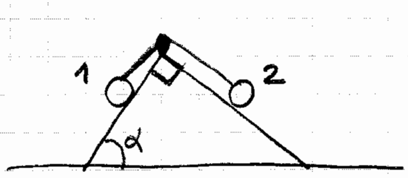
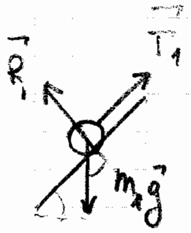
Soluzioni

1)



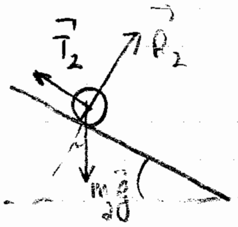
$\alpha = 60°$

(a) Forze su 1



$$m_1 \vec{a}_1 = \vec{R}_1 + \vec{T}_1 + m_1 \vec{g}$$

Forze su 2



$$m_2 \vec{a}_2 = \vec{R}_2 + \vec{T}_2 + m_2 \vec{g}$$

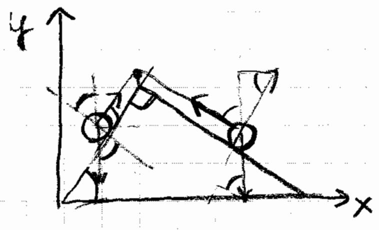
$$\begin{cases} T_1 = m_1 g \sin\alpha \\ R_1 = m_1 g \cos\alpha \end{cases}$$

$$\begin{cases} T_2 = m_2 g \sin\left(\frac{\pi}{2} - \alpha\right) = m_2 g \cos\alpha \\ R_2 = m_2 g \cos\left(\frac{\pi}{2} - \alpha\right) = m_2 g \sin\alpha \end{cases}$$

$\Rightarrow$ poiché $T_1 = T_2$ $\quad m_1 g \sin\alpha = m_2 g \cos\alpha$

$$\boxed{\frac{m_1}{m_2} = \frac{1}{\operatorname{tg}\alpha} = 0{,}577}$$

(b) m_1 e m_2 dati $\Rightarrow$



$$\begin{cases} m_1 a_{1y} = -m_1 g + R_1 \cos\alpha + T_1 \sin\alpha \\ m_1 a_{1x} = -R_1 \sin\alpha + T_1 \cos\alpha \end{cases}$$

$$\begin{cases} m_2 a_{2y} = -m_2 g + R_2 \sin\alpha + T_2 \cos\alpha \\ m_2 a_{2x} = R_2 \cos\alpha - T_2 \sin\alpha \end{cases}$$

Eq. complicate !! Ulteriori condizioni

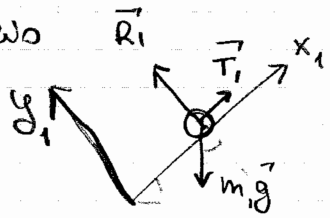
filo inest. $\longrightarrow$ $T_1 = T_2$

moto sul piano $\longrightarrow$ $\dfrac{a_{1y}}{a_{1x}} = tg\,\alpha$, $\dfrac{a_{2x}}{a_{2y}} = -tg\,\alpha$

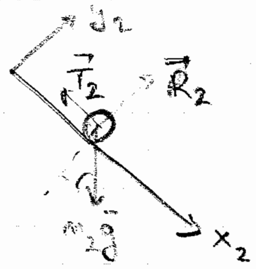
filo inest. $\longrightarrow$ $|a_1| = |a_2|$

8 incognite e 8 eqn. ... ma brutte !

Rifaccio



$$\begin{cases} m_1 a_{1,x} = -m_1 g \sin\alpha + T_1 \\ 0 = R_1 - m_1 g \cos\alpha \end{cases}$$



$$\begin{cases} m_2 a_{2x} = m_2 g \cos\alpha - T_2 \\ 0 = R_2 - m_2 g \sin\alpha \end{cases}$$

$T_1 = T_2$

$|a_{1x}| = |a_{2x}|$ $\Rightarrow$ $a_{1x} \equiv a$

$$\begin{cases} m_1 a = T - m_1 g \sin\alpha \\ m_2 a = m_2 g \cos\alpha - T \end{cases} \Rightarrow (m_1 + m_2) a = (m_2 \cos\alpha - m_1 \sin\alpha) g$$

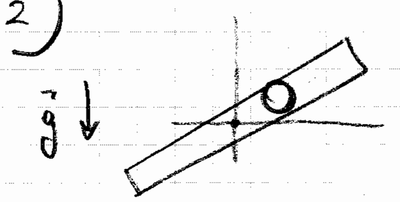
$$\Downarrow$$

$$\begin{cases} R_1 = m_1 g \cos\alpha \\ R_2 = m_2 g \sin\alpha \end{cases} \qquad a = \frac{m_2 \cos\alpha - m_1 \sin\alpha}{m_1 + m_2}\, g$$

$$T = m_1 a + m_1 g \sin\alpha = g\, \frac{m_1 m_2 \cos\alpha - \cancel{m_1^2 \sin\alpha} + \cancel{m_1^2 \sin\alpha} + m_1 m_2 \sin\alpha}{m_1 + m_2}$$

$$= m_1 m_2\, g\, \frac{\cos\alpha + \sin\alpha}{m_1 + m_2} . \Rightarrow \begin{cases} a = 0.438\ m/s^2 \\ T = 8.92\ N \end{cases}$$

2)



m

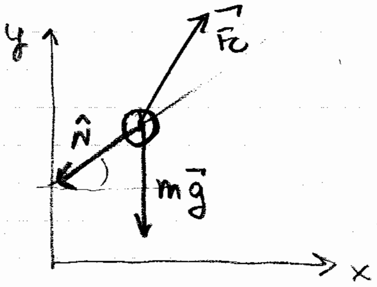
ω

$$\vec{F} = m\,\vec{a}$$

$$\vec{a} = \frac{dv}{dt}\,\hat{T} + \frac{v^2}{R}\,\hat{N}$$

$$\vec{v} = \omega R\,\hat{T} \quad \Rightarrow \quad \vec{a} = \frac{\omega^2 R^{\cancel{2}}}{\cancel{R}}\,\hat{N}$$

$$\vec{F} = m\,\omega^2 R\,\hat{N}$$



$$\vec{F_c} + m\vec{g} = m\,\omega^2 R\,\hat{N}$$

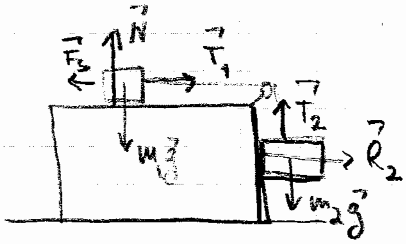
$$F_{c_x} = -\,m\,\omega^2 R\cos\theta$$

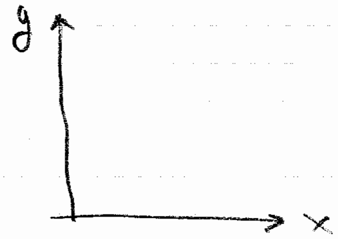
$$F_{c_y} - mg = -\,m\,\omega^2 R\sin\theta$$

$$\Rightarrow \quad F_{c_x} = -\,m\,\omega^2 R\cos\theta \qquad = -1.5\ N$$

$$F_{c_y} = mg - m\,\omega^2 R\sin\theta \qquad = 4.35\ N$$

3)





(a)

$$\begin{cases} m_1\vec{g} + \vec{T_1} + \vec{N} + \vec{F_s} = m_1\vec{a}_1 = 0 \\ m_2\vec{g} + \vec{T_2} + \vec{R_2} = m_2\vec{a}_2 = 0 \end{cases}$$

$$\begin{cases} -m_1 g + N = 0 \\ T_1 - F_S = 0 \end{cases} \qquad \begin{cases} -m_2 g + T_2 = 0 \\ R_2 = 0 \end{cases}$$

$$N = m_1 g \qquad T_2 = m_2 g$$

$$F_S = T_1$$

$$\Rightarrow \quad F_S = m_2 g \leq F_S^{max} = \mu_S N$$

$$\Rightarrow \quad m_2 \cancel{g} \leq \mu_S m_1 \cancel{g}$$

$$\mu_S \geq \frac{m_2}{m_1} = 2$$

(b)
$$\begin{cases} m_1 \vec{g} + \vec{T}_1 + \vec{N} = m_1 \vec{a}_1 \\ m_2 \vec{g} + \vec{T}_2 + \vec{R}_2 = m_2 \vec{a}_2 \\ M\vec{g} + \vec{R} + \vec{F} - \vec{N} - \vec{R}_2 + \vec{F}_c = M\vec{a} \end{cases}$$

forza della carrucola sul blocco

$$m_c \vec{a} = -\vec{F}_c - \vec{T}_1 - \vec{T}_2 \simeq 0 \quad -\vec{F}_c = \vec{T}_1 + \vec{T}_2$$

$$\begin{cases} -m_1 g + N = 0 \\ T_1 = m_1 a_{1x} \\ -m_2 g + T_2 = 0 \\ R_2 = m_2 a_{2x} \\ -Mg + R - N - T_2 = 0 \\ F - R_2 - T_1 = Ma \end{cases}$$

6 equ. e 9 incognite!

In più so che:
$$\begin{cases} T_1 = T_2 \\ a_{1x} = a_{2x} = a \end{cases}$$

$$N = m_1 g$$

$$\left.\begin{aligned} m_1 a &= T \\ T &= m_2 g \end{aligned}\right\} \rightarrow \quad a = \frac{m_2}{m_1} g$$

$$R_2 = m_2 a$$

$$R = Mg + N + T$$

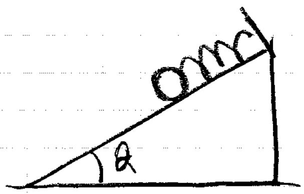
$$F = Ma + T + R_2$$

$$F = Ma + m_2 g + m_2 a \Rightarrow$$

$$F = (M + m_2) \frac{m_2}{m_1} g + m_2 g$$

$$= \frac{m_2 g}{m_1} (M + m_1 + m_2) = 137.2\ N$$

4)

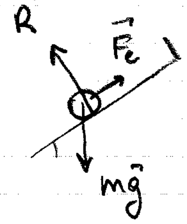


k, l_0

m

θ

(a)



$$m\vec{g} + \vec{F_e} + \vec{R} = 0$$

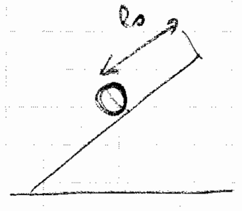
$$\Rightarrow mg \sin\theta - F_e = 0$$

$$F_e = mg \sin\theta$$

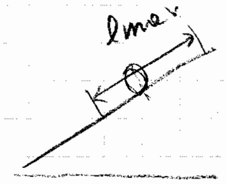
$$k(l - l_0) = mg \sin\theta$$

$$x_{eq} = l_{eq} = l_0 + \frac{mg}{k} \sin\theta = 0.395\ m$$

(b)



$t=0$



$$E_{in} = -mg\, l_0 \sin\theta$$

$$E_{fin} = \frac{1}{2} k (l_m - l_0)^2 - mg\, l_m \sin\theta$$

$$\Rightarrow \quad E_{in} = E_{fin}$$

$$\downarrow$$

$$-mg\, l_0 \sin\theta = \frac{1}{2} k (l_m - l_0)^2 - mg\, l_m \sin\theta$$

$$\frac{1}{2} k (l_m - l_0)^2 - mg (l_m - l_0) \sin\theta = 0$$

Soluzioni $\quad l_m = l_0$

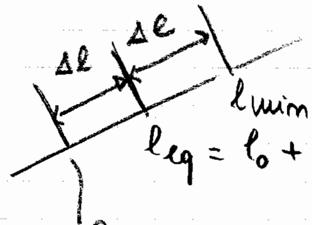
$$\frac{1}{2} k (l_m - l_0) = mg \sin\theta$$

$$l_m = l_0 + 2 \frac{mg}{k} \sin\theta$$

$\Rightarrow \quad l_0$ è l_{min}

$l_0 + 2\frac{mg}{k}\sin\theta$ è l_{max}

$\Rightarrow$



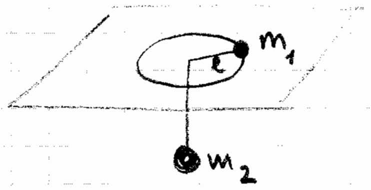
$$\Delta l = \frac{mg}{k} \sin\theta$$

$$l_{min} = l_{eq} - \Delta l$$

$$l_{eq} = l_0 + \frac{mg}{k} \sin\theta$$

$$l_{max} = l_{eq} + \Delta l = 0.590 \text{ m}$$

5)

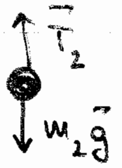


m_1

m_2

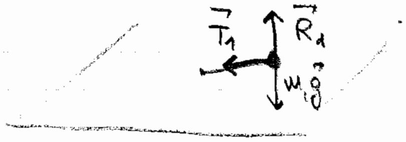
ℓ

(a) Forze su 2



Poiché 2 non cade $\quad T_2 = m_2 g$

Forze su 1



$$\vec{T}_1 + \vec{R}_1 + m_1\vec{g} = m_1\vec{a}_1$$

Il moto è circ. unif. $\Rightarrow \quad \vec{a}_1 = \hat{N}\,\dfrac{v_1^2}{\ell}$

$$\Rightarrow \quad R_1 - m_1 g = 0$$

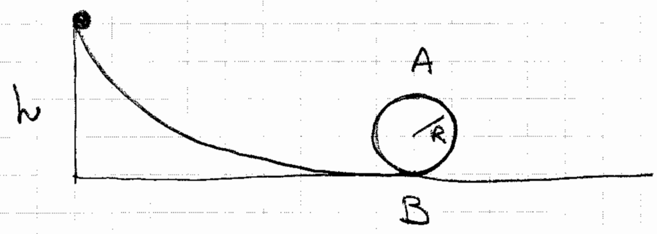
$$T_1 = m_1\frac{v_1^2}{\ell}$$

$$T_1 = T_2 \Rightarrow \quad m_2 g = m_1\frac{v_1^2}{\ell}$$

$$\Rightarrow v_1 = \sqrt{\frac{m_2}{m_1}\,g\,\ell} = 3.5 \text{ m/s}$$

(b) $\quad T = m_2 g = 9.8 \text{ N}$

6)



m

R

(a) Cons. dell' energia $\Rightarrow$

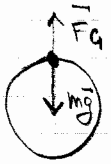
$$E_{in} = E_A$$

$$\cancel{m} g h = \frac{1}{2} \cancel{m} v_A^2 + \cancel{m} g 2R$$

$$v_A = \sqrt{2g(h-2R)} = \sqrt{3gR} = 3.84 \text{ m/s}$$

(b)

$$a_N = \frac{v_A^2}{R}$$

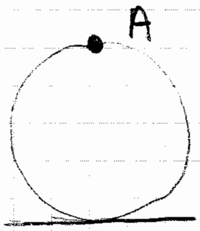


$$\Rightarrow -m\frac{v_A^2}{R} = -mg + F_G$$

$$\Rightarrow F_G = -m\frac{3g\cancel{R}}{\cancel{R}} + mg = -2mg$$

$\Rightarrow F_G$ è diretta in basso ! $\qquad F_G = 9.8$ N

(c)



$$m a_c = mg - F_G$$

$$m\frac{v^2}{R} = mg - F_G \leftarrow \text{vera sempre}$$

Per fare il giro, occorre che sulla pallina ci sia una forza opportuna per farle proseguire il moto circolare

Tale forza è esercitata o dalle guide e peso, o al limite dal solo peso $\Rightarrow$

$$F_q = mg - m\frac{v^2}{R} \geq 0$$

Dalla cons. dell'energia (v.d. punto (e))

$$v = \sqrt{2g\,(h-2R)} \Rightarrow$$

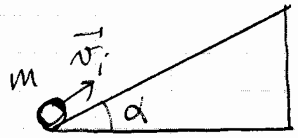
$$mg - m\,\frac{2g}{R}\,(h-2R) \geq 0$$

$$\cancel{mg} - 2mg\,\frac{h}{R} + \overset{5}{\cancel{4}}\,mg \geq 0$$

$$h \leq \frac{5\,mg}{2\,mg}\,R = \frac{5}{2}\,R$$

$$\Rightarrow h_{min} = \frac{5}{2}R = 1{,}25\ m$$

7)



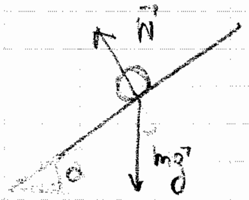
Teorema delle forze vive (o cons. dell'en. mec.)

$$E_{c\,fin} - E_{c\,in} = \sum \mathcal{L}_i$$

$$0 - \frac{1}{2}\,m\,v_i^2 = \mathcal{L}_{peso} + \mathcal{L}_{attrito}$$

$$-\frac{1}{2}\,m\,v_i^2 = -mg\,h_{max} - \mu_D N\,\frac{h_{max}}{\sin\alpha}$$

$N = m g \cos\alpha$



$\Rightarrow$

$$\frac{1}{2} \cancel{m} v_i^2 = \cancel{m} g h_{max} + \mu_D \cancel{m} g \frac{\cos\alpha}{\sin\alpha} h_{max}$$

$$\Rightarrow \quad h_{max} = \frac{v_i^2}{2g\left(1 + \frac{\mu_D}{tg\alpha}\right)} =$$