Workshop: Supersymmetry, Supergravity, Superstrings

Integrability of AdS/CFT

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AdS/CFT correspondence

Maldacena'97
Gubser,Klebanov,Polyakov'98
Witten'98

$\mathcal{N}=4$ SYM	Strings on $AdS_5\times S^5$
't Hooft coupling: $\lambda = g_{YM}^2 N$	String tension: $T=\frac{1}{2\pi\alpha'}=\frac{\sqrt{\lambda}}{2\pi}$
Number of colors: N	String coupling: $g_s=\frac{\lambda}{4\pi N}$
Large-N limit	Free strings
Strong coupling	Classical strings
Local operators Scaling dimension: Δ	String states Energy: E

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AdS/CFT-Integrability

Minahan,Zarembo'02

It seems to be a solvable (=integrable) theory,
at least for non-interacting strings (g_s=0),
or planar SU($N_c \to \infty$) N=4 SYM!

All modern means of 2d integrability applied:
- Bethe ansatz
- Finite gap method
- Factorizable S-matrix in 2d, etc

We are now able to calculate the dimensions of (long) operators
In N=4 SYM at any coupling!

Metsaev-Tseytlin superstring

- It is a sigma model on the coset

$$\mathrm{AdS}_5 \times \mathrm{S}^5 \sim \mathrm{PSU}(2,2|4)/\left(\mathrm{Sp}(2,2) \times \mathrm{Sp}(4)\right)$$

- Supergroup element *g* : (4|4) × (4|4) supermatrix of PSU(2,2|4)

Isometry of sphere

$SU(4) \sim SO(6)$

$B = C^\dagger \cdot \mathrm{diag}(1,1,-1,-1)$

$$g = \left(\begin{array}{c|c} g_{11} & g_{12} \\ \hline g_{21} & g_{22} \end{array} \right) \in \mathrm{PSU}(2,2|4)$$

16 complex fermions

Isometry of Anti de Sitter (conformal group)

$SU(2,2) \sim SO(2,4)$

sphere

$$\frac{SU(4)}{Sp(4)} \sim \frac{SO(6)}{SO(5)} = S^5$$

Anti de Sitter

$$\frac{SU(2,2)}{Sp(2,2)} \sim \frac{SO(2,4)}{SO(1,5)} = AdS_5$$

- Decompose current: $$J = -g^{-1}dg = \left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right)$$

$$J^{(0,2)} = \frac{1}{2}\begin{pmatrix} A \pm EA^TE & 0 \\ 0 & D \pm ED^TE \end{pmatrix}$$

$$E = \left(\begin{array}{c|c} 0 & 1 \\ \hline -1 & 0 \end{array}\right)_{4\times 4}$$

$$J^{(1,3)} = \frac{1}{2}\begin{pmatrix} 0 & B \mp iEC^TE \\ C \pm iEB^TE & 0 \end{pmatrix}$$

$$J = J^{(0)} + J^{(1)} + J^{(2)} + J^{(3)}$$

- Bosons: $J^{(2)} \in AdS_5 \oplus S^5,$
- To factor out: $J^{(0)} \in \mathrm{sp}(2,2) \oplus \mathrm{sp}(4),$
- Fermions: $J^{(1,3)}$

Metsaev-Tseytlin String Action

$$S_{MT} = \frac{\sqrt{\lambda}}{4\pi} \mathsf{str} \int_{\mathcal{M}_2} \left[J^{(2)} \wedge *J^{(2)} - J^{(1)} \wedge J^{(3)} \right]$$

$$J = J^{(0)} + J^{(1)} + J^{(2)} + J^{(3)}$$

- Bosons: $J^{(2)} \in AdS_5 \times S^5,$
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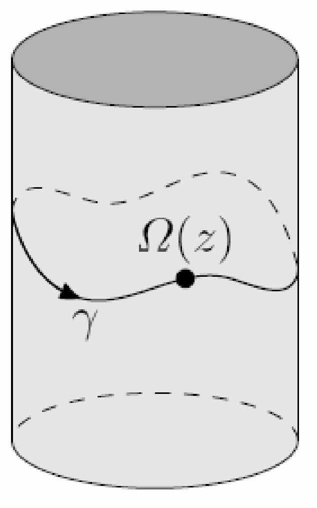
Classical integrability of MT superstring



- All Bianchi identities and eqs. of motion (current conserv.) are packed into a Lax eq.:

Bena,Roiban,Polchinski'02

$$(d + A(z)) \wedge (d + A(z)) = 0,$$

with the connection

$$A(z) = J^{(0)} + \frac{1}{2}\left(z^{-2} + z^{2}\right) J^{(2)} + \frac{1}{2}\left(z^{-2} - z^{2}\right) * J^{(2)} + z^{-1} J^{(1)} + z\, J^{(3)}$$

- Monodromy matrix: $\Omega(z) = P \exp \oint A(z) d\sigma$

Conserved quantities: eigenvalues of $\Omega(z)$

$$\{e^{i\tilde{p}_1(z)}, e^{i\tilde{p}_2(z)}, e^{i\tilde{p}_3(z)}, e^{i\tilde{p}_4(z)} || e^{i\hat{p}_1(z)}, e^{i\hat{p}_2(z)}, e^{i\hat{p}_3(z)}, e^{i\hat{p}_4(z)}\}$$

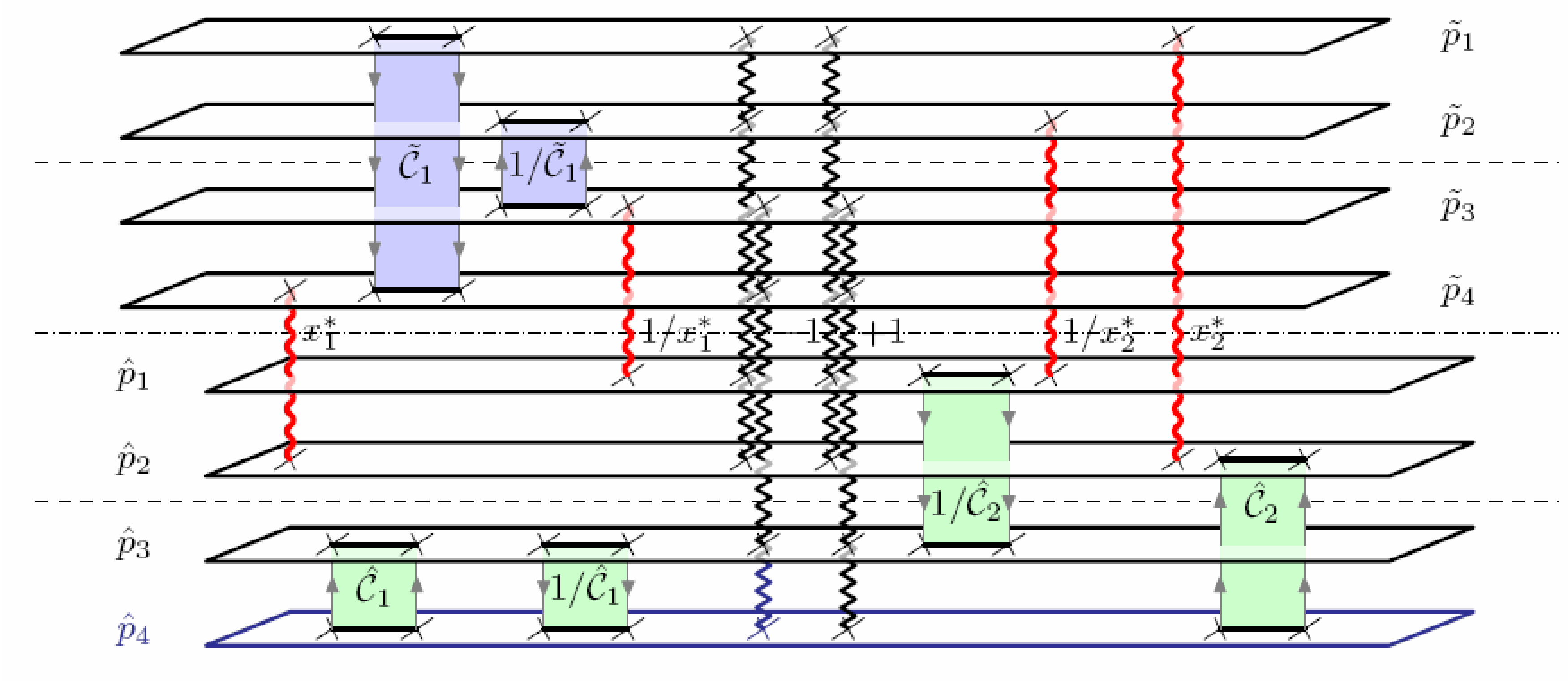
Eigenvalues are found by solving a characteristic equation for Ω .
They define an algebraic curve and Riemann surface.

8-Sheet Riemann Surface ("Finite Gap")

V.K., Marshakov,Minahan,Zarembo'04
Beisert, V.K, Sakai,Zarembo'05

$$y_k(z) = -izp'_k(z)$$

Sphere quasi-momenta



Anti de Sitter quasi-momenta

$$x = \frac{1+z^2}{1-z^2}$$

Algebraic curve of quasi-momentum

$y_k(z) = -izp_k'(z)$ - good variables, having only:

- branch cuts at $\tilde{z}_i, \hat{z}_j$ where same grading e.v.'s cross;

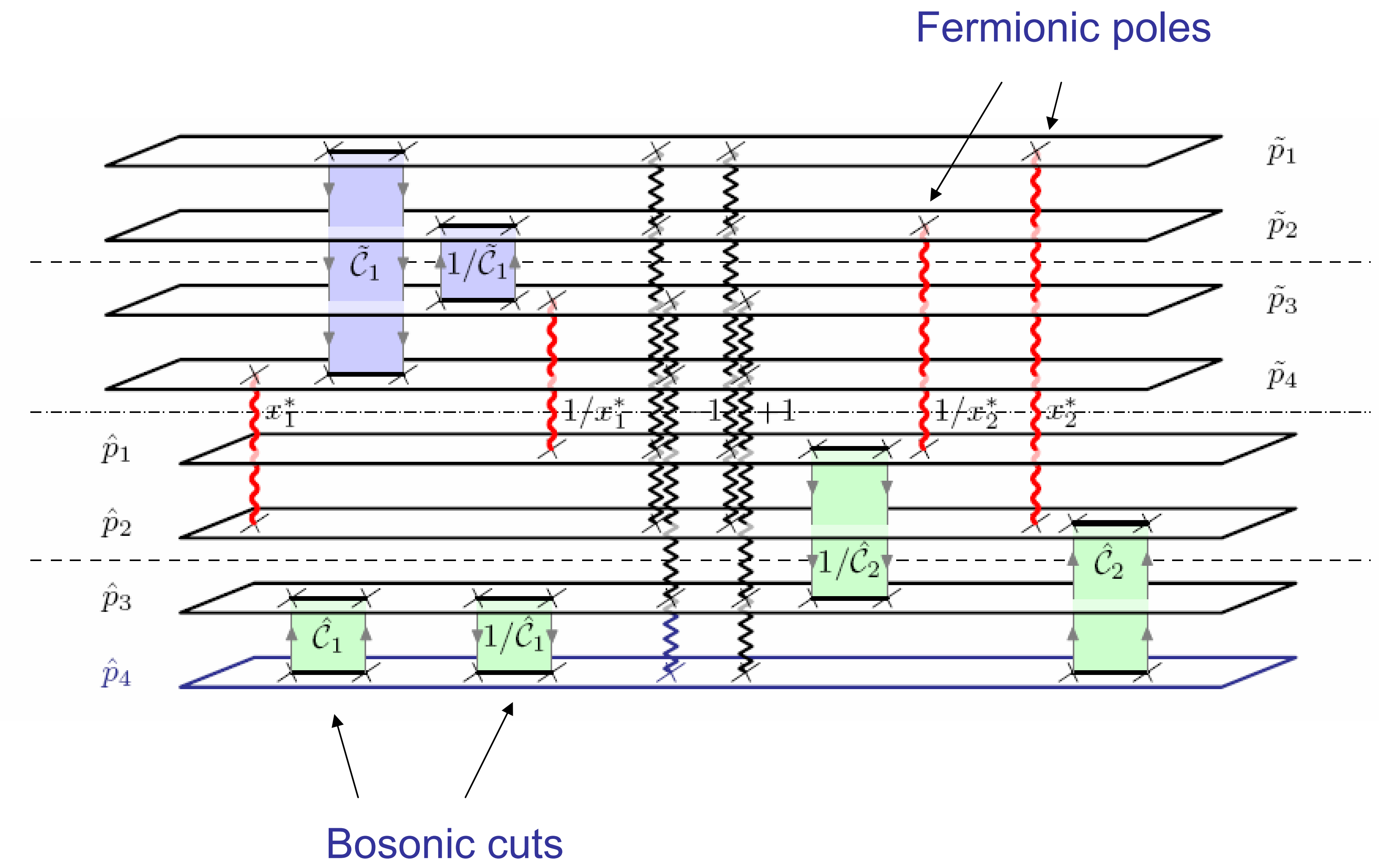
Cut C_{ij} : $p_i^+ - p_j^- = 2\pi n_{ij}$ (KMMZ-eqs.)

- poles at z_j^* where opposite grading e.v.'s cross.

Corresponding (1|1)x(1|1) sub-supermatrix of $\Omega(z)$

$$\left(\begin{array}{c|c} a & b \\ \hline c & d \end{array}\right) = u(z) \left(\begin{array}{c|c} \frac{bc}{a-d} + a & 0 \\ \hline 0 & \frac{bc}{a-d} + d \end{array}\right) u^{-1}(z)$$

Riemann surface



Inversion symmetry and Virasoro

- Important symmetry: $\Omega(1/x) = C\Omega^{-st}(x)C^{-1}$

Induces a monodromy:

$$\tilde{y}_k(1/x) = -\tilde{y}_{k'}(x), \qquad (1,2,3,4) \leftrightarrow (2,1,4,3)$$

$$\hat{y}_k(1/x) = -\hat{y}_{k'}(x), \qquad (5,6,7,8) \leftrightarrow (6,5,8,7)$$

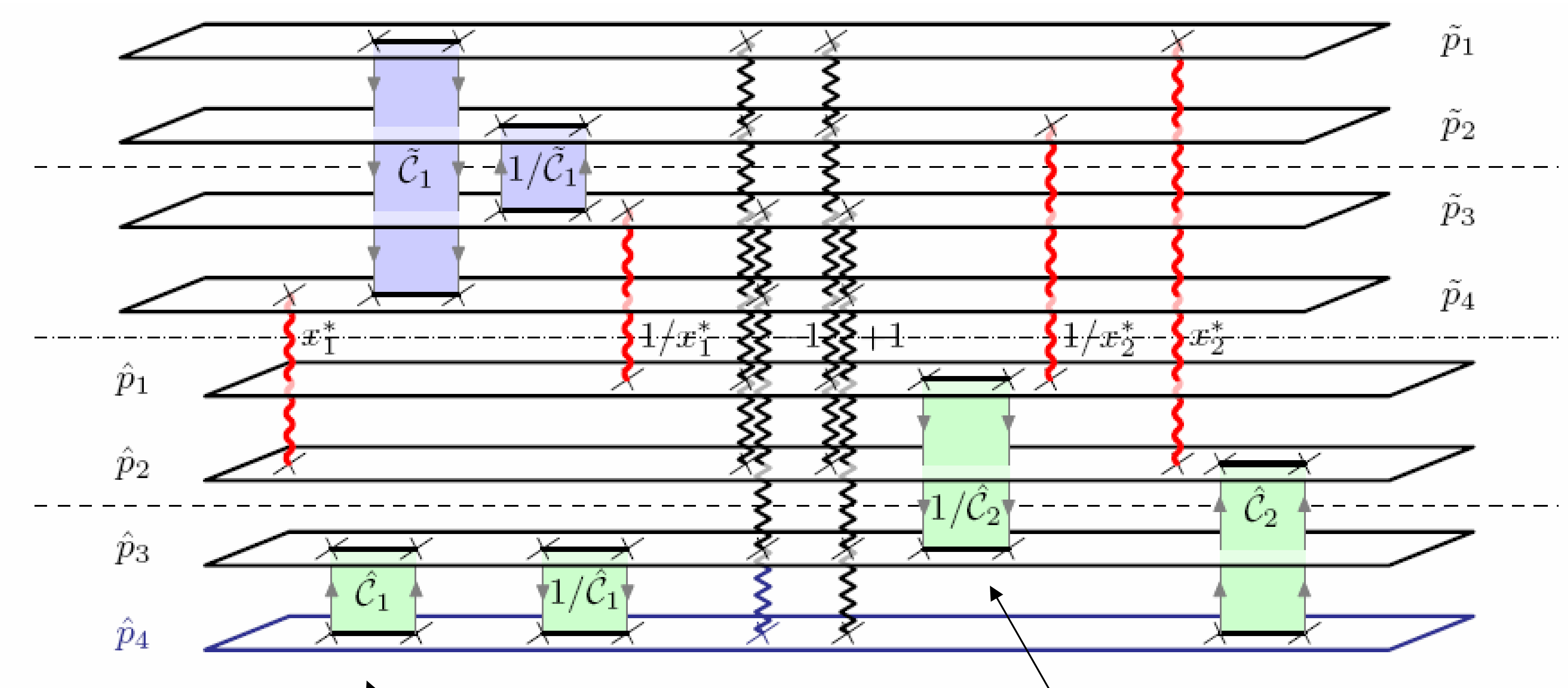
Virasoro constraints fix the poles at $x=\pm 1$

$$\text{str}\ (J^{(2)} \pm * J^{(2)})^2 = 0$$

This synchronises the poles of the S^5 and AdS_5 quasimomenta

Riemann surface

Poles at x=±1 are the synchronised



Related by x to 1/x symmetry

Related by x to 1/x symmetry

Conserved Charges

- Conserved harges: angular momenta, spins J_1, J_2, J_3, S_1, S_2

and energy *E*,

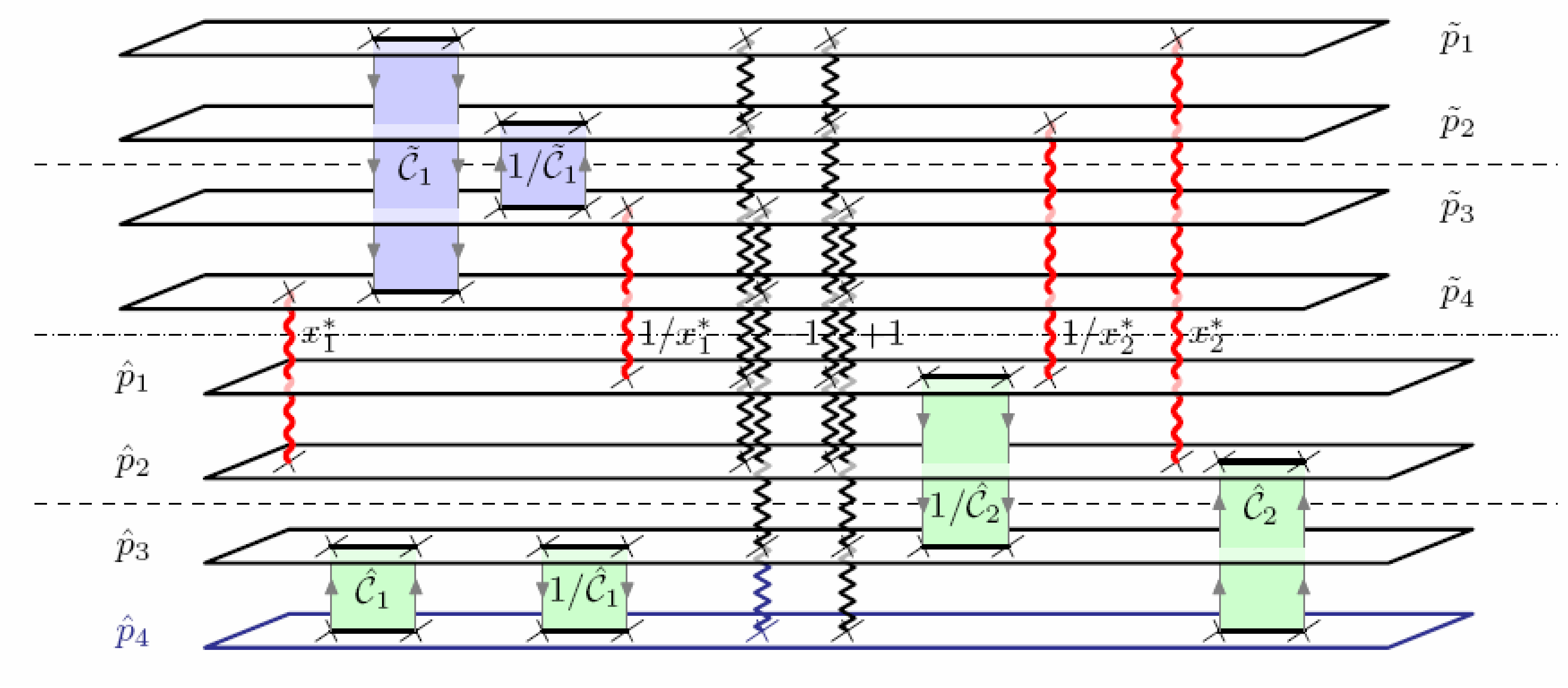
fix the asymptotics at x=0,∞.

$$\mathbf{S^5}: \quad \tilde{p}_1(x) = -\frac{2\pi}{\sqrt{\lambda}}(J_1 + J_2 - J_3)\frac{1}{x} + \dots, \quad etc.$$

$$\mathbf{AdS^5}: \quad \hat{p}_1(x) = \frac{2\pi}{\sqrt{\lambda}}(E + S_1 - S_2)\frac{1}{x} + \dots, \quad \text{etc.}$$

- All these date fix completely the algebraic curve and define Energy E of a state → dimension Δ of operator in SYM.

Riemann surface of the curve



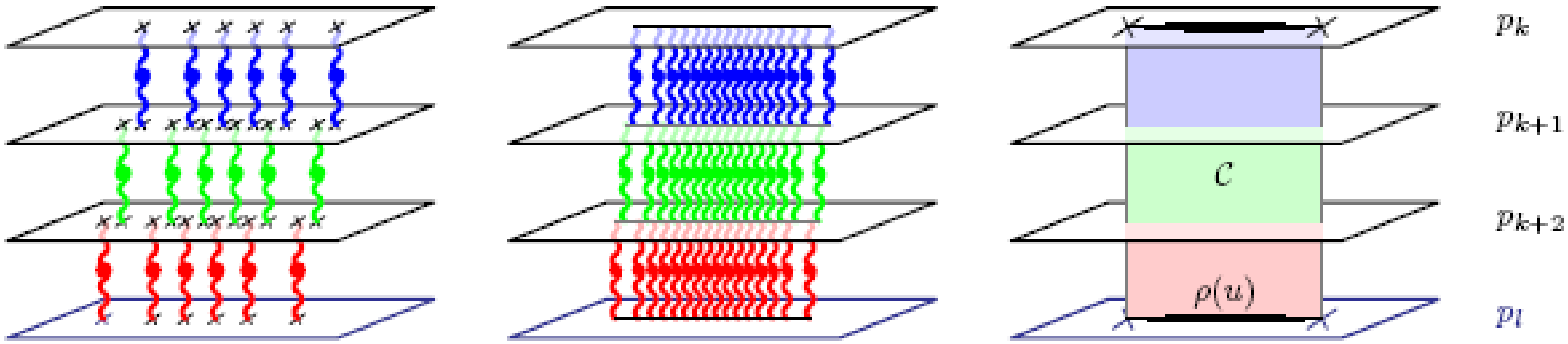
- Algebraic curve encodes all “action” variables;
- “Angle” variables defined by holomorphic integrals.
  (possible to restore corresponding classical string motion).
- Good start for quantization (non-pert. symmetry x→1/x important!)

How to quantize this superstring?

• Condensation of poles (Bethe roots) matches the string cuts

Beisert, V.K., Sakai,Zarembo'05

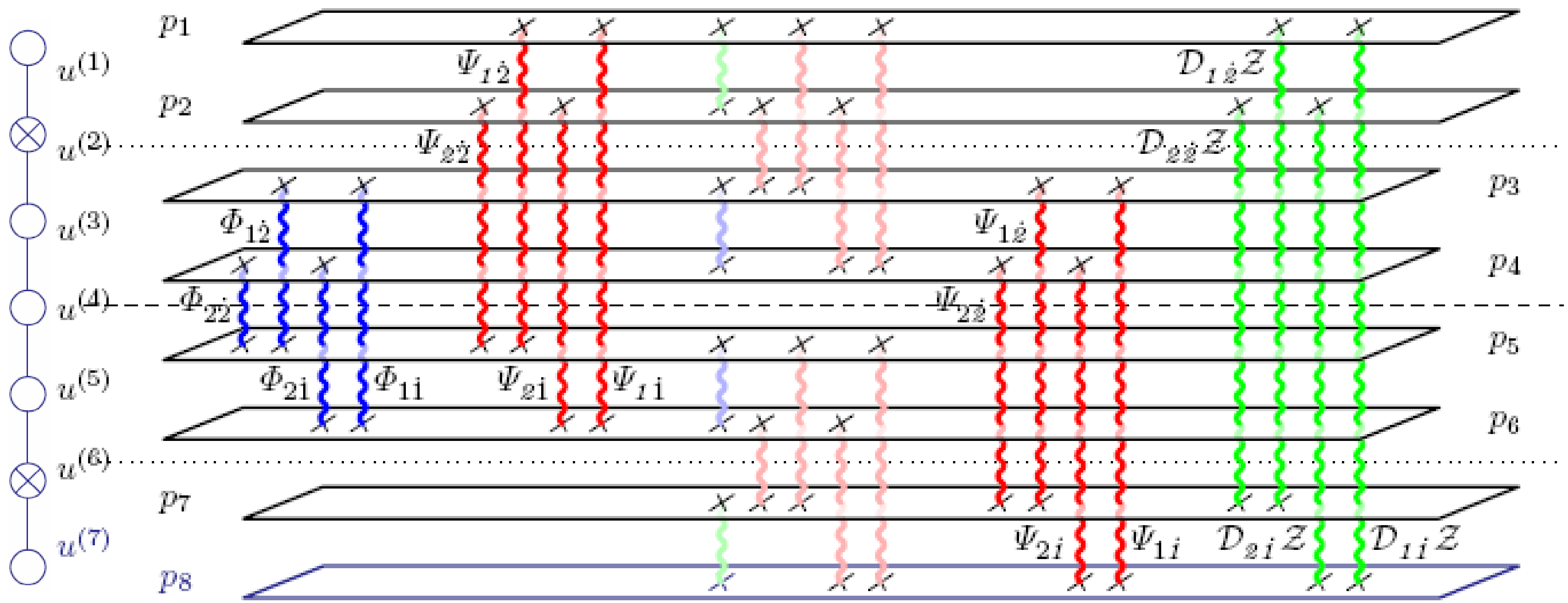
• Formation of cuts from strings of stacks:



• Using single roots as perturbations of finite gap solution one can perform the WKB quantization directly from algebraic curve

Gromov,Vieira '07

Dictionary: Poles - SYM Fields



- To each field of SYM corresponds a Bethe root or stack of roots.
- Bosonic roots with the same mode number n_k condense into cuts in the scaling limit of long operators.
- Fermionic roots stay apart.

- Algebraic curves of string and SYM coincide by the appropriate identification of parameters: AdS/CFT correspondence!

N=4 Supersymmetric Yang-Mills Theory

Gliozzi,Scherk,Olive'77

Action:

$$S = \frac{1}{\lambda} \int d^4x \ \mathrm{tr} \left\{ \frac{1}{4} F^2 + \frac{1}{2} (D_\mu \Phi_I)^2 - \frac{1}{4} [\Phi_I, \Phi_J]^2 + \dot{\Psi} \not{\nabla} \Psi - \frac{1}{2} \Psi [\not{\Phi}, \Psi] - \frac{1}{2} \dot{\Psi} [\not{\Phi}, \dot{\Psi}] \right\}$$

All fields of SYM in ajoint of SU(Nc):

$$\chi \in \{ \mathcal{D}_{\dot{\alpha}\beta}, \Phi_{ab}, \Psi_{\dot{\alpha}b}, \dot{\Psi}^b_{\dot{\alpha}}, \mathcal{F}_{\alpha\beta}, \dot{\mathcal{F}}_{\dot{\alpha}\dot{\beta}} \}$$

- Local operators: $\mathcal{O}(x) = \mathrm{tr}\, [\chi_1(x)\chi_2(x) \ldots \chi_L(x)]$

Superconformal Symmetry

Generators of global superconformal psu(2,2|4) symmetry:

R-symmetry so(6) (scalars) → R^a_b

Poincare: $L^\alpha_\beta, \dot L^{\dot\alpha}_{\dot\beta}, P_{\dot\alpha\beta}$

spec. conf. → $K^{\alpha\dot b}$

Dilatation ("Energy"=Dim.) → D

$$\mathcal{J} = \{R^a_b, \underbrace{\overbrace{L^\alpha_\beta, \dot L^{\dot\alpha}_{\dot\beta}, P_{\dot\alpha\beta}}^{\text{Poincare}}, K^{\alpha\dot b}, D}_{\text{conf. so(2,4)}} | \underbrace{Q^a_{\dot\beta}, \dot Q_{\dot a\beta}}_{\text{SUSY}}, \underbrace{S^\alpha_b, \dot S^{\alpha\dot\beta}}_{\text{superconf.}}\}$$

Algebra relations:

$$\{Q, Q\} = \{S, S\} = \{Q, \dot S\} = \{\dot Q, S\} = 0$$

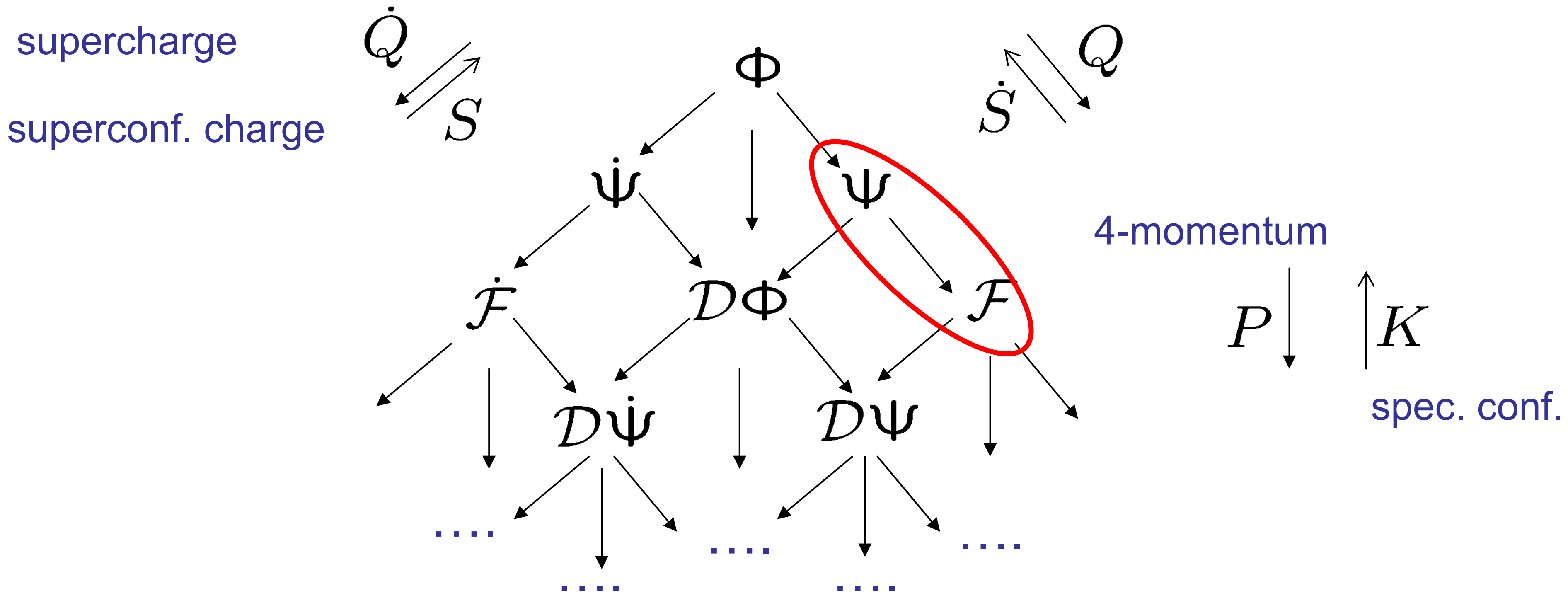
$$\{Q, \dot Q\} = P, \qquad \{S, \dot S\} = K$$

$$\{Q, Q\} = D + R + L$$

$$\{Q, K\} = S$$

Transformation of fields

SYM symmetric under global superconformal symmetry: PSU(2,2|4)



Example:

$$\delta_\epsilon \Psi_{\alpha a} = -\frac{1}{2}\sigma^{\mu}_{\alpha\dot\beta}\varepsilon^{\dot\beta\dot\gamma}\sigma^{\nu}_{\dot\gamma\gamma}\epsilon^{\gamma}_{a}\mathcal{F}_{\mu\nu} + \frac{i}{2}\sqrt{\lambda}\sigma^{m}_{ab}\sigma^{bc}_{n}\varepsilon_{\alpha\beta}\epsilon^{\beta}_{c}\left[\Phi_m, \Phi^n\right]$$

Superalgebra and Beisert`s S-matrix

- Choice of BPS vacuum:

$$\mathrm{Tr}(\,.\,.\,.\,.\,.\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,.\,.\,.\,.\,)$$

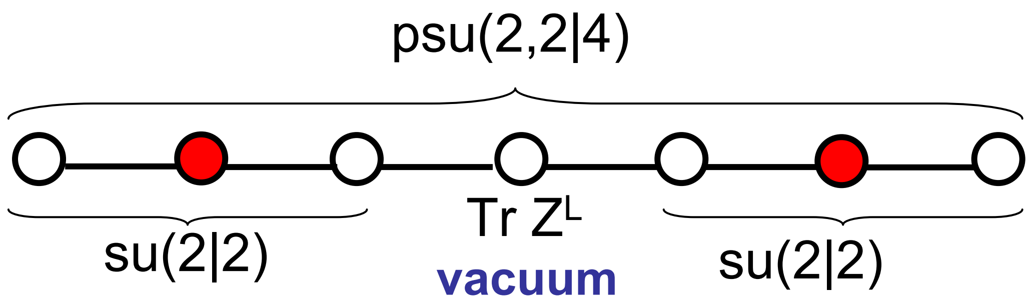
breaks the symmetry:

$$PSU(2,2|4) \;\rightarrow\; SU(2|2) \times SU(2|2)$$

- An operator $\chi = (\Phi, \nabla, \mathcal{F}, \psi)$ inserted with momentum p:

$$\Sigma_k \;\mathrm{Tr}(\ldots\ldots Z\,Z\,Z\,Z\,Z\,Z\,Z\,\overset{k}{\chi}\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z \ldots\ldots) \quad \exp(ikp)$$

Extended su(2|2) Algebra



Algebra relations: $su(2|2) \rightarrow su(2|2) \times R^3$

$$\{Q^\alpha_a, S^b_\beta\} = \delta^b_a \mathcal{L}^\alpha_\beta + \delta^\alpha_\beta \mathcal{R} + \delta^b_a \delta^\alpha_\beta \mathcal{C}$$

$$\{Q^\alpha_a, Q^\beta_b\} = \epsilon^{\alpha\beta}\epsilon_{ab}\mathcal{P}$$

$$\{S^a_\alpha, S^\beta_b\} = \epsilon^{ab}\epsilon_{\alpha\beta}\mathcal{K}$$

central charges

Action on States

- State : a supervector $(\phi_1, \phi_2 | \psi_1, \psi_2)$

$$\begin{aligned} Q_a^\alpha |\phi^b\rangle &= a\, \delta_a^b |\psi^\alpha\rangle \\ Q_a^\alpha |\psi^\beta\rangle &= b\, \epsilon^{ab} \epsilon_{\alpha\beta} |\phi^b \mathcal{Z}^+\rangle \\ S_\alpha^a |\phi^b\rangle &= c\, \epsilon^{ab} \epsilon_{\alpha\beta} |\psi^\beta \mathcal{Z}^-\rangle \\ S_\alpha^a |\psi^\beta\rangle &= d\, \epsilon^{ab} \delta_\alpha^\beta |\phi^a\rangle \end{aligned}$$

- Closure of algebra: $ad - bc = 1$

$$\begin{aligned} \mathcal{C}|\chi\rangle &= \frac{1}{2}(ad + bc)|\chi\rangle \\ \mathcal{P}|\chi\rangle &= ab|\chi \mathcal{Z}^+\rangle \\ \mathcal{K}|\chi\rangle &= cd|\chi \mathcal{Z}^-\rangle \end{aligned}$$

- Without central charges the representation is shortened: ab=cd=0.

Dispersion relation

- Algebra closure on the state

Σ_k Tr(..... Z Z Z Z Z Z Z χ Z Z Z Z Z Z Z Z Z ......) exp(ikp)

fixes

$$\mathcal{P} = \sqrt{\frac{\lambda}{2}}(e^{-ip} - 1)$$

$$\mathcal{K} = \sqrt{\frac{\lambda}{2}}(e^{ip} - 1)$$

$$E = \mathcal{C} = \frac{1}{2}\sqrt{1 + 8\lambda \sin^2\left(\frac{p}{2}\right)}$$

- For multiple insertions of operators:

$$\delta = \Delta - \Delta_0 = \sum_{k=1}^{J}\left(\sqrt{1 + 8\lambda \sin^2\frac{p_k}{2}} - 1\right)$$

Scattering on the SYM Spin Chain

- Scattering of two operators χ_1 , χ_2 and asymptotic S-matrix:

$$\Sigma_{k,m} \, \mathrm{Tr}(\ldots\ldots \; Z \overset{k}{\chi} Z\,Z\,Z \ldots\ldots Z\,Z\,Z\,Z \overset{m}{\chi} Z\,Z \ldots\ldots) \quad \exp(ik\, p_1 - im\, p_2) \; +$$

$$+S_{12}(p_1,p_2) \times \Sigma_{k,m} \, \mathrm{Tr}(\ldots\ldots \; Z \overset{k}{\chi} Z\,Z\,Z \ldots\ldots Z\,Z\,Z\,Z \overset{m}{\chi} Z\,Z \ldots\ldots) \; \exp(ik\, p_2 - im\, p_1)$$

- S-matrix of elementary operator insertions factorizes:

$$S_{PSU(2,2|4)} \rightarrow \sigma^2 \; S_{SU(2|2)} \times S_{SU(2|2)}$$

- Suffices to construct $S_{SU(2|2)}$ and the phase σ^2

Fixing Scattering Matrix

- Action of S-matrix on matrix elements

$$\mathcal{S}_{12}|\phi_2^a\phi_1^b\rangle = A_{12}|\phi_2^{\{a}\phi_1^{b\}}\rangle + B_{12}|\phi_2^{[a}\phi_1^{b]}\rangle + \frac{1}{2}\epsilon^{ab}\epsilon_{\alpha\beta}C_{12}|\psi_2^\alpha\psi_1^\beta\mathcal{Z}^-\rangle$$

$$\mathcal{S}_{12}|\psi_1^\alpha\psi_2^\beta\rangle = D_{12}|\psi_2^{\{\alpha}\psi_1^{\beta\}}\rangle + E_{12}|\psi_2^{[\alpha}\psi_1^{\beta]}\rangle + \frac{1}{2}F_{12}\epsilon^{ab}\epsilon_{\alpha\beta}|\phi_2^a\phi_1^b\mathcal{Z}^+\rangle$$

$$\mathcal{S}_{12}|\phi_1^a\psi_2^\beta\rangle = G_{12}|\psi_2^\beta\phi_1^a\rangle + H_{12}|\phi_2^a\psi_1^{\beta]}\rangle$$

$$\mathcal{S}_{12}|\psi_1^\alpha\phi_2^b\rangle = K_{12}|\psi_2^\alpha\phi_1^b\rangle + L_{12}|\phi_2^b\psi_1^{\alpha]}\rangle$$

- Commutation with any su(2|2) symmetry generator J

$$[J_1 \otimes I + I \otimes J_2,\ S_{12}] = 0$$

fixes the S-matrix completely up to a scalar dressing factor

S-matrix and dressing factor

$$S_{PSU(2,2|4)}(p_1,p_2) \;=\; \sigma^2 \; S_{SU(2|2)} \times S_{SU(2|2)}$$

- Coefficients :

$$A_{12} = S^0_{12}\frac{x_2^+ - x_1^-}{x_2^- - x_1^+}$$

Beisert'06

$$B_{12} = S^0_{12}\frac{x_2^+ - x_1^-}{x_2^- - x_1^+}\left(1 - 2\frac{x_2^- - 1/x_1^+}{x_2^- - 1/x_1^-}\,\frac{x_2^+ - x_1^+}{x_2^+ - x_1^-}\right)$$

....

where $S^0(p_1,p_2) = \frac{1 - 1/(x_2^+ x_1^-)}{1 - 1/(x_2^- x_1^+)}\;\sigma^2_{12}(p_1,p_2)$

- Zhukovsky parametrization used:

$$u = x + 1/x, \qquad x(u) = u + \sqrt{u^2 - 4}, \qquad x^{\pm} = x(u \pm i/2)$$

- Energy and momentum: $e^{ip} = \frac{x^+}{x^-}, \qquad \delta = \sqrt{\lambda}\left(\frac{i}{x^+} - \frac{i}{x^-}\right)$

Dressing Factor and Crossing

- Janik`s crossing equation

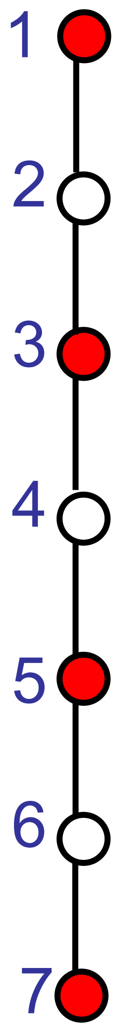
$$\bar{S}_0(p_1,-p_2) = S_0(p_1,p_2)\,\frac{x_2^+ - x_1^+}{x_2^+ - x_1^-}\,\frac{x_2^- - 1/x_1^+}{x_2^- - 1/x_1^-}$$

Janik’05

Does not fix S-matrix but with the extra physical and analytical input it was determined!

Beisert,Ernandes,Lopez’06

Beisedrt,Eden,Staudacher’06



psu(2,2|4)

Beisert-Staudacher eqs.

Beisedrt,Staudacher'05

$$1 = \prod_k e_{-2}\left(u_j^{(2)} - u_k^{(2)}\right) e_{+1}\left(u_j^{(2)} - u_k^{(3)}\right)$$

$$1 = \prod_k e_{-1}\left(u_j^{(3)} - u_k^{(2)}\right) r_{+}\left(u_j^{(3)}, u_k^{(4)}\right)$$

$$\left(\frac{x_j^{(4)+}}{x_j^{(4)-}}\right)^L = \prod_k \sigma^2(x_j^{(4)}|x_k^{(4)}) r_{-}\left(u_j^{(5)}, u_k^{(4)}\right) e_{+2}\left(u_j^{(4)} - u_k^{(4)}\right) r_{-}\left(u_j^{(3)}, u_k^{(4)}\right)$$

$$1 = \prod_k r_{+}\left(u_j^{(5)}, u_k^{(4)}\right) e_{-1}\left(u_j^{(5)} - u_k^{(6)}\right)$$

$$1 = \prod_k e_{+1}\left(u_j^{(6)} - u_k^{(5)}\right) e_{-2}\left(u_j^{(6)} - u_k^{(6)}\right)$$

$$e_k(u) = \frac{u + ik/2}{u - ik/2}, \quad r_{\pm}(u, \tilde{u}) = \frac{x - \tilde{x}^{\pm}}{x - \tilde{x}^{\mp}}.$$

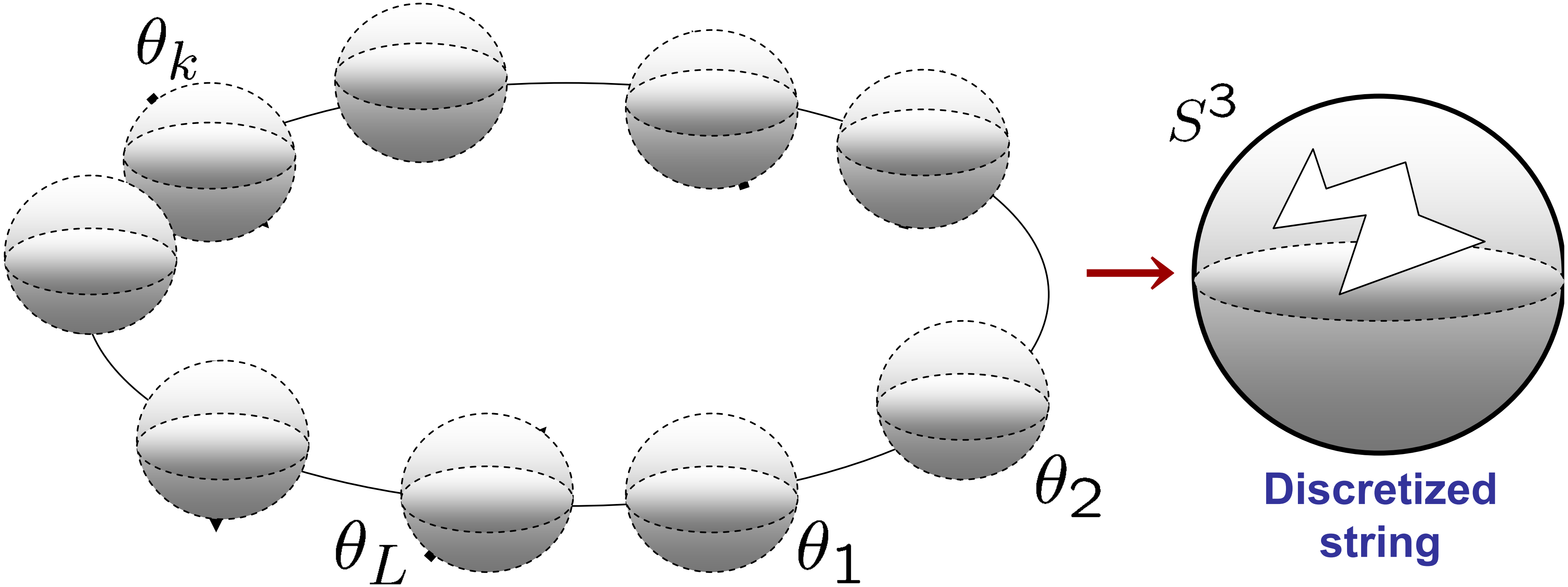
Completely fixes dimensions of long operators of N=4 SYM!

(by rapidities of the middle node)

Can we represent it by inhomogeneous spin chain?

Mann,Polchinski'05
Gromov, V.K.,Sakai,Vieira'06

String σ-Model on $S^3 \times R1$ as a chain of particles



$E = M \cosh\theta$

$p = M \sinh\theta$

- isotopic degrees of freedom: SO(4)

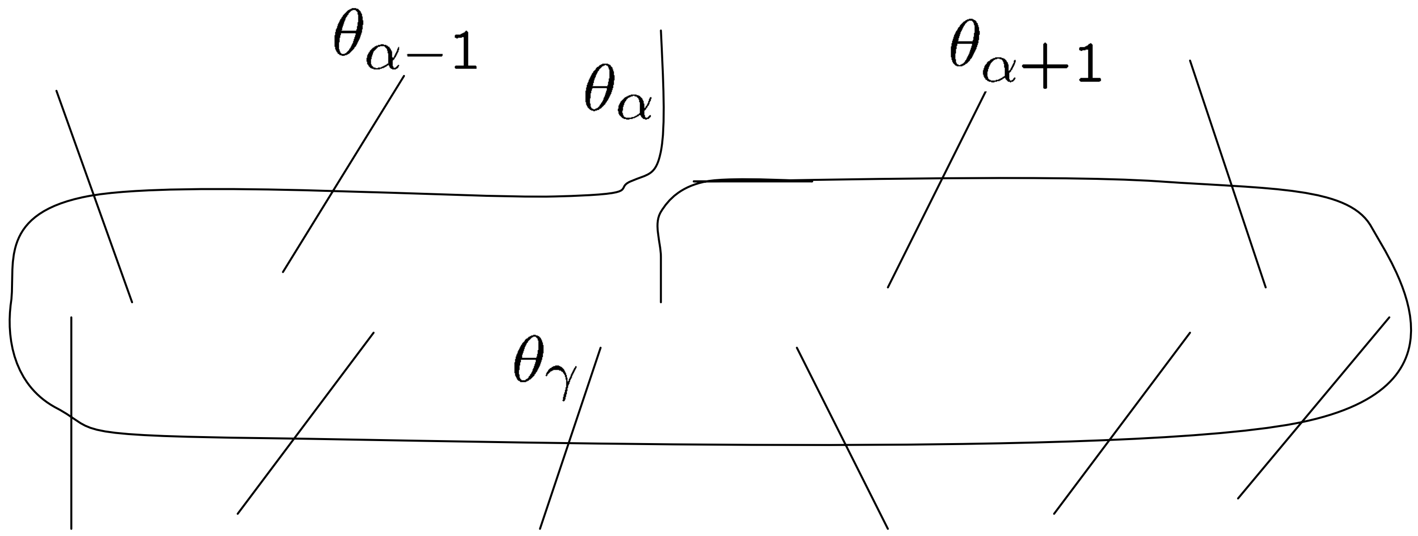
S-matrix $\hat{S}(\theta) \in SO(4)$

Fixed from Yang-Baxter eqs., unitarity, crossing and analyticity:

Zamolodchikovs'79

• Periodicity condition defining the states:

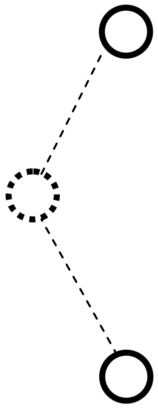
$$e^{-i\mu \,\mathrm{sinh}\, \pi\theta_\alpha} \; |\psi >= \prod_{\beta=\alpha+1}^{L} \hat{S}\left(\theta_\alpha - \theta_\beta\right) \prod_{\gamma=1}^{\alpha-1} \hat{S}\left(\theta_\alpha - \theta_\gamma\right) \; |\psi >$$



- Periodicity condition defining the states:

$$e^{-i\mu \sinh \pi\theta_\alpha} \; |\psi> = \prod_{\beta=\alpha+1}^{L} \hat{S}\left(\theta_\alpha - \theta_\beta\right) \prod_{\gamma=1}^{\alpha-1} \hat{S}\left(\theta_\alpha - \theta_\gamma\right) \; |\psi>$$

- Bethe equations (diagonalization of periodicity condition):



$$\begin{aligned} 1 &= \prod_{\beta}^{J_u} \frac{u_j - \theta_\beta - i/2}{u_j - \theta_\beta + i/2} \prod_{i\neq j}^{J_u} \frac{u_j - u_i + i}{u_j - u_i - i}\,, \\ e^{-i\mu \sinh \pi\theta_\alpha} &= \prod_{\beta\neq\alpha}^{L} S_0^2\left(\theta_\alpha - \theta_\beta\right) \prod_{j}^{J_u} \frac{\theta_\alpha - u_j + i/2}{\theta_\alpha - u_j - i/2} \prod_{k}^{J_v} \frac{\theta_\alpha - v_k + i/2}{\theta_\alpha - v_k - i/2}\,, \\ 1 &= \prod_{\beta}^{J_v} \frac{v_k - \theta_\beta - i/2}{v_k - \theta_\beta + i/2} \prod_{l\neq k}^{J_v} \frac{v_k - v_l + i}{v_k - v_l - i}\,, \end{aligned}$$

- θ -variables describe longitudinal motions of string, u,v “magnon” variables – the transverse.

Gromov,V.K.'06

- Excluding θ 's we reproduce the asymptotic (L, λ→∞) AFS eq. conjectured in

Arutyunov,Frolov,Staudacher'04

$$e^{i\mathrm{p}_k L} \equiv \left(\frac{y^+(u_k)}{y^-(u_k)}\right)^L = \prod_{j=1}^{J} \frac{u_k - u_j + i}{u_k - u_j - i}\, \sigma^2(u_k, u_j)$$

where

$$\sigma(u_k, u_j) = \frac{1 - 1/(y_j^+ y_k^-)}{1 - 1/(y_j^- y_k^+)} \left(\frac{(y_j^- y_k^- - 1)}{(y_j^- y_k^+ - 1)} \frac{(y_j^+ y_k^+ - 1)}{(y_j^+ y_k^- - 1)}\right)^{i(u_j - u_k)}$$

With Zhukovski parametrization:

$$z = x + 1/x, \quad x(2\pi\sqrt{\lambda}\ z) \equiv \frac{1}{2}\left(z + \sqrt{z^2 - 4}\right), \quad y_j^{\pm} = x(u \pm i/2)$$

Energy:

$$\Delta \;=\; L + \sum_{j=1}^{J}\left(\sqrt{1 + \frac{\lambda}{\pi^2}\sin^2\left(p_j/2\right)} - 1\right)$$

(same as for the full AdS/CFT!)

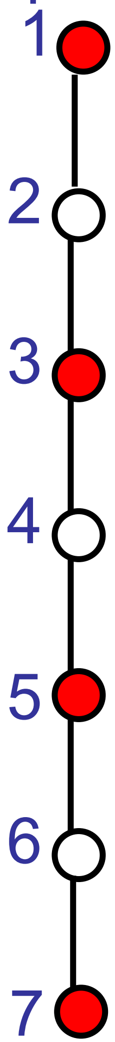
- Reminds old covariant quantization: θ's are not excited (Virasoro cond.)

Full theory

Naively, write the BS eqs. as follows:

Gromov,V.K.,Sakai,Vieira'06
Gromov,V.K.'06

psu(2,2|4)



$$1 = \prod_{\beta=1}^{L} S_0(\theta_\alpha - \theta_\beta) \prod_k e_{+1}(\theta_\alpha - u_k^{(4)})$$

$$-\sqrt{\lambda} < \theta < \sqrt{\lambda}$$

$$1 = \prod_k e_{-2}\left(u_j^{(2)} - u_k^{(2)}\right) e_{+1}\left(u_j^{(2)} - u_k^{(3)}\right)$$

$$1 = \prod_k e_{-1}\left(u_j^{(3)} - u_k^{(2)}\right) r_+\left(u_j^{(3)}, u_k^{(4)}\right)$$

$$\prod_\beta e_{+1}\left(u_j^{(4)} - \theta_\beta\right) = \prod_k r_-\left(u_j^{(5)}, u_k^{(4)}\right) e_{+2}\left(u_j^{(4)} - u_k^{(4)}\right) r_-\left(u_j^{(3)}, u_k^{(4)}\right)$$

$$1 = \prod_k r_+\left(u_j^{(5)}, u_k^{(4)}\right) e_{-1}\left(u_j^{(5)} - u_k^{(6)}\right)$$

$$1 = \prod_k e_{+1}\left(u_j^{(6)} - u_k^{(5)}\right) e_{-2}\left(u_j^{(6)} - u_k^{(6)}\right)$$

The BS eqs are reproduced in the limit $L \longrightarrow \infty.$

For finite L the supersymmetry among multiplets is broken.
May be, a useful building block for the future.