41.92° < \underbrace{90° - 37° = 53°}$$

D'altra parte

$$\vec{a} = \dot{v}\,\hat{T} + \frac{v^2}{R}\,\hat{N}$$



$$\begin{cases} \hat{T} = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath} \\ \hat{N} = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath} \end{cases} \Rightarrow$$

$$\vec{a} = \dot{v}\,(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) + \frac{v^2}{R}\,(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$$

$$= \left(\dot{v}\cos\theta - \frac{v^2}{R}\sin\theta\right)\hat{i} + \left(\dot{v}\sin\theta + \frac{v^2}{R}\cos\theta\right)\hat{j}$$

$$= (-22.5\,\hat{i} + 20.2\,\hat{j})\ m/s^2$$

$$\Rightarrow \quad \dot{v}\cos\theta - \frac{v^2}{R}\sin\theta = -22.5\ m/s^2 \equiv -a_x$$

$$\dot{v}\sin\theta + \frac{v^2}{R}\cos\theta = 20.2\ m/s^2 \equiv a_y$$

Dalla seconda eqn. $\dot{v} = \frac{a_y}{\sin\theta} - \frac{v^2}{R}\frac{\cos\theta}{\sin\theta} \Rightarrow$

$$\Rightarrow \quad a_y\frac{\cos\theta}{\sin\theta} - \frac{v^2}{R}\left(\frac{\cos^2\theta}{\sin\theta} + \sin\theta\right) = -a_x$$

$$\Rightarrow \quad \frac{v^2}{R} = \frac{a_y\cos\theta + a_x\sin\theta}{\cancel{\sin\theta}}\,\cancel{\sin\theta} = 29.67\ m/s^2$$

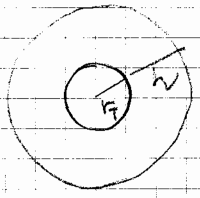
$$\Rightarrow \quad v = 6.67\ m/s$$

$$\vec{v} = v\cos\theta\,\hat{i} + v\sin\theta\,\hat{j} =$$

$$= (5.33\,\hat{i} + 4.01\,\hat{j})\ m/s$$

(b) $a_\perp = \frac{v^2}{R} = 29.67\ m/s^2$

7)




$r_T = 6400$ km $= 6.4 \times 10^6$ m

$h = 200$ km $= 2 \times 10^5$ m

$T = 88.2$ m $= 5292$ sec

(a) $a_\perp = \dfrac{v^2}{R} = \omega^2 R$

$\omega = \dfrac{2\pi}{T} \Rightarrow a_\perp = \dfrac{4\pi^2}{T^2}(r_T + h) = 9.30\ m/s^2$

(b) $v = \omega R = \dfrac{2\pi(r_T + h)}{T} = 7836\ m/s = \dfrac{7836}{1000}\ \dfrac{km}{s}$

$= 7.84\ \dfrac{km}{s} = 7.84\ \dfrac{km}{1h} \cdot 3600$ se

$= 28210$ km/h !