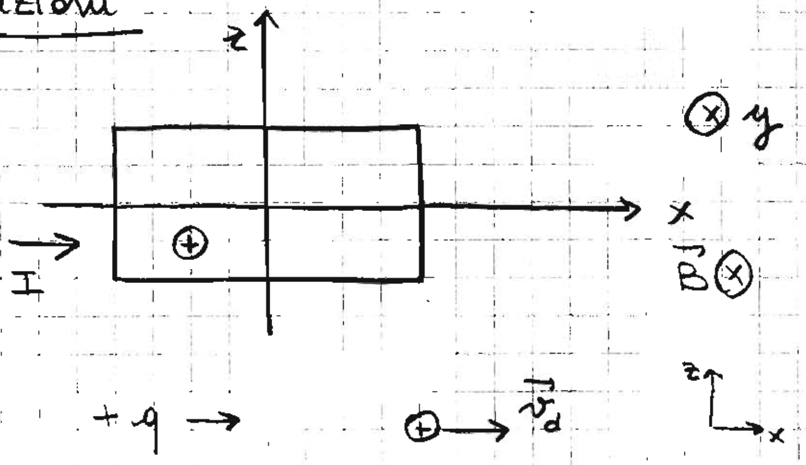
Soluzioni

1)



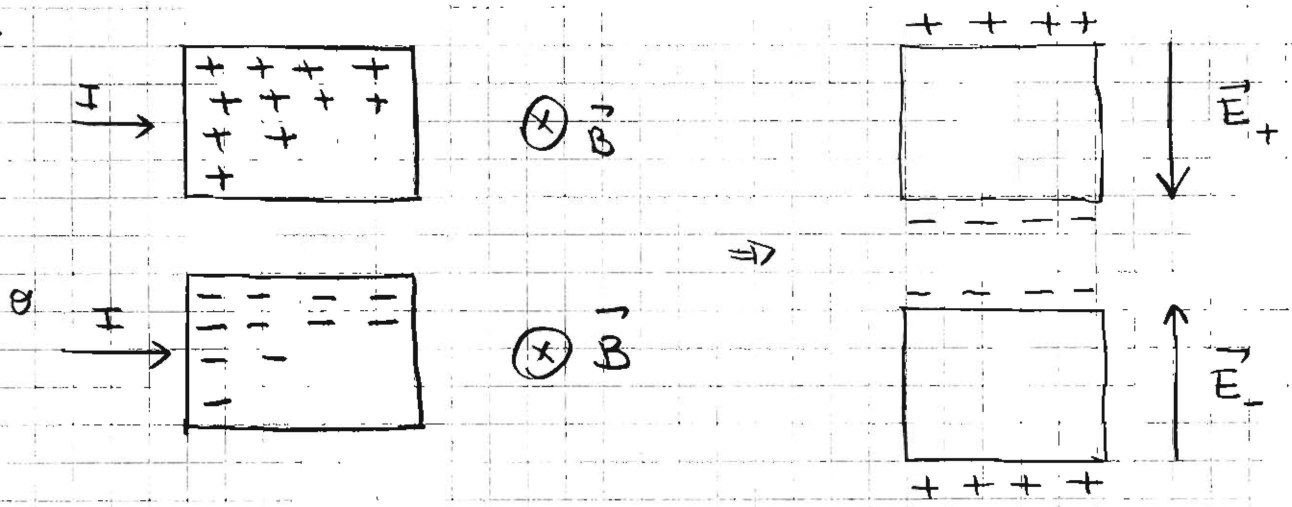
(a) $+q \rightarrow$ $\oplus \xrightarrow{} \vec{v}_d$

$$\vec{F}_+ = +q\,\vec{v}_d \times \vec{B} = q\,v_d\,B\,\hat{z}$$

$-q \rightarrow$ $\vec{v}_d \leftarrow \ominus$

$$\vec{F}_- = -q\,\vec{v}_d \times \vec{B} = -q\,v_d\,B\,(-\hat{z}) = q\,v_d\,B\,\hat{z}$$

$\Rightarrow$



(b) $\vec{F}_{el} = +q\,\vec{E} = qE(-\hat{z})$ $\quad q>0$

$= -q\,\vec{E} = -qE(\hat{z})$ $\quad q<0$ $\Rightarrow$

all'equilibrio $\cancel{q}E = \cancel{q}v_d B$ $\qquad E = v_d B$

$$V_c - V_a = \Delta V = -\int_a^c \vec{E}\cdot d\vec{s} = -\int_a^c (\mp v_d B\,\hat{z}\cdot dz\,\hat{z})$$

$\Rightarrow \Delta V = +v_d B d \qquad q>0$

$\qquad = -v_d B d \qquad q<0$

(c) $N = \frac{\#\ \text{port carica}}{\text{Vol}}$?

$$\mathcal{E} \equiv |\Delta V|$$

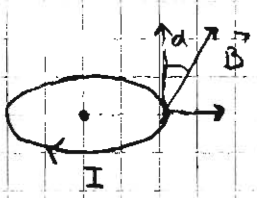
$$v_d = \frac{|\Delta V|}{Bd} = \frac{\mathcal{E}}{Bd}$$

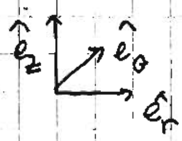
$$I = N \underbrace{s\, d\, v_d}_{\text{Vol/tempo}}\, q \longleftarrow (I = N A v_d q)$$

$$\Rightarrow \quad I = N s \cancel{d} \frac{\mathcal{E}}{B \cancel{d}} q \Rightarrow$$

$$N = \frac{B I}{\mathcal{E} s q} \quad //$$

2)





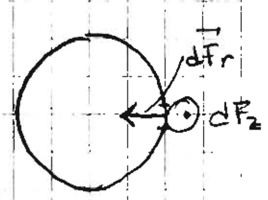
$$\vec{B} = B \sin\alpha\ \hat{e}_r + B \cos\alpha\ \hat{e}_z$$

$$d\vec{l} = -r\, d\theta\ \hat{e}_\theta$$

$$d\vec{F} = I\, d\vec{l} \times \vec{B} = -I\, r\, d\theta\, B \left[\sin\alpha\, \underbrace{(\hat{e}_\theta \times \hat{e}_r)}_{-\hat{e}_z} + \cos\alpha\, \underbrace{(\hat{e}_\theta \times \hat{e}_z)}_{+\hat{e}_r}\right]$$

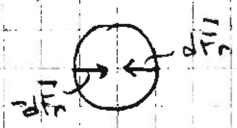
$$d\vec{F} = \underbrace{+ I r B \sin\alpha\, d\theta\ \hat{e}_z}_{d\vec{F}_z} \underbrace{- I r B \cos\alpha\, d\theta\ \hat{e}_r}_{d\vec{F}_r}$$

Visto dall'alto



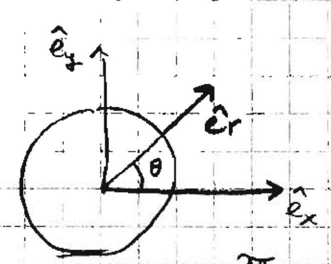
$\sum d\vec{F}_r$ su tutta la spira è $= 0$

Infatti



Matematicamente questo lo si vede anche così:

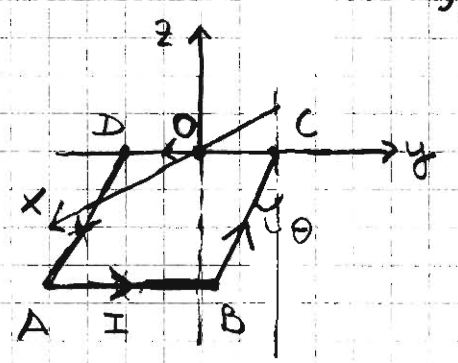
$$\hat{e}_r = \cos\theta\,\hat{e}_x + \sin\theta\,\hat{e}_y$$



$$\vec{F}_r \equiv \int d\vec{F}_r = -IrB\cos\alpha \underbrace{\left[\int_0^{2\pi}\cos\theta\, d\theta\,\hat{e}_x + \int_0^{2\pi}\sin\theta\, d\theta\,\hat{e}_y\right]}_{=0}$$

$$\Rightarrow \quad \vec{F} = IrB\sin\alpha\,\hat{e}_z \int_0^{2\pi} d\theta = 2\pi r IB\sin\alpha\,\hat{e}_z$$

3)



$$\vec{B} = B\hat{z}$$
$$\vec{g} = -g\hat{z}$$

$$\sum \vec{F}_{ext} = 0$$

$$\sum \vec{M}_{O,ext} = 0$$

$$\vec{F}_{ext} = \begin{cases} M\vec{g} & \text{peso} \\ \vec{R} & \text{reazione vincolare} \\ \vec{F}_{Magn} & \end{cases}$$

$$\vec{F}_{Magn} = \vec{F}_{AB} + \vec{F}_{BC} + \vec{F}_{CD} + \vec{F}_{DA}$$

$$\vec{F}_{AB} = IaB(+\hat{x})$$

$$\vec{F}_{BC} = I\,\vec{CB} \wedge B\hat{z} = IBa\,(\hat{x}\sin\theta + \hat{z}\cos\theta) \wedge \hat{z}$$
$$= +IBa\sin\theta\,\hat{y}$$

$$\vec{F}_{DC} = -IaB\,\hat{x}$$

$$\vec{F}_{DA} = -IBa\sin\theta\,\hat{y}$$

Che $\sum \vec{F}_{ext} = 0 \Rightarrow \vec{R} + m\vec{g} = 0 \quad \vec{R} = -m\vec{g}$

Per avere θ_{eq}, impongo

$$\sum \vec{M}_{O_{ext}} = 0$$

$$\vec{M}_{O\,ext} = \begin{cases} mg\,\frac{a}{2}\sin\theta\,\hat{y} \\ \vec{M}_{O,R} = 0 \\ \vec{M}_{O,magn} \end{cases}$$

$$\vec{M}_{O,AB} = +IaB\left(-a\cos\theta\,\hat{z} + a\sin\theta\,\hat{x}\right)\wedge\hat{x}$$

$$= -Ia^2B\cos\theta\,\hat{y}$$

$$\vec{M}_{O,BC} = IBa\sin\theta\left(-\frac{a}{2}\cos\theta\,\hat{z} + \frac{a}{2}\sin\theta\,\hat{x}\right)\wedge\hat{y}$$

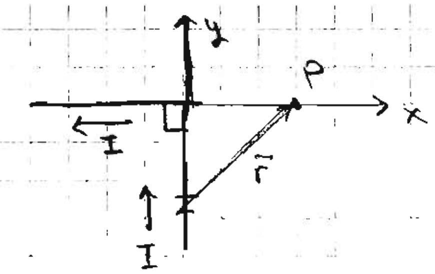
$$\vec{M}_{O,DE} = 0$$

$$\vec{M}_{O,OA} = -IBa\sin\theta\left(-\frac{a}{2}\cos\theta\,\hat{z} + \frac{a}{2}\sin\theta\,\hat{x}\right)\wedge\hat{y}$$

$$= -\vec{M}_{O,BC}$$

$$\Rightarrow mg\,\frac{a}{2}\sin\theta\,\cancel{\hat{y}} - Ia^2B\cos\theta\,\cancel{\hat{y}} = 0$$

$$\mathrm{tg}\,\theta_{eq} = \frac{2IaB}{mg}$$

4) 

Poichè $d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \hat{r}}{r^2}$, il tratto di filo // a x non darà alcun contributo, ⇒ rimane

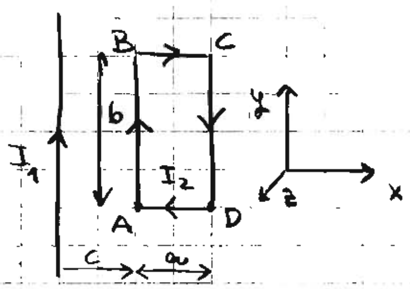
$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \wedge \vec{r}}{r^3} = \frac{\mu_0}{4\pi} I \frac{d\vec{y} \wedge (x\hat{x} + y\hat{y})}{(x^2+y^2)^{3/2}}$$

$$= \frac{\mu_0}{4\pi} I \frac{x\,dy}{(x^2+y^2)^{3/2}} (-\hat{z})$$

$$\vec{B} = \frac{\mu_0}{4\pi} I\, x\, (-\hat{z}) \int_{-\infty}^{0} \frac{dy}{(x^2+y^2)^{3/2}} = \frac{\mu_0}{4\pi} I \cancel{x} (-\hat{z}) \left.\frac{y}{x^{\cancel{2}}\sqrt{x^2+y^2}}\right|_{-\infty}^{0}$$

$$= \frac{\mu_0}{4\pi} \frac{I}{x} (-\hat{z}) \left(0 - \lim_{y\to-\infty} \frac{y}{|y|}\right)$$

$$= \frac{\mu_0}{4\pi} \frac{I}{x} (-\hat{z})$$

5) 

$$\vec{B}_1(P) = \frac{\mu_0}{2\pi} \frac{I_1}{r} (-\hat{z}) \quad \forall P \in \{xy\}$$

$$\vec{F}_{AB} = \frac{\mu_0}{2\pi} \frac{I_1}{c} I_2\, b(-\hat{x}) \qquad \vec{F}_{DA} = \frac{\mu_0}{2\pi} I_1 I_2 \left(\int_c^{c+a} \frac{1}{x}dx\right)(-\hat{y})$$

$$\vec{F}_{CD} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{(c+a)} b(+\hat{x}) \qquad \vec{F}_{BC} = \frac{\mu_0}{2\pi} I_1 I_2 \left(\int_c^{c+a} \frac{1}{x}dx\right)(+\hat{y})$$

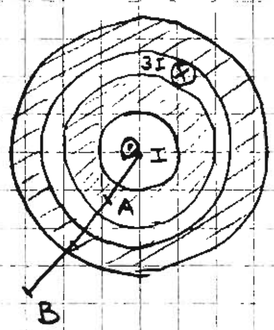
$$\Rightarrow \vec{F}_{DA} + \vec{F}_{BC} = 0$$

$$\Rightarrow \vec{F} = \vec{F}_{AB} + \vec{F}_{CD} = \frac{\mu_0}{2\pi} I_1 I_2 b \left(\frac{1}{c+a} - \frac{1}{c}\right)\hat{x}$$

$$= \frac{\mu_0}{2\pi} \frac{I_1 I_2 b}{(c+a)(c)} (-a\,\hat{x})$$

$$= \frac{\mu_0}{2\pi} I_1 I_2 \frac{a\,b}{c\,(c+a)} (-\hat{x})$$

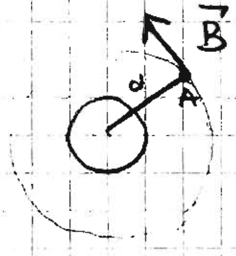
6)



$\vec{B}(A)$?

$\vec{B}(B)$?

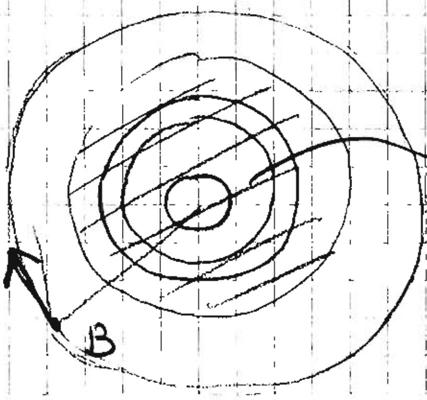
Simmetria cilindrica $\Rightarrow$



(come per il filo ∞)

$$2\pi d\, B = \mu_0 I \quad \Rightarrow \quad B(A) = \frac{\mu_0}{2\pi} \frac{I}{d}$$

Fuori dal cavo



$|I - 3I|$

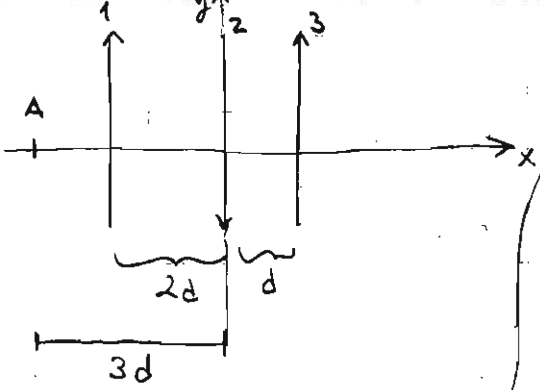
$= 2I$ $\otimes$

$\Rightarrow \vec{B}$ come in fig.

$$\cancel{2}\pi(3d)\, B = \mu_0 \cancel{2} I \quad \Rightarrow$$

$$B(B) = \frac{\mu_0}{\pi} \frac{I}{3d}$$

7)



$$\vec{B} = \sum_i \vec{B}_i$$

Ampère

$$2\pi d\, B_1 = \mu_0 I \Rightarrow \vec{B}_1 = \frac{\mu_0}{2\pi}\frac{I}{d}(+\hat{z})$$

Ampère

$$2\pi\, 3d\, B_2 = \mu_0 I \Rightarrow \vec{B}_2 = \frac{\mu_0}{2\pi}\frac{I}{3d}(-\hat{z})$$

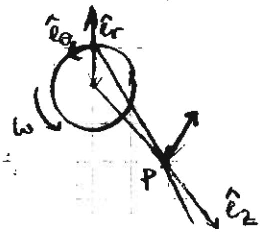
Amp.

$$2\pi\, 4d\, B_3 = \mu_0 I \Rightarrow \vec{B}_3 = \frac{\mu_0}{2\pi}\frac{I}{4d}(+\hat{z})$$

$$\Leftarrow$$

$$\vec{B} = \frac{\mu_0}{2\pi}\frac{I}{d}\hat{z}\left(1 - \frac{1}{3} + \frac{1}{4}\right) = \frac{\mu_0}{2\pi}\frac{I}{d}\frac{11}{12}(+\hat{z})$$

8)



$$\frac{+q}{2\pi a} = \lambda$$

$$I = \frac{dq}{dt} = \lambda\frac{dl}{dt} = \frac{q}{2\pi a}\, a\frac{d\theta}{dt} = \frac{q}{2\pi}\omega$$

$$d\vec{B}(P) = \frac{\mu_0}{4\pi} I \frac{d\vec{l}\wedge\hat{r}}{r^2} = \frac{\mu_0}{4\pi} I \frac{d\vec{l}\wedge\vec{r}}{r^3}$$

$$\left.\begin{aligned} d\vec{l} &= a\,d\theta\;\hat{e}_\theta \\ \vec{r} &= -a\hat{e}_r + \frac{a}{2}\hat{e}_z \end{aligned}\right\} \quad d\vec{l}\wedge\vec{r} = -a^2 d\theta\,(-\hat{e}_z) + \frac{a^2}{2}d\theta\;\hat{e}_r$$

$$= +a^2 d\theta\;\hat{e}_z + \underbrace{\frac{a^2}{2}d\theta\;\hat{e}_r}$$

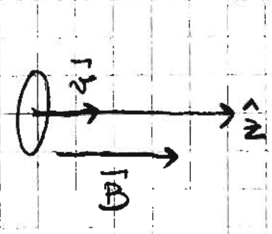
integrato su dθ non dà contributo (vd. pb. 2)

$$\Rightarrow \quad d\vec{B}(P) = \frac{\mu_0}{4\pi} I \frac{(+a^2 d\theta)}{r^3}\hat{e}_z$$

$$\Rightarrow \quad \vec{B}(P) = \frac{\mu_0}{4\pi}\frac{I a^2\, 2\pi}{\left(a^2 + \frac{a^2}{4}\right)^{3/2}}(+\hat{e}_z) = \frac{\mu_0}{\cancel{4}\pi}\frac{q\omega}{\cancel{2\pi}}\frac{\cancel{a^2}\,\cancel{2\pi}}{a^{\cancel{3}}\,\frac{5\sqrt{5}}{\cancel{8}\,2}}\hat{e}_z$$

$$= \frac{\mu_0\, q\omega}{\frac{5}{2}\sqrt{5}\,\pi a}.$$

9)



$$\vec{v} = v\,\hat{z}$$
$$\vec{B} = \beta z\ \hat{z}$$

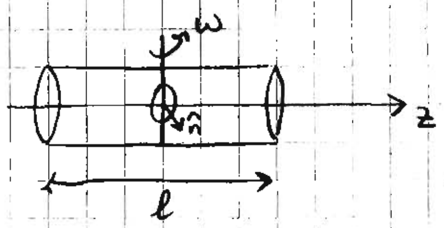
$$P = \frac{\mathcal{E}^2}{R} = \frac{1}{R}\left(\frac{d\phi_B}{dt}\right)^2$$

$$\phi_B = \pi r^2 \beta z$$

$$\frac{d\phi_B}{dt} = \pi r^2 \beta \frac{dz}{dt} = v \pi r^2 \beta$$

$$\Rightarrow \quad P = \frac{1}{R}\left(v \pi r^2 \beta\right)^2$$

10)



N, i

$$\vec{B}_{in} = \mu_0 \frac{N}{\ell} i\ \hat{z}$$

angolo a $t=0$

$$\phi_B = \pi r^2 \left(\mu_0 \frac{N}{\ell} i\right) \hat{z}\cdot\hat{n} = \pi r^2 \left(\mu_0 \frac{N}{\ell} i\right) \cos(\omega t + \varphi)$$

$$\frac{d\phi_B}{dt} = -\pi r^2 \left(\mu_0 \frac{N}{\ell} i\right) \omega \sin(\omega t + \varphi)$$

$$P = \frac{\mathcal{E}^2}{R} = \frac{1}{R}\left[\pi r^2 \mu_0 \frac{N}{\ell} i\, \omega \sin(\omega t + \varphi)\right]^2$$

$$P_{max} \Rightarrow \sin(\omega t + \varphi) = \pm 1 \quad \Rightarrow$$

$$P_{max} = \frac{\left(\pi r^2 \mu_0 N/\ell\ i \omega\right)^2}{R}$$