Orientifold planar equivalence.

An overview and some hints from the lattice**.

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** Based on:
- A. Patella. *An Insight on the proof of orientifold planar equivalence on the lattice.* Phys. Rev. D74 (034506), 2006.
- B. Lucini, A. Patella and C. Pica. *Baryon currents in QCD with compact dimensions.* [arXiv:hep-th/0702167].
- work in progress, in collaboration with L. Del Debbio, B. Lucini, C. Pica.

Outline

1. Orientifold planar equivalence.
   - Large-N limits.
   - Some predictions.
2. The proof of orientifold planar equivalence.
   - The basic ideas.
   - Some assumptions.
3. The role of charge conjugation.
   - AsQCD at small volume.
   - C-breaking from lattice simulations.

Early works.

- The idea of orientifold planar equivalence was born in string theory.

  A. Sagnotti. Some properties of open string theories. arXiv:hep-th/9509080.

  A. Sagnotti. Surprises in open-string perturbation theory. Nucl. Phys. Proc. Suppl. 56B (1997) 332.

- Then, it was transposed to the framework of pure field theory.

  A. Armoni, M. Shifman and G. Veneziano. Exact results in non-supersymmetric large N orientifold field theories. Nucl. Phys. B 667, 170 (2003).

  A. Armoni, M. Shifman and G. Veneziano, From super-Yang-Mills theory to QCD: Planar equivalence and its implications. arXiv:hep-th/0403071.

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’t Hooft large-N limit.

1flavour QCD
SU(3) gauge theory + 1 Dirac fermion in the fundamental representation.

generalizing to N colours...

N-QCD
SU(N) gauge theory + 1 Dirac fermion in the fundamental representation.

$\xrightarrow[]{N\to\infty}$

Quenched QCD
SU(N) gauge theory + 1 quenched Dirac fermion in the fundamental representation.

Orientifold large-N limit.

1flavour QCD
SU(3) gauge theory + 1 Dirac fermion in the fundamental representation.

generalizing to N colours...

As-QCD
SU(N) gauge theory + 1 Dirac fermion in the antisymmetric representation. $\xRightarrow{N\to\infty}$

Orientifold large-N limit.

1flavour QCD
SU(3) gauge theory + 1 Dirac fermion in the fundamental representation.

generalizing to N colours...

As-QCD
SU(N) gauge theory + 1 Dirac fermion in the antisymmetric representation. $\overset{N\to\infty}{\Longrightarrow}$

Orientifold planar equivalence

Each Dirac (even massive) fermion in the antisymmetric representation can be replaced with a Majorana fermion in the adjoint representation in the planar limit.

Orientifold large-N limit.

1flavour QCD
SU(3) gauge theory + 1 Dirac fermion in the fundamental representation.

generalizing to N colours...

As-QCD
SU(N) gauge theory + 1 Dirac fermion in the antisymmetric representation.

$\stackrel{N\to\infty}{\Longrightarrow}$

Super Yang-Mills
SU(N) gauge theory + 1 Majorana fermion in the adjoint representation.

Orientifold planar equivalence

Each Dirac (even massive) fermion in the antisymmetric representation can be replaced with a Majorana fermion in the adjoint representation in the planar limit.

Orientifold large-N limit.

N_f flavours QCD
SU(3) gauge theory + N_f Dirac fermions
in the fundamental representation.

generalizing to N colours...

Orientifold-QCD
SU(N) gauge theory +
1 Dirac fermion in the antisymmetric +
$(N_f - 1)$ fermions in the fundamental.

$\xrightarrow{N\to\infty}$

Super Yang-Mills
SU(N) gauge theory + 1 Majorana
fermion in the adjoint representation.

Orientifold planar equivalence

Each Dirac (even massive) fermion in the antisymmetric representation can be replaced with a Majorana fermion in the adjoint representation in the planar limit.

Some predictions.

- The chiral condensate from the gluino condensate.

$$\langle\bar{\Psi}\Psi\rangle_{\mu} = -\frac{3}{2\pi}\mu^3\lambda(\mu)^{-\frac{\gamma}{\beta_0}-\frac{3\beta_1}{\beta_0}}\exp\left(-\frac{9}{\beta_0\lambda(\mu)}\right)\mathcal{K}(n_f,1/N)$$

$$\langle\bar{\Psi}\Psi\rangle_{2GeV} \simeq -(270\pm 30MeV)^3 \quad \text{for one flavour}$$

A. Armoni, M. Shifman, and G. Veneziano. Phys. Lett. B579 (384-390), 2004.
A. Armoni, G. Shore, and G. Veneziano. Nucl. Phys. B740 (23-35). 2006.

... to be compared with SU(3) lattice simulations.

$$\langle\bar{\Psi}\Psi\rangle_{2GeV} \simeq -(269\pm 9MeV)^3$$

T. DeGrand, R. Hoffmann, S. Schaefer, and Z. Liu. Phys. Rev. D74 (054501), 2006.

- Degeneration of the $\mathcal{P}$-even and $\mathcal{P}$-odd bosonic states.

  ... until now, no numerical results are available in this direction (this is computationally a hard problem).

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The basic ideas.

A. Armoni, M. Shifman, and G. Veneziano. Phys. Rev. D71 (045015), 2005.
A. Patella. Phys. Rev. D74 (034506), 2006.

- The free energy $W_R(\lambda, m)$ of the theory with fermions in the representation R follows:

$$e^{-W_R(\lambda,m)} = \int e^{-S_{YM}(U)} \,\mathrm{Det}\, \left(D_R + m\right)^{N_f} \mathcal{D}U$$

where

$$S_{YM} = -\frac{2N}{\lambda} \sum_p \mathrm{Re}\,\mathrm{tr}\, U_p$$

$$D^{(R)}_{xy} = \sum_\mu \eta_\mu(x) \left\{ R[U_\mu(x)]\delta_{x,y+\hat{\mu}} - R[U_\mu(y)]^\dagger \delta_{x,y-\hat{\mu}} \right\}$$

The basic ideas.

- The fermionic effective action can be expanded as a sum of Wilson loops in the representation R.

$$\ln \mathrm{Det}\ (D_R + m) \to - \sum_{\alpha \in \mathcal{C}} \frac{c(\alpha)}{m^{L(\alpha)}} \mathcal{W}_R(\alpha)$$

- The free energy is finally obtained as a parturbation of the pure Yang-Mills one.

$$W_R(\lambda, m) = W_{YM}(\lambda) + \sum_{n=0}^{\infty} \frac{N_f^n}{n!} \sum_{\alpha_1 \ldots \alpha_n} \frac{c(\alpha_1) \cdots c(\alpha_n)}{m^{\sum_i L(\alpha_i)}} \langle \mathcal{W}_R(\alpha_1) \cdots \mathcal{W}_R(\alpha_n) \rangle_{YM,c}$$

Some assumptions.

$$W_R(\lambda, m) = W_{YM}(\lambda) + \sum_{n=0}^{\infty} \frac{N_f^n}{n!} \sum_{\alpha_1 \dots \alpha_n} \frac{c(\alpha_1) \cdots c(\alpha_n)}{m^{\sum_i L(\alpha_i)}} \langle \mathcal{W}_R(\alpha_1) \cdots \mathcal{W}_R(\alpha_n) \rangle_{YM,c}$$

- Let's suppose that YM at $N = \infty$ doesn't break $\mathcal{C}$-parity.
  From standard formulas:

$$\mathcal{W}_{Adj} = |\mathrm{tr} U|^2 - 1 \qquad \mathcal{W}_{As} = \frac{1}{2} \left\{ (\mathrm{tr} U)^2 - \mathrm{tr} U^2 \right\}$$

Thanks to the factorization property of the planar limit, the following identity holds:

$$\left(\frac{1}{2}\right)^n \langle \mathcal{W}_{Adj}(\alpha_1) \cdots \mathcal{W}_{Adj}(\alpha_n) \rangle_{YM,c} = \langle \mathcal{W}_{As}(\alpha_1) \cdots \mathcal{W}_{As}(\alpha_n) \rangle_{YM,c} \quad \text{as } N \to \infty$$

Some assumptions.

- Let's suppose that the loop expansion converges for some values $\bar{m}$ of the mass. In this case, the planar limit and the loop expansion can be exchanged for masses higher than $\bar{m}$. The orientifold planar equivalence holds in this regime and the free energy is an analitic function of the mass.

$$\lim_{N\to\infty} \frac{1}{N^2} W_{Adj}(\lambda, m) = \lim_{N\to\infty} \frac{1}{N^2} W_{As}(\lambda, m)$$

  A. Armoni, M. Shifman and G. Veneziano. arXiv:hep-th/0701229, 2007.

- Finally, if no phase transitions occur as the mass decreases, the orientifold planar equivalence is proven. Moreover it can be shown that the only relevant phase transition is the break-down of $\mathcal{C}$-parity.

  M. Unsal and L. G. Yaffe. Phys. Rev. D74 (105019), 2006.

- At present, there are strong indications to believe that $\mathcal{C}$-parity is not broken... also from lattice simulations.

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AsQCD at small volume.

Timothy J. Hollowood and Asad Naqvi. hep-th/0609203, 2006.

T. DeGrand and R. Hoffmann. JHEP 0702 (022), 2007.

B. Lucini, A. Patella and C. Pica. arXiv:hep-th/0702167, 2007.

M. Unsal. arXiv:hep-th/0703025, 2007.

- Let us consider SU(N) AsQCD on $T^3 \times \mathbf{R}$, with periodic boundary conditions for fermions. For small enough compact dimensions, thanks to asymptotic freedom, AsQCD enters in the perturbative regime.
- Two order parameters for the charge conjugation symmetry can be defined: the Polyakov loop Ω winding around a compact dimension and the spatial components of the Noether current associated with the baryonic number symmetry $\bar{\psi}\vec{\gamma}\psi$ (the baryonic current).
- Under the action of $\mathcal{P}$, $\mathcal{C}$, $\mathcal{T}$, $\mathcal{CPT}$ symmetries, these order parameters transform according to:

$$\langle \mathrm{tr}\Omega \rangle \longrightarrow \langle \mathrm{tr}\Omega \rangle^*$$
$$\langle \bar{\psi}\vec{\gamma}\psi \rangle \longrightarrow -\langle \bar{\psi}\vec{\gamma}\psi \rangle$$

AsQCD at small volume.

- The Polyakov loop is computed at one-loop level...

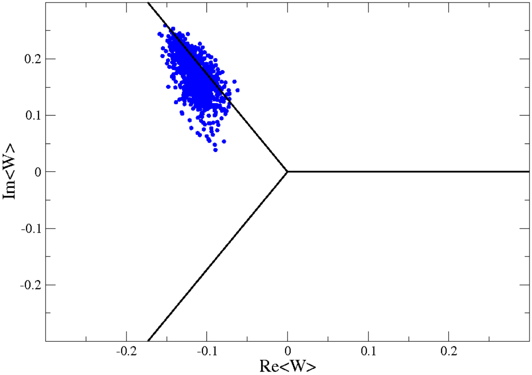
$$\frac{1}{N}\langle \mathrm{tr}\,\Omega_m\rangle = \exp(i v_m) = \begin{cases} \pm i & \text{for N=0 mod 4} \\ \exp\left(\pm i \frac{(N-1)\pi}{2N}\right) & \text{for N=1 mod 4} \\ \exp\left(\pm i \frac{(N\pm 1)\pi}{2N}\right) & \text{for N=2 mod 4} \\ \exp\left(\pm i \frac{(N+1)\pi}{2N}\right) & \text{for N=3 mod 4} \end{cases}$$

- ... and so the baryonic current.

$$\langle \bar{\psi}\vec{\gamma}\psi\rangle = -\frac{N_f N(N-1)}{2L^3}\left(\frac{mL}{\pi}\right)^2 \sum_{\substack{\vec{k}\in \mathbf{Z}^3 \\ \vec{k}\neq 0}} \frac{K_2(mLk)}{k^2}\sin(2\vec{k}\vec{v})\vec{k}$$

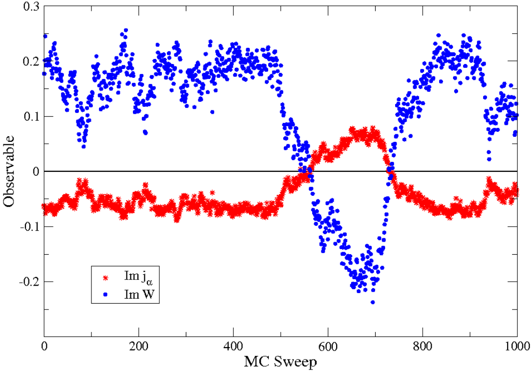
- Thus, $\mathcal{P}$, $\mathcal{C}$, $\mathcal{T}$, $\mathcal{CPT}$ symmetries are spontaneously broken.
- Since the charge conjugation symmetry-breaking seems deeply connected in this theory with the breaking of the $\mathcal{CPT}$-parity and the Lorentz symmetry, it should be restored for large volumes.

C-breaking from lattice simulations.



Data obtained for SU(3) QCD with $\beta = 5.5$ and $am = 0.1$ on a 24×4^3 lattice, corresponding to $a = 0.125$ fm and $L = 0.5$ fm.

C-breaking from lattice simulations.



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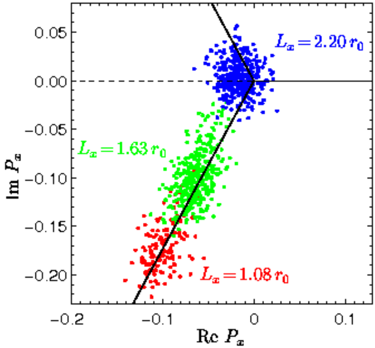
C-breaking from lattice simulations.



Plot from DeGrand and Hoffmann (JHEP 0702 022). It shows the restoration of the charge conjugation symmetry at about $L = 1\,fm$.

Conclusions.

1. The orientifold planar equivalence is a useful tool to understand QCD in an unconventional planar limit, and to relate it to a supersymmetric theory.
2. Testing directly the orientifold planar equivalence by lattice simulations can be a hard task.
3. The orientifold planar equivalence could fail if the charge conjugation symmetry were spontaneously broken.
4. Recent numerical simulations (and theorical arguments) suggest that the charge conjugation symmetry is not broken (in the case of infinite volume).