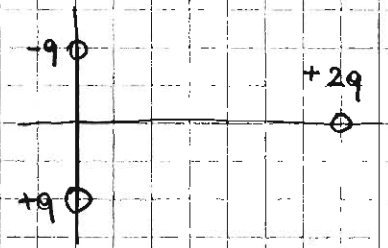
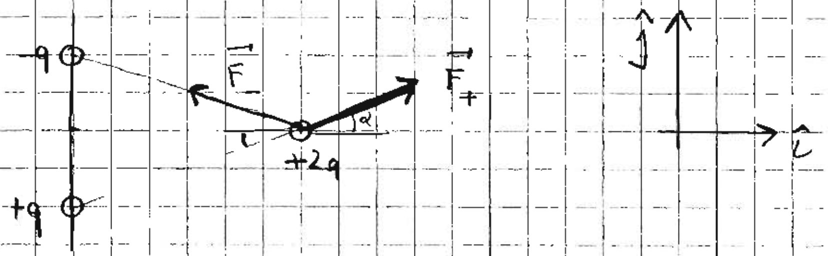
Soluzioni

1)



(a)



$$\vec{F}_+ = k_e \frac{2q^2}{(a^2+d^2)} (\cos\alpha \, \hat{i} + \sin\alpha \, \hat{j})$$

$$= k_e \frac{2q^2}{(a^2+d^2)} \left(\frac{d}{\sqrt{a^2+d^2}} \hat{i} + \frac{a}{\sqrt{a^2+d^2}} \hat{j} \right)$$

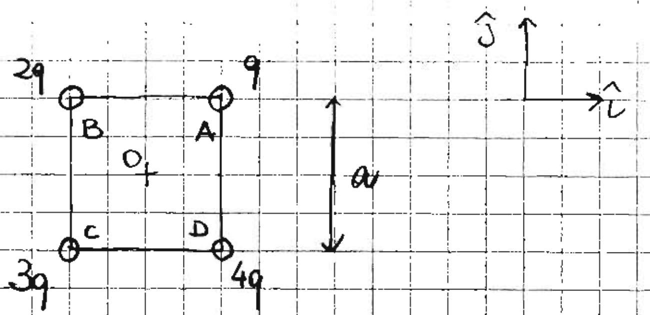
$$\vec{F}_- = k_e \frac{2q^2}{(a^2+d^2)} (-\cos\alpha \, \hat{i} + \sin\alpha \, \hat{j})$$

$$= k_e \frac{2q^2}{(a^2+d^2)} \left(\frac{-d}{\sqrt{a^2+d^2}} \hat{i} + \frac{a}{\sqrt{a^2+d^2}} \hat{j} \right)$$

$$\vec{F} = \vec{F}_+ + \vec{F}_- = k_e \frac{2q^2}{(a^2+d^2)} \frac{2a}{\sqrt{a^2+d^2}} \hat{j}$$

(b) $\vec{F} = m\vec{a} \Rightarrow \vec{a} = k_e \dfrac{2q^2}{m} \dfrac{2a}{(a^2+d^2)^{3/2}} \hat{j}$

2)



(a) $$\vec{E}(A) = \vec{E}_{2q} + \vec{E}_{3q} + \vec{E}_{4q}$$

$$= k_e \frac{2q}{a^2}\hat{\imath} + k_e \frac{3q}{2a^2}\left(\frac{\sqrt{2}}{2}\hat{\imath} + \frac{\sqrt{2}}{2}\hat{\jmath}\right)$$

$$+ k_e \frac{4q}{a^2}\hat{\jmath}$$

$$= k_e \frac{q}{a^2}\left[\left(2 + \frac{3\sqrt{2}}{4}\right)\hat{\imath} + \left(\frac{3\sqrt{2}}{4} + 4\right)\hat{\jmath}\right]$$

$$= k_e \frac{q}{a^2}\left(\frac{8+3\sqrt{2}}{4}\hat{\imath} + \frac{16+3\sqrt{2}}{4}\hat{\jmath}\right)$$

(b) $$\vec{F}_q = q\vec{E}(A) = k_e \frac{q^2}{a^2}\left(\frac{8+3\sqrt{2}}{4}\hat{\imath} + \frac{16+3\sqrt{2}}{4}\hat{\jmath}\right)$$

(c) $$\vec{E}(O) = \vec{E}_q + \vec{E}_{2q} + \vec{E}_{3q} + \vec{E}_{4q} =$$

$$= k_e \frac{q}{\frac{a^2}{2}}\left(-\frac{\sqrt{2}}{2}\hat{\imath} - \frac{\sqrt{2}}{2}\hat{\jmath}\right)$$

$$+ k_e \frac{2q}{\frac{a^2}{2}}\left(\frac{\sqrt{2}}{2}\hat{\imath} - \frac{\sqrt{2}}{2}\hat{\jmath}\right)$$

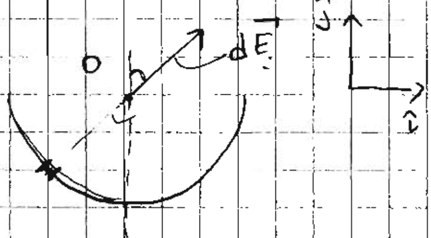
$$+ k_e \frac{3q}{\frac{a^2}{2}}\left(\frac{\sqrt{2}}{2}\hat{\imath} + \frac{\sqrt{2}}{2}\hat{\jmath}\right)$$

$$+ k_e \frac{4q}{\frac{a^2}{2}}\left(-\frac{\sqrt{2}}{2}\hat{\imath} + \frac{\sqrt{2}}{2}\hat{\jmath}\right) =$$

$$= k_e \frac{2q}{a^2} \left\{ \hat{\imath} \left[-\frac{\sqrt{2}}{2} + \frac{2\sqrt{2}}{2} + \frac{3}{2}\sqrt{2} - 4\frac{\sqrt{2}}{2} \right] + \hat{\jmath} \left[-\frac{\sqrt{2}}{2} - 2\frac{\sqrt{2}}{2} + \frac{3}{2}\sqrt{2} + 4\frac{\sqrt{2}}{2} \right] \right\}$$

$$= k_e \frac{2q}{a^2} \cancelto{2}{4} \frac{\sqrt{2}}{\cancel{2}} \hat{\jmath} = k_e \, 4\sqrt{2} \, \frac{q}{a^2} \hat{\jmath}$$

3)



$$\pi r = l$$

$$\lambda = \frac{Q}{\pi r} = \frac{dq}{dl}$$

$$d\vec{E} = k_e \frac{dq}{r^2} \left(\sin\theta \, \hat{\imath} + \cos\theta \, \hat{\jmath} \right)$$

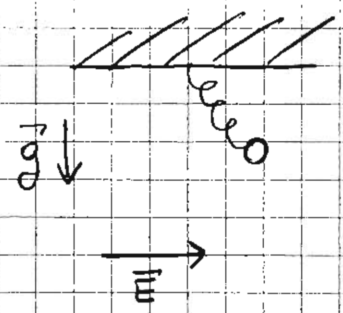
$$dq = \lambda \, dl = \lambda \, r \, d\theta \quad \Rightarrow$$

$$d\vec{E} = k_e \frac{\lambda r \, d\theta}{r^2} \left(\sin\theta \, \hat{\imath} + \cos\theta \, \hat{\jmath} \right)$$

$$\Rightarrow \vec{E} = k_e \frac{\lambda}{r} \int_{-\pi/2}^{+\pi/2} \left[\sin\theta \, \hat{\imath} + \cos\theta \, \hat{\jmath} \right] d\theta$$

$$= k_e \frac{\lambda}{r} \left(-\cos\theta \, \hat{\imath} + \sin\theta \, \hat{\jmath} \right) \Big|_{-\pi/2}^{\pi/2}$$

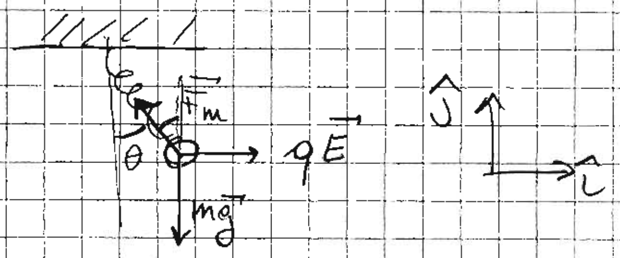
$$= k_e \frac{\lambda}{r} \hat{\jmath} \, 2 = 2 k_e \frac{\lambda}{r} \hat{\jmath}$$

4) 

k, l_0

m

Diagramma delle forze su m



Equilibrio $\Rightarrow$ $\vec{F}_m + m\vec{g} + q\vec{E} = 0$

$$\Rightarrow \begin{cases} -F_m \sin\theta + qE = 0 \\ F_m \cos\theta - mg = 0 \end{cases}$$

$$F_m = k(l_{eq} - l_0) \Rightarrow$$

$$k(l_{eq} - l_0)\sin\theta = qE$$

$$k(l_{eq} - l_0)\cos\theta = mg$$

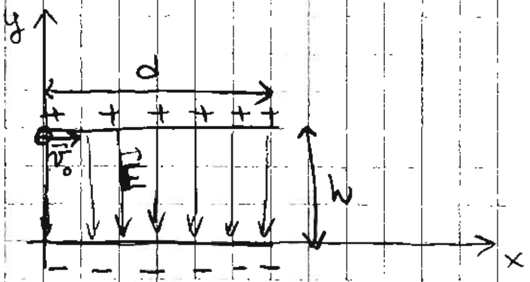
$$\operatorname{tg}\theta_{eq} = \frac{qE}{mg}$$

$$k(l_{eq} - l_0) = \frac{mg}{\cos\theta_{eq}} = mg\sqrt{1 + \operatorname{tg}^2\theta_{eq}}$$

$$= mg\sqrt{1 + \frac{q^2E^2}{m^2g^2}} \Rightarrow$$

$$l_{eq} = l_0 + \frac{mg}{k}\sqrt{1+\frac{q^2E^2}{m^2g^2}}$$

5)



$m,\ v_0,\ q$
$Q,\ C,\ h,\ d$

$$\Delta V = \frac{Q}{C} = Eh \Rightarrow E = \frac{Q}{Ch} \Rightarrow$$

$$m\vec{a} = q\vec{E} \Rightarrow$$

$$\vec{a} = -\frac{q}{m}\frac{Q}{Ch}\,\hat{y}$$

$$\begin{cases} x(t) = v_0 t \\ y(t) = h - \frac{1}{2}at^2 \end{cases}$$

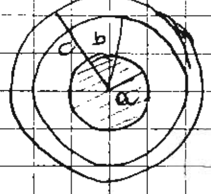
Se $\underbrace{0 = h - \frac{1}{2}at^2 \quad , \quad v_0 t \geq d}$

$$= h = \frac{1}{2}at^2 \geq \frac{1}{2}a\frac{d^2}{v_0^2} \Rightarrow$$

$$v_0^2 \geq \sqrt{\frac{a\,d^2}{2h}}$$

$$v_{0\,min} = \sqrt{\frac{qQ}{mCh}\frac{d^2}{2h}} = \frac{d}{h}\sqrt{\frac{qQ}{2mC}}$$

6)



$a,\ Q>0$

$b,\ c$

a) Applichiamo ripetutamente il teorema di Gauss, sapendo che $\vec{E}$ deve essere radiale, perché il problema ha simmetria sferica $\Rightarrow$

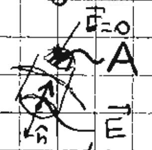
$r<a \qquad 4\pi r^2 E = \frac{1}{\varepsilon_0} q_{in} = \frac{1}{\varepsilon_0} \frac{Q}{\frac{4}{\cancel{3}}\cancel{\pi} a^3} \frac{4}{\cancel{3}}\cancel{\pi} r^3$

$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0} Q \frac{r}{a^3}$

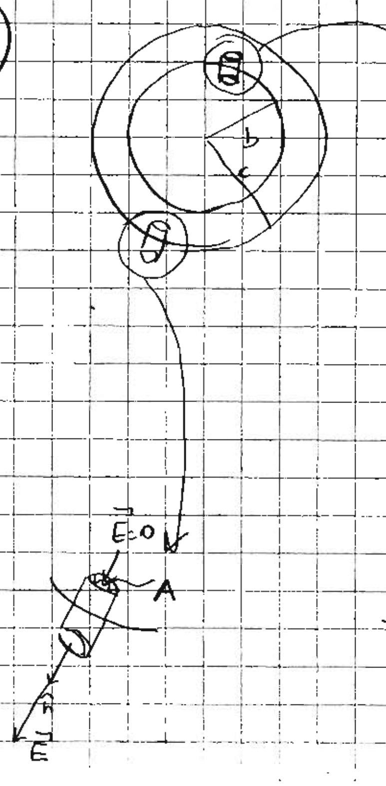
$a<r<b \qquad 4\pi r^2 E = \frac{1}{\varepsilon_0} Q \quad \Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$

$b<r<c$ ho un conduttore $\Rightarrow E = 0$

$r>c \qquad 4\pi r^2 E = \frac{1}{\varepsilon_0} Q \quad \Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$



b)



$-E_{r=b^-} \cancel{A} = \frac{1}{\varepsilon_0} \sigma_{in} \cancel{A} \quad \Rightarrow$

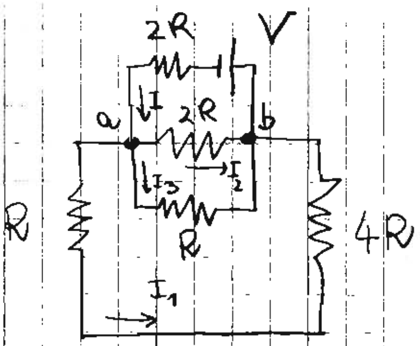
$-\frac{1}{4\pi\varepsilon_0} \frac{Q}{b^2} = \frac{1}{\varepsilon_0} \sigma_{in} \quad \Rightarrow$

$\sigma_{in} = \frac{-Q}{4\pi b^2}$

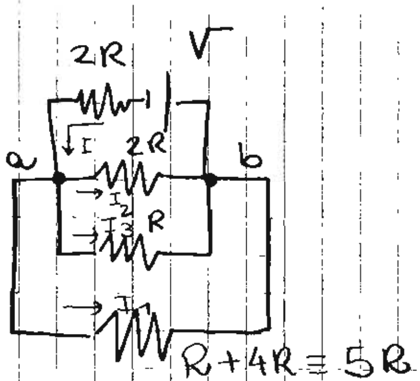
$+E_{r=c^+} \cancel{A} = \frac{1}{\varepsilon_0} \sigma_{out} \cancel{A} \quad \Rightarrow$

$\sigma_{out} = \frac{1}{4\pi\varepsilon_0} \varepsilon_0 \frac{Q}{c^2} = \frac{Q}{4\pi c^2}$

7)



$\equiv$

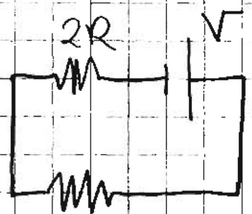


a) $I = I_1 + I_2 + I_3$ (nodo a o b)

$\Delta V_{ab} = 2R\, I_2 = R\, I_3 \Rightarrow I_3 = 2 I_2$

$\Delta V_{ab} = R\, I_3 = 5R\, I_1 \Rightarrow I_3 = 5 I_1$ $\Rightarrow I_2 = \frac{5}{2} I_1$

Calcolo I: il circuito è equivalente a:



$$\left(\frac{1}{2R} + \frac{1}{R} + \frac{1}{5R}\right)^{-1} = \left(\frac{5+10+2}{10R}\right)^{-1} = \frac{10}{17} R$$

$$\Rightarrow I\left(2R + \frac{10}{17} R\right) = V$$

$$\Rightarrow I = \frac{V}{\frac{44}{17} R} = \frac{17}{44} \frac{V}{R} = 1.932 \text{ A}$$

$$I = I_1 + \frac{5}{2} I_1 + 5 I_1 = \frac{2+5+10}{2} I_1 = \frac{17}{2} I_1$$

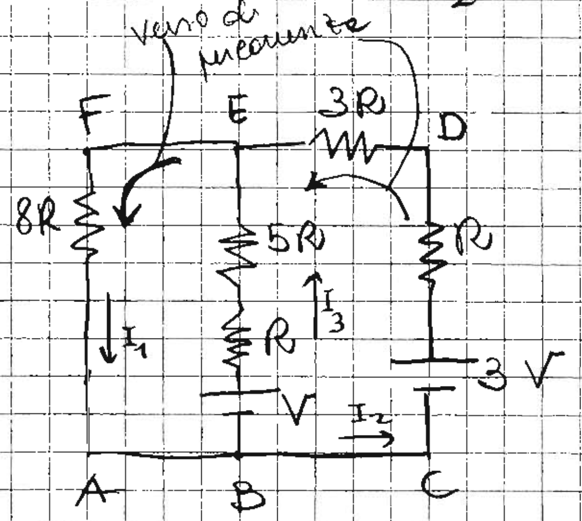
$$\Rightarrow I_1 = \frac{17}{44} \frac{V}{R} \frac{2}{17} = \frac{1}{22} \frac{V}{R} = 0.227 \text{ A}$$

$$I_2 = \frac{5}{44} \frac{V}{R} = 0.568 \text{ A}$$

$$I_3 = 5/22\ V/R = 1.136 \text{ A}$$

b) $\Delta V_{ab} = 5RI_1 = \frac{5}{22} V = 5.68 V$

8)



$R = 1\Omega$

$V = 4 V$

1ª legge di Kirchhoff $\Rightarrow$ $I_1 = I_2 + I_3$

2ª legge applicata ad ABEF

$$-8RI_1 + V - RI_3 - 5RI_3 = 0 \qquad (1)$$

2ª legge applicata a BCDE

$$3V - I_2R - 3I_2R + 5RI_3 + RI_3 - V = 0 \qquad (2)$$

(1) + (2) $\Rightarrow$

$$\begin{cases} V - 8RI_1 - 6RI_3 = 0 \\ 2V - 4RI_2 + 6RI_3 = 0 \end{cases}$$

(Nota: la somma delle due mi dà l'eq. x la maglia ACDF)

$I_1 = I_2 + I_3 \Rightarrow$

$$V - 8R(I_2 + I_3) - 6RI_3 = 0$$

$$\begin{cases} V - 8RI_2 - 14RI_3 = 0 \\ 2V - 4RI_2 + 6RI_3 = 0 \end{cases} \xrightarrow{(\times 2)} \begin{cases} V - 8RI_2 - 14RI_3 = 0 \\ 4V - 8RI_2 + 12RI_3 = 0 \end{cases}$$

Faccio la differenza delle equ. $\Rightarrow$

$$-3V - 26 R I_3 = 0 \quad \Rightarrow \quad I_3 = -\frac{3}{26}\frac{V}{R} = 0.462A$$

Vuol dire che I_3 è nel verso opposto a quello scelto!

$$8 R I_2 = +V - 14 \cancel{R} \overbrace{\left(-\frac{3}{26} \frac{V}{\cancel{R}} \right)}^{I_3} =$$

$$= +V + \frac{42}{26} V = + \frac{\cancel{68}^{\,34}}{\cancel{26}_{\,13}} V = + \frac{34}{13} V$$

$$\Rightarrow I_2 = + \frac{17}{52} \frac{V}{R} = 1.308 \text{ A}$$

$$I_1 = \left(\frac{17}{52} - \frac{3}{26} \right) \frac{V}{R} = \frac{17-6}{52} \frac{V}{R} = \frac{11}{52} \frac{V}{R} = 0.846 \text{ A}$$