Thermal production of gravitinos

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Supersymmetry, Supergravity, Superstrings
March 19-21 2007

*hep-ph/0701104 with Alessandro Strumia (to appear in PhysRevD)

Outline

- Gravitino in Cosmology

- Physics of Gravitino Production at High T

- Applications

Gravitino basics

- SUSY predicts new particles a) superpartners of SM particles (sparticles) b) **gravitino**.

- **Gravitino** ψ_μ couples to particles/sparticles via **supercurrent** S^μ with gravitational strength, cannot be seen at colliders:

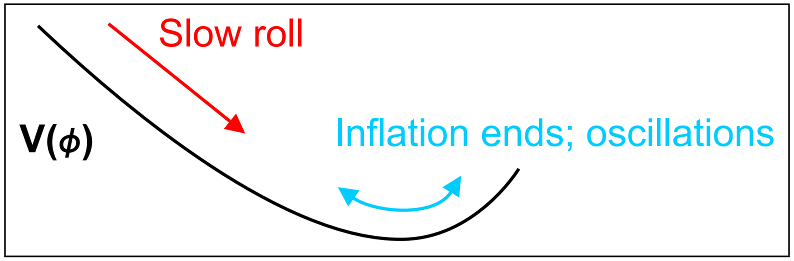
$$\boxed{\mathcal{L}_{\text{int}} = \frac{1}{2M_{\text{Pl}}} \bar{\psi}_\mu \, S^\mu}$$

However: will be produced at high T in the Early Universe (more precisely, during/after **reheating**)

Reheating Reminder

- Slow-roll inflation ends when **inflaton** ϕ reaches minimum of its potential $V(\phi)$ and starts coherent oscillations

- Oscillation amplitude decreases while **inflaton** ϕ decays into SM particles and thermalizes them



- **Reheating temperature** $T_{\mathrm{RH}} \equiv$ temperature when radiation density of SM starts dominating the expansion of the Universe (transition to radiation dominated FRW).

- T_{RH} is the maximal *observable* temperature of the Universe: prior history has been washed out by entropy released during the inflaton decay.

Gravitino Cosmology

- In SUSY world, a SUSY version of SM (like MSSM) gets thermalized, and gravitinos start being produced.

- Gravitino **overproduction** has to be watched:
  - If **LSP**, has to satisfy $\Omega_{3/2} < \Omega_{\text{DM}}^{\text{observed}} \approx 20\%$
  - If **not LSP**, decay during or after BBN, with damaging consequences

- Gravitino **production rate** estimate: $\boxed{\gamma = \frac{dn_{3/2}}{dt} = \gamma_0 \frac{T^6}{M_{\text{Pl}}^2}}$

  Efficient production during/immediately after reheating, when T maximal

- The energy density of gravitinos after reheating:

$$\frac{\Omega_{3/2}}{\Omega_{\text{DM}}^{\text{observed}}} \approx 1.5\gamma_0 \left(\frac{m_{3/2}}{100\,\text{GeV}}\right)\left(\frac{T_{\text{RH}}}{10^{10}\text{GeV}}\right)$$

- *Today's problem:* **compute** γ_0.

Gravitino production I. Scatterings

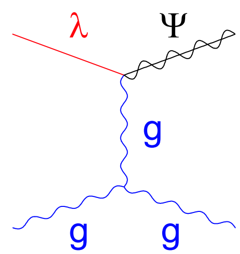
- Interaction Lagrangian $\mathcal{L}_{\text{int}} = \dfrac{1}{2M_{\text{Pl}}} \bar{\psi}_\mu S^\mu$

- Super-QCD supercurrent $S^\mu_{\text{SQCD}} =$ (gluon)×(gluino)+(quark)×(squark).

- There are 10 SQCD $2 \rightarrow 2$ processes giving gravitino production:

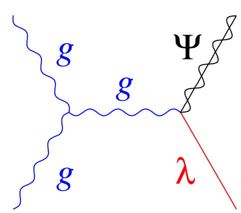
  – IR-divergent (with gluon t-channel exchange).

    E.g.: gluon+gluino→gluon+Gravitino



  – IR-convergent.

    E.g.: gluon+gluon→gluino+Gravitino



- Best result up to now was [Bolz,Brandenburg,Buchmuller 2000]

$$\gamma_{\text{scattering}} = \gamma_0 \frac{T^6}{M_{\text{Pl}}^2}$$

$$\boxed{\gamma_0 = 0.5 g_3^2 \log \frac{1.2}{g_3} \left(1 + \frac{M_{\text{gluino}}^2}{3 m_{3/2}^2} \right) + O(g_3^4)}$$

Here g_3 is SU(3) gauge coupling, $g_3 \approx 0.85$ at $T \sim 10^{10}$ GeV.

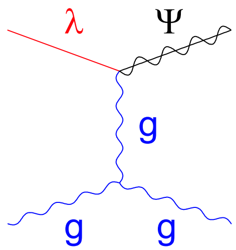
- *Comments*

  - $M_{\text{gluino}}^2 / m_{3/2}^2$ term: related to goldstino ($\equiv$ longitudinal massive gravitino). Dominates for $m_{3/2} \ll M_{\text{gluino}}$

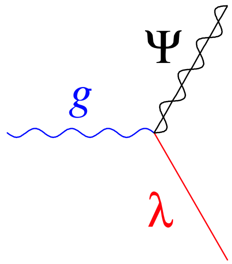
  - $\log g_3$ dependence: related to how the IR divergence is regulated by gluon thermal mass $m_{\text{gluon}} \sim g_3 T$. (Debye screening of electric fields in plasma)

Gravitino production II. Decays

- We identify an important subclass of $O(g_3^4)$ corrections. Let's compare typical production rate due to **scattering** and **decay**, keeping track of phase space factors of π.



$$\gamma_{\text{scat}} \sim \frac{1}{\pi^4}\sigma T^6 \sim \frac{g_3^2}{\pi^5}\frac{T^6}{M_{\text{Pl}}^2}$$



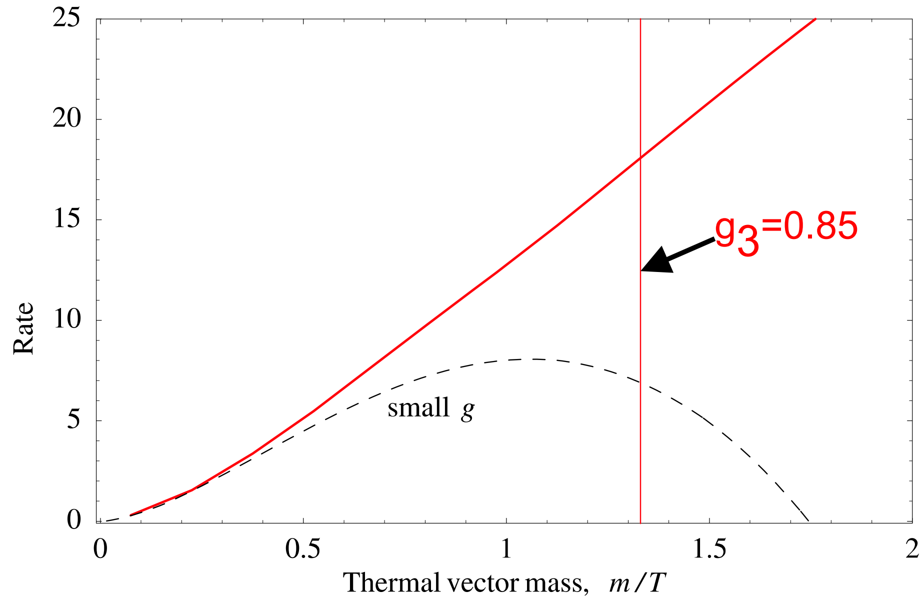
$$\gamma_{\text{decay}} \sim \frac{1}{\pi^2}\Gamma T^3 \sim \frac{g_3^4 T^6}{\pi^3 M_{\text{Pl}}^2} = O(g_3^2\pi^2)\,\gamma_{\text{scat}}$$

where we used

$$\Gamma \sim \frac{m}{T}\Gamma_{\text{rest}}, \quad \Gamma_{\text{rest}} \sim \frac{m^3}{\pi M_{\text{Pl}}}, \quad m \sim g_3 T$$

- γ_{decay}, although formally higher order, may be sizeable. **Needs to be included**.

Our result



NB: Once $\gamma_{\rm decay}$ is added, further contributions are expected to be suppressed by powers of g/π (usual expansion parameter in field theory at finite T).

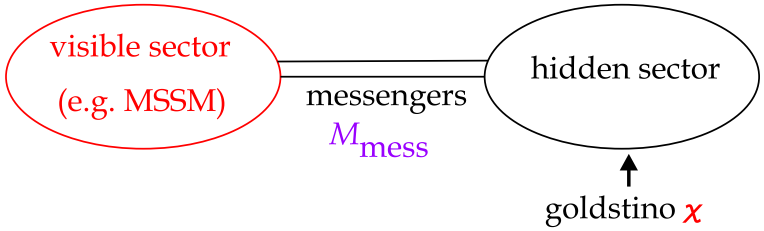
Perturbative expansion does not fail, simply some new effect appears only at higher order.

Calculation ingredients

- Gravitino and goldstino couplings - *equivalence theorem*
- Dispersion relations at finite temperature - *thermal spectral densities*
- Avoiding double counting - *subtracted scattering rates*

Goldstino and Super-Higgs mechanism

- Phenomenology assumes existence of heavy **hidden sector** where SUSY is broken spontaneously.



- At $E \ll M_{\text{mess}}$

$$\mathcal{L}_{\text{low energy}} = \mathcal{L}_{\text{visible}} + \mathcal{L}_{\text{soft-SUSY-breaking}} + \frac{1}{\sqrt{2}F}\bar{\chi}\partial_\mu S^\mu_{\text{visible}}$$

(where F is a SUSY-breaking vev in **hidden sector**, $m_{\text{soft}} \sim F/M_{\text{mess}}$)

- After coupling to SUGRA, **gravitino eats goldstino** χ and becomes massive (super-Higgs mechanism [Deser,Zumino 1977]):

$$\Psi_\mu = \psi_\mu - \sqrt{\frac{2}{3}}\frac{\partial_\mu \chi}{m_{3/2}} \qquad m_{3/2} = \frac{F}{\sqrt{3}M_{\text{Pl}}}$$

$\Longrightarrow$ in **gravity-mediation** models $m_{3/2} \sim m_{\text{soft}}$

$\Longrightarrow$ in **gauge-mediation** $m_{3/2} \ll m_{\text{soft}}$ possible.

Equivalence theorem

- Although goldstino is the longitudinal component of massive gravitino, at energies $E \gg m_{3/2}$ **gravitino** and **goldstino** do not mix $\Longrightarrow$ can study their production **independently**, and in the **massless approximation**:

$$\gamma(\Psi) \approx \gamma(\psi) + \gamma(\chi), \qquad \mathcal{L}_{\text{int}} = \frac{1}{2M_{\text{Pl}}} \bar{\psi}_\mu S^\mu_{\text{vis}} + \frac{1}{\sqrt{2}F} \bar{\chi}\, \partial_\mu S^\mu_{\text{vis}}$$

- Goldstino has dim-6 coupling. Naively could expect production rate

$$\gamma(\chi) \sim \frac{T^8}{F^2} \quad \text{(wrong)}$$

This is wrong because **Goldstino must decouple** in the limit when SUSY breaking goes to zero. In fact,

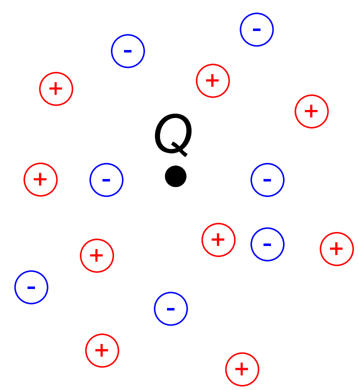
$$\partial_\mu S^\mu_{\text{visible}} \propto m_{\text{soft}}$$

and thus [Leigh, Rattazzi 1995], [Ellis et al 1995]

$$\gamma(\chi) \sim \frac{m^2_{\text{soft}} T^6}{F^2} \sim \frac{m^2_{\text{soft}}}{m^2_{3/2}} \gamma(\psi)$$

Thermal effects

Particles in thermal plasma get masses $m_{\text{thermal}} \sim gT$ (g any coupling). *Reminder*: **Debye screening of electric field in QED**.



Electric potential ϕ creates unequal chemical potentials $\mu = \pm e\phi$ for $e^{\pm} \Longrightarrow$ induced charge density

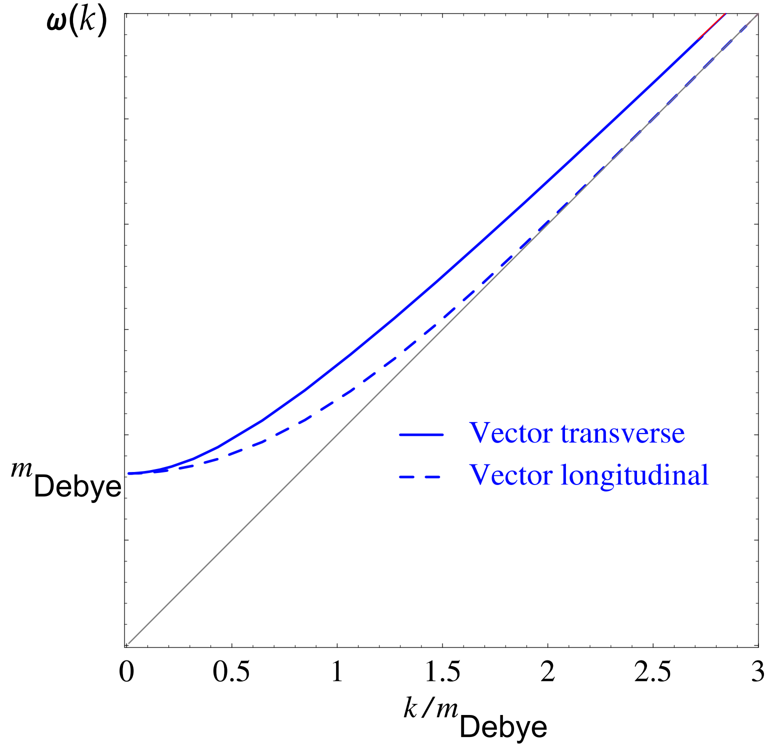
$$\rho_{\text{ind}} = e\left(n_{e^+} - n_{e^-}\right) = \frac{1}{3}e\left(\frac{\mu}{T}\right)T^3 = \frac{1}{3}e^2T^2\phi$$

This induced density gives **mass term** in the Poisson equation:

$$\Delta\phi + m_{\text{Debye}}^2\phi = -Q\delta(r), \quad m_{\text{Debye}} = \frac{1}{\sqrt{3}}eT.$$

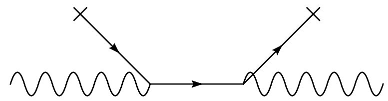
$\Longrightarrow$ Static electric field decays away **exponentially** on scale m_{Debye}^{-1}.

- Analogous kinetic-theory calculation can be done applying time-dependent EM fields to the plasma. $\Longrightarrow$ modified dispersion relation for EM wave propagation.

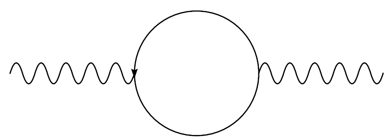


NB: longitudinal waves ($\vec{E} \| \vec{k}$) can propagate at finite T.

- *Equivalently*: can find modified dispersion by computing **Compton scattering corrections** to photon propagator in plasma:



- In thermal field theory formalism, these are computed as 1-loop corrections to photon self-energy



where we have to use **finite-T propagators**

$$S_F(p) = (\not{p} + m)\left(\frac{1}{p^2 - m^2 + i\varepsilon} + i2\pi n_F(p_0)\delta(p^2 - m^2)\right)$$

with corrected imaginary parts to account for presence of particles in the thermal bath.

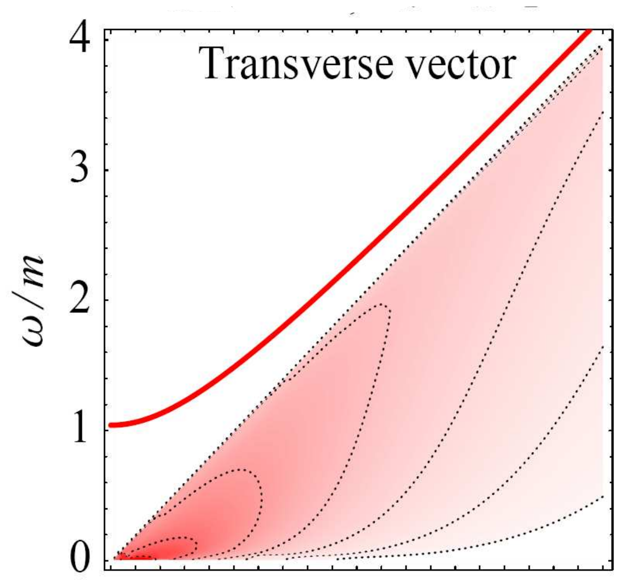
- Resumming these 1-loop corrections, we get photon propagator at finite temperature:

$$D_{\mu\nu} = \sum_{i=T,L} \frac{P^i_{\mu\nu}}{p^2 - \Pi_i(\omega, k) + i\varepsilon}$$

where $P^{T,L}_{\mu\nu}$ are **transverse** and **longitudinal** projectors.
At $T = 0$ Lorentz invariance fixes $\Pi_T = \Pi_L$, however at finite-T Lorentz invariance is broken by thermal bath $\Longrightarrow$ more general structure.

- Propagator poles give **dispersion relations**. Self-energies $\Pi_{T,L}$ also have **imaginary parts** for $\omega/k < 1$ related to Landau damping (energy transfer between EM waves and electrons in the plasma) $\Longrightarrow$ **continuum spectral densities** below the light cone.

$$\rho_{T,L}(\omega, k) = \mathrm{Im} \frac{1}{p^2 - \Pi_{T,L} + i\varepsilon} = Z\,\delta(\omega - \omega_{T,L}(k)) + \rho_{\text{cont}}(\omega, k)$$



- Spectral densities ≈ parton densities, i.e. relevant photon distribution becomes

$$\frac{d^3\vec{k}}{2\omega} n_B(\omega) d\omega \rightarrow \frac{d^3\vec{k}}{2\omega} \rho(\omega, k) n_B(\omega) d\omega$$

- Spectral densities satisfy 'sum rules' meaning essentially that total number of photons remain the same.

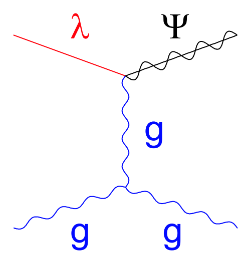
Summary of thermal effects

Here we discussed **photon** in QED plasma, but similar effects exist for **electrons** and **other particles in other plasmas**. In particular in QCD and in Super-QCD (with different values of thermal masses).

- **Modified dispersion relation** - screening of IR divergences
- **Modified quasiparticle distribution** *a là* parton distribution functions in QCD
- Spectral densities (imaginary parts of 1-loop resummed propagators) accurately encode both effects.

Gravitino production

Come back to our problem of gravitino production from Super-QCD plasma. **If we don't take finite-T effects on particle propagation into account**, the leading effect is due to $2 \to 2$ scatterings like *gluon+gluino→gluon+Gravitino*:



However, these give IR divergent production rate

$$\gamma_{\text{scat}} \sim {g_3}^2 \frac{T^6}{M_{\text{Pl}}^2} \log \frac{T}{\mu}$$

where μ is the IR regulator. Can guess $\mu \sim m_{\text{Debye}} \sim g_3 T$. If we want to do better than this estimate, have to use the full thermal gluon propagator.

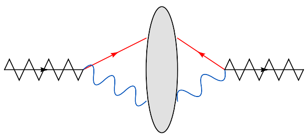
$\gamma \sim \mathrm{Im}\,\Pi$

This useful piece of formalism relates gravitino production rate to **imaginary part of its propagator**:

$$\gamma = \dot{n}_{3/2} = -\int \frac{d^3\vec{k}}{(2\pi)^3 E} n_F(E) \mathrm{Im}\ \Pi(p)$$

$$\Pi = \frac{1}{4M_{\mathrm{Pl}}^2} \mathrm{Tr}[\Pi^{3/2}_{\mu\nu} \langle S^\nu(p) S^\mu(-p) \rangle_T]$$

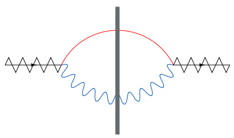
($\Pi^{3/2}_{\mu\nu} = -\frac{1}{2}\gamma_\mu \not{p} \gamma_\nu - \not{p}\eta_{\mu\nu}$ is the tensor used to sum over gravitino polarizations)

$\gamma \sim \mathrm{Im}$ 

NB: True to all orders in the plasma couplings; to leading order in $\frac{1}{M_{\mathrm{Pl}}}$.

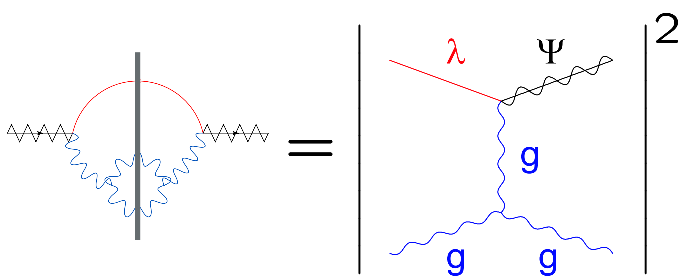
Cutting rules

Thermal field theory cutting rules allow to connect $\gamma \sim \mathrm{Im}\Pi$ formalism with direct calculation of production rate via scatterings.

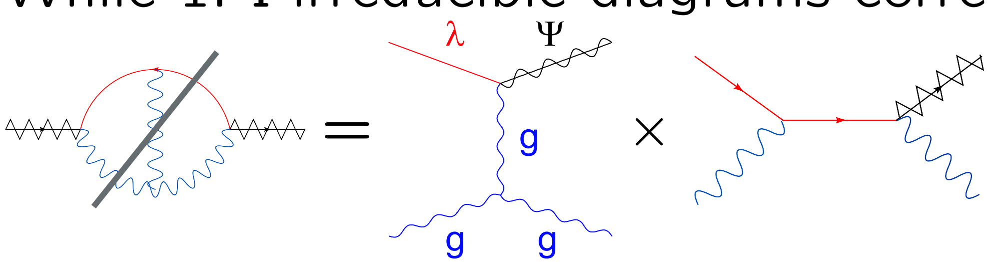
-  $= 0$ (no phase space)

Various cuts of 2-loop diagrams correspond to terms in scattering amplitude squared.

- self-energy corrections describe squares of individual diagrams:

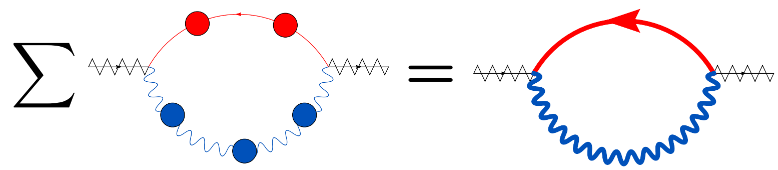


- While 1PI irreducible diagrams correspond to cross terms:

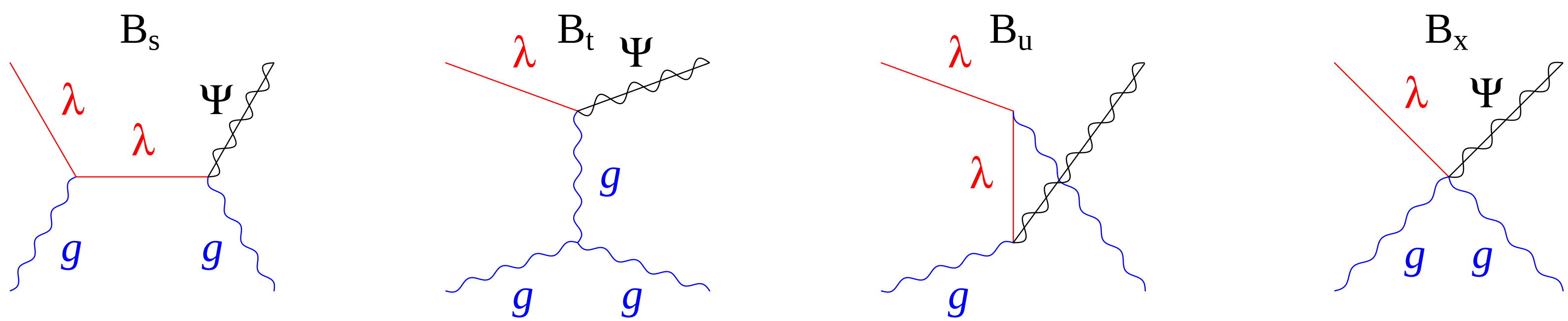


Resummation: Decay diagram

- **Decay diagram**, has to be computed from finite temperature propagators:



- **Decay diagram** overcounts some contributions which are part of $\gamma_{\text{scattering}}$. **These are all individual** $2 \rightarrow 2$ **diagrams squared.** *E.g.* gluon+gluino→gluon+Gravitino is the sum of 4 diagrams:



$$\gamma_{\text{scattering}} \sim |B_s + B_t + B_u + B_x|^2$$

Subtracted scattering rate

- It turns out that terms $|B_{s,t,u}|^2$ are all contained in **Decay diagram**. Thus we define **s**ubtracted scattering rate

$$\gamma_{\text{subtracted}} \sim |B_s + B_t + B_u + B_x|^2 - |B_s|^2 - |B_t|^2 - |B_u|^2$$

- So defined γ_{sub} is **IR-finite**! This is because the only IR-divergent term $|B_t|^2$ is subtracted away.

  $\Longrightarrow$ Removing double counting also removes IR divergence.

- Full scattering rate is defined as

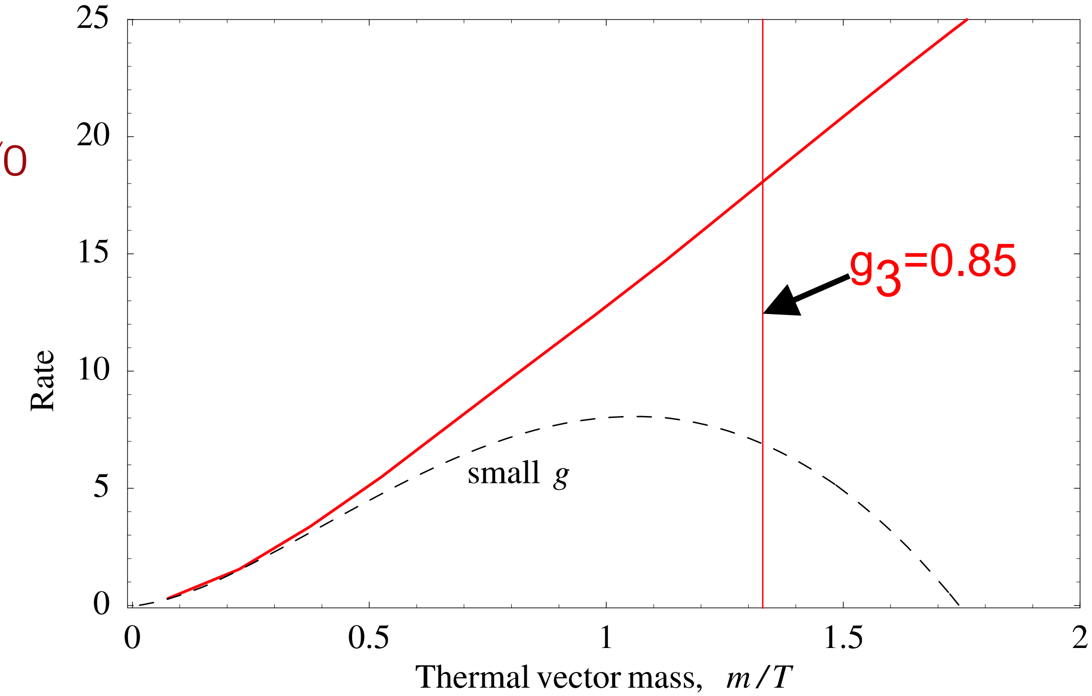
$$\boxed{\gamma = \gamma_{\text{Decay}} + \gamma_{\text{subtracted}}}$$

Result and applications

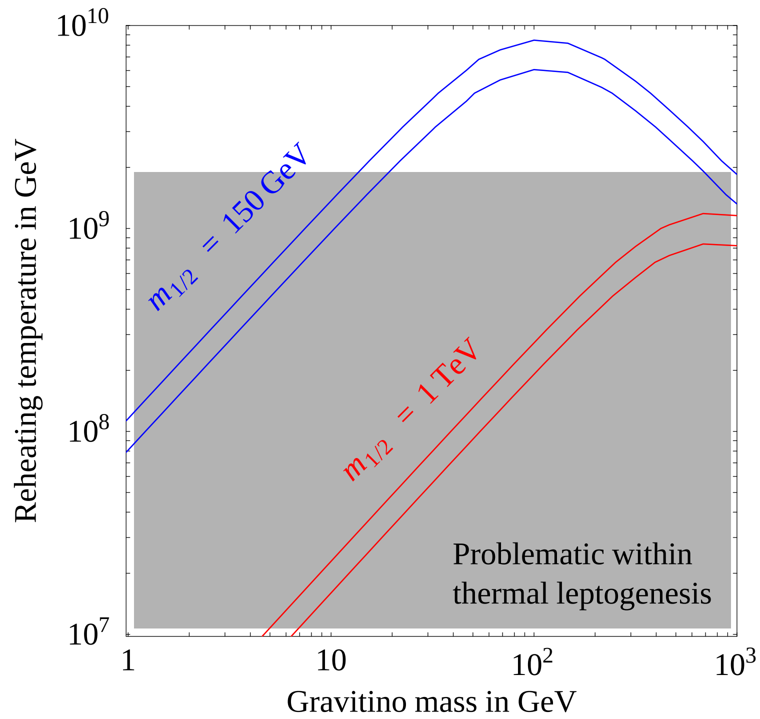
- Gravitino production rate coefficient γ_0 as a function of SQCD coupling:

$$\gamma = \frac{dn_{3/2}}{dt} = \gamma_0 \frac{T^6}{M_{\rm Pl}^2}\left(1 + \frac{M_{\rm gluino}^2}{3m_{3/2}^2}\right)$$

γ_0



- Bounds for gravitino Dark Matter and Reheating temperature



  - Contours show regions of $(m_{3/2}, T_{\mathrm{RH}})$ plane where **gravitino abundance** equals observed **DM abundance** (3σ levels).

  - Shaded region is problematic for standard thermal leptogenesis.

Conclusions

- Most accurate to date computation of gravitino abundance in post-inflationary universe

- Main contributions: scatterings and decays in Super-QCD plasma

- Also included: $O(g_2^2)$, $O(g_Y^2)$, $O(\lambda_{\text{top}}^2)$ effects.

- Stronger constraints on coexistence of Gravitino DM + Thermal Leptogenesis