1th Year Talk - PhD Course XXX

QCD Confinement and Center Symmetry

Ver 0.6

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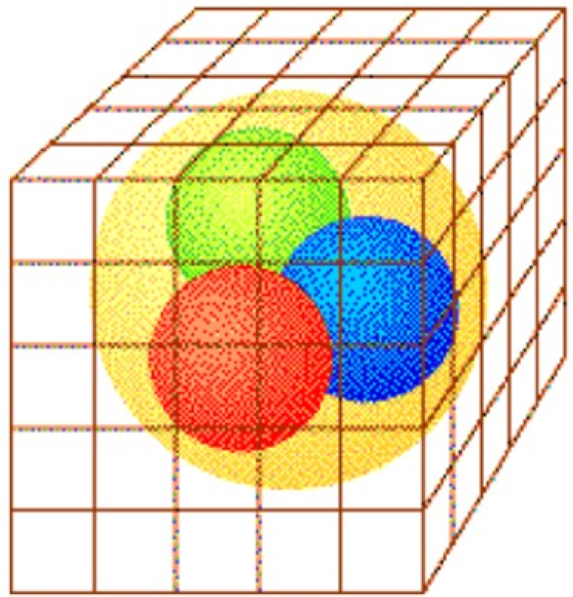
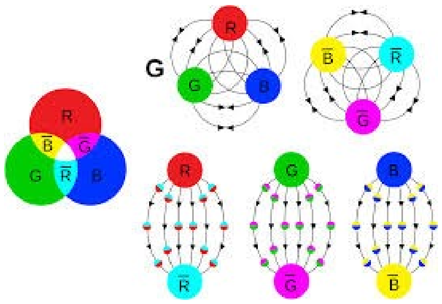
Confinement



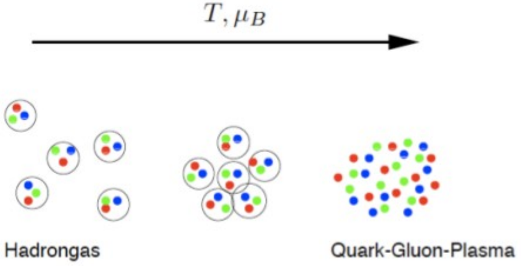
No free quarks are observed in Nature



All the known hadrons are singlets under the SU(3) color group

$$\psi_\pi = \sum_i \bar{q}_i q_i,$$
$$\psi_{p,n} = \sum_{ijk} \epsilon_{ijk} q_i q_j q_k$$

At temperature T=270 Mev, Yang-Mills theory goes through a "deconfinement" transition: the quark free energy (measured by the Polyakov loop) become finite and hadrons dissolve into their constituents.



Deconfinement phase transition

Chiral condensate: $\bar{\psi}\psi = \bar{L}R + \bar{R}L$

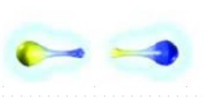
Polyakov loop: $P = Tr\, Pe^{-ig\oint A_4(\vec{x})\, d\tau}$

- T=0
  - quarks are confined:
  - chiral symmetry is broken
  - Z symmetry is restored

$$\langle\bar{\psi}\psi\rangle \sim -(240\ Mev)^2 \neq 0$$
$$\langle P\rangle = 0$$

$$F_q = \infty$$

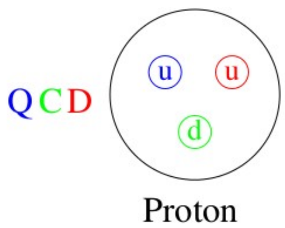
- T=Tc
  - quarks become deconfined
  - chiral symmetry is restored
  - Z symmetry is broken



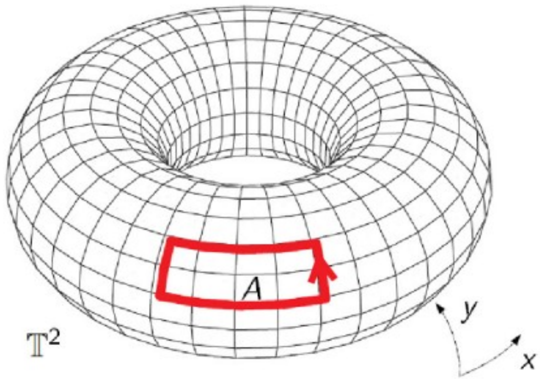
$$F_q < \infty$$

$$\langle\bar{\psi}\psi\rangle = 0$$
$$\langle P\rangle \neq 0$$

Overview

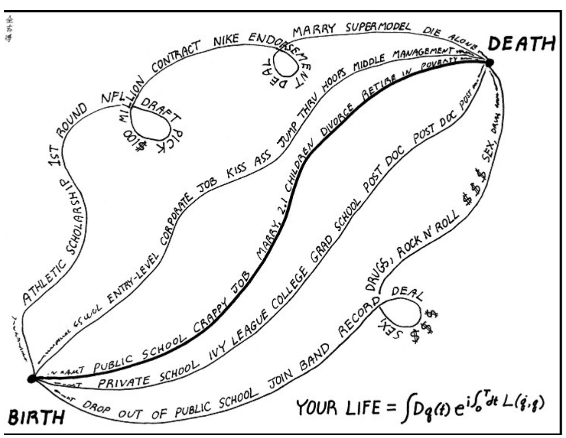


- Fields
- Lagrangian
- Gauge transforms



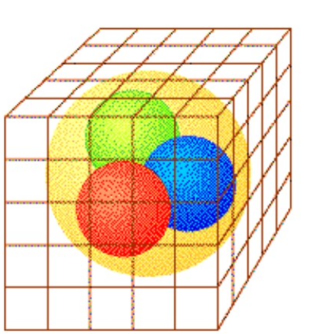
- Lattice basic
- Monte Carlo
- Chiral condensate
- Polyakov loop

QCD



$$\langle O\rangle = \frac{\int \mathcal{D}A\, \mathcal{D}[\bar\psi,\psi]\, e^{-S[\psi,A]}\, O[\psi,A]}{\int \mathcal{D}A\, \mathcal{D}[\bar\psi,\psi]\, e^{-S[\psi,A]}}$$

- Center Symmetry
- Symmetry Breaking
- Confinement



$$z = e^{i\frac{2\pi}{N}k}$$

Quantum ChromoDynamics (QCD)

Quantum ChromoDynamics (QCD) is the established fundamental theory of strong force. It describes the fundamental interaction of elementary particles named quarks and gluons.

$$\mathcal{L}_{QCD} = \frac{1}{2} Tr\, F_{\mu\nu} F_{\mu\nu} + \bar{\psi}(D+m)\psi$$

Dirac operator

$$D = \gamma_\mu (\partial_\mu + igA_\mu)$$

$T^a: \; a = 1 \ldots N_c^2 - 1$ — $SU(N)$ group generators

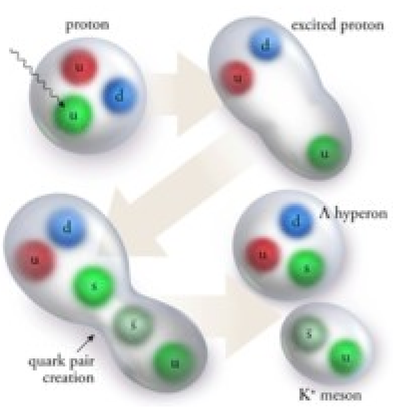
$A_\mu = \sum_a T^a A_\mu^a(x)$ — Gauge fields $(4 \, x \, N_c \, x \, N_c)$

$\psi(x) \equiv \psi_{\text{spin, color,flavor, ...}}(x)$ — Quark Dirac spinor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

Lie Algebra:

$$\begin{cases} Tr[T^a T^b] = \frac{1}{2}\delta_{ab} \\ [T^a, T^b] = i f^{abc} T^b T^c \end{cases}$$



Gauge transforms

$$\Omega(x) = e^{-i\sum_a T^a \omega_a(x)}$$

$$\psi \rightarrow \Omega\psi, \quad \bar{\psi} \rightarrow \bar{\psi}\Omega^{-1},$$

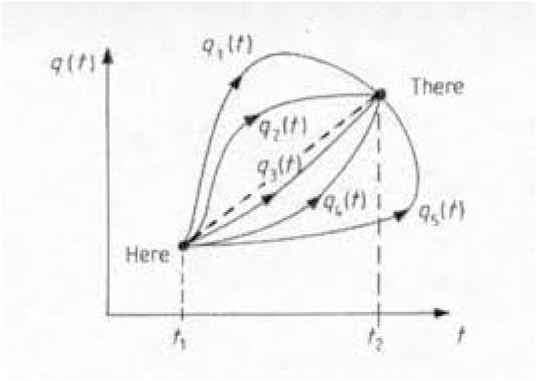
$$A_\mu \rightarrow \Omega A_\mu \Omega^{-1} + \frac{1}{ig}\Omega \partial_\mu \Omega^{-1}$$

Path-Integral formulation

- Effective "gluon" partition function

$$Z=\int \mathcal{D}[A]e^{-S_{eff}[A]}$$

The Euclidean Path integral is the basic tool for quantizing fields on the lattice.



- Where:

$$e^{-S_{eff}[A]}=e^{-S_G[A]}\cdot|M|$$

$$S_G[A]=\frac{1}{2}\int d^4x\, Tr\, F_{\mu\nu}F^{\mu\nu}$$

Pure gauge part

$$(\bar\psi_1\ \ \bar\psi_2\ \cdots)\begin{pmatrix} M_{11} & M_{12} & \cdots \\ M_{21} & \ddots & M_{31} \\ \cdots & M_{32} & \ddots \end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\\ \vdots\end{pmatrix}$$

Gaussian integration of the fermionic part

$$\int \mathcal{D}\psi\mathcal{D}\bar\psi\, e^{-\int d^4x\,\bar\psi(D+m)\psi}=|M|=|m+D|$$

$$|M|=e^{Tr\ln[M]}=e^{Tr\ln[m+D]}=e^{-\sum_{k=0}^{\infty}\frac{(-1)^k}{k\,m^k}Tr\,D^k}$$

Fermionic determinant expansion (highly non local)

$$|M|>0$$

$$S_{eff}[A]=S_G[A]+\sum_{k=1}^{\infty}\frac{(-1)^k}{m^k k}Tr\,D^k[A]$$

$\frac{1}{m_q}$ expansion (static limit)

$$\mathcal{D}\bar\psi\mathcal{D}\psi=\prod_{x,\text{color},\text{flavor},\ldots}d\bar\psi_x d\psi_x$$

$$\mathcal{D}[A]=\prod_{x,\mu,a}dA^a_{x,\mu}$$

Euclidean formulation:

$$t=-ix_4$$
$$d^4x=-id^4x_E$$
$$x_\mu x^\mu=-x^E_\mu x^E_\mu$$

$$\gamma_0=\gamma_i^E,\ \gamma_i=-i\gamma_i^E$$
$$\{\gamma^E_\mu,\gamma^E_\nu\}=2\delta_{\mu\nu}$$

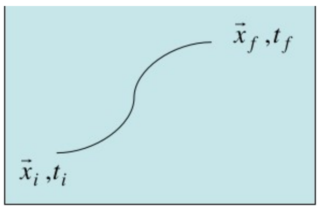
Lattice: basic

LatticeQCD is a non-perturbative approach to the QCD gauge theory, based on a discretization of the QCD action which preserves gauge invariance at all stages.

$$\psi_x \sim U_{xy}\,\psi_y$$
$$U \rightarrow \Omega \cdot U \cdot \Omega^{-1}$$

$$\bar{\psi}_x \; \overbrace{U_{xy}\,\psi_y}^{\text{transform as } \psi_x}$$

gauge invariant



$$U_{x,\mu} = P e^{-ig\int_x^{x+a\hat{\mu}} A_\mu(x)\,dx^\mu}$$

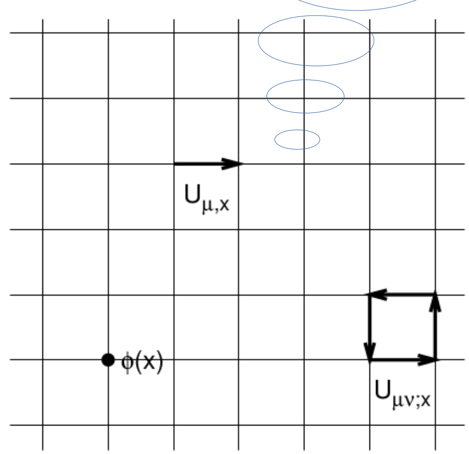
Parallel transporter
x→ y (as in RG)

Quarks pick up an appropriate non-abelian phase when hopping from a site to the other.

$$U_\mu \approx_{a\to 0} e^{-ig\,a\,A_\mu}$$

Lattice:

$$\Lambda = a\mathbb{Z}^4 = \{x \,|\, \frac{x_\mu}{a} \in \mathbb{Z}\}$$
$$\psi(x), A_\mu(x) \;\in\; \Lambda$$



$U_\mu \in SU(N)$

$$f'(x) \approx \frac{f(x+a) - f(x)}{a}$$

Covariant derivatives:

$$\nabla_\mu \psi \simeq \frac{1}{a}(U_\mu \psi_{x+a\hat{\mu}} - \psi_x)$$
$$\nabla_\mu^* \psi \simeq \frac{1}{a}(\psi_x - U_{-\mu}\psi_{x-a\hat{\mu}})$$

Momentum cutoff:

$$\psi(x) = -1, +1, -1, +1, \ldots$$
$$\lambda > 2a \;\rightarrow\; -\frac{\pi}{a} \le p \le \frac{\pi}{a}$$

Wilson operator:

$$D_W = \frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla_\mu^*) + r \cdot a \cdot \nabla_\mu \cdot \nabla_\mu^*$$

Improvements:

Twisted mass, staggered , DMF, etc...

Lattice Action

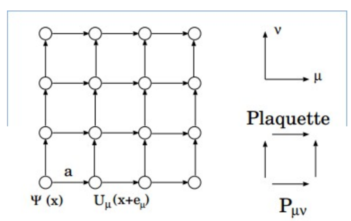
$$S_{eff}[U] = \underbrace{-\beta_{lat}\frac{1}{N}\sum_P \Re\, Tr[U_P]}_{S_G} + \underbrace{\sum_k \frac{(-1)^k}{k}\frac{Tr\, D^k[U]}{m^k}}_{S_F}$$

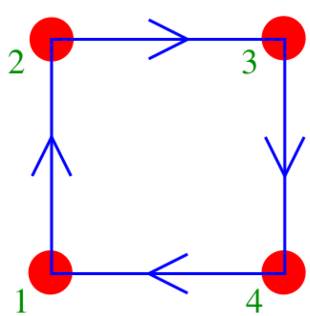
$$\beta_{lat} = \frac{2N_C}{g^2}$$

$$e^{-S_{eff}} = e^{-S_G}|M|$$

Gluonic part

- *Gluonic part* contains only the trace of *plaquette* operators





$U_p = U_{1,2}U_{2,3}U_{3,4}U_{4,1}$

$$U_P = e^{-ig\oint_P A_\mu(x)\, dx_\mu}$$

$a \to 0$

$$e^{-iga^2 F_{\mu\nu}}$$

Stokes theorem

$$\frac{1}{2}\int d^4x\, Tr\, F_{\mu\nu}F^{\mu\nu}$$

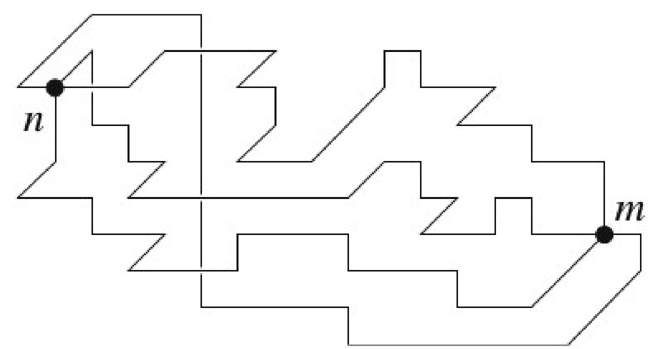
Fermionic part

- *Fermion Kinetic operator espansion:*

$$M_{x,y} = m\cdot\delta_{x,y} + \frac{1}{2}[\gamma_\mu U_\mu \delta_{x+a\hat{\mu},y} - \gamma_\mu U_{-\mu}\delta_{x-a\hat{\mu},y}]$$

Discretized form

$$|M| = e^{Tr\ln[M]} = e^{Tr\ln[m+D]} \propto e^{-\sum_{k=0}^{\infty}\frac{(-1)^k}{km^k}Tr\, D^k}$$



Static quark limit $m \to \infty$

D^k

- Is made up of parallel transports U, connecting lattice lattice points n-m
- Trace means closed paths n=m

$$Tr[D^k] = Tr[\underbrace{D\, D\, D\, \ldots .\, D}_{\text{k times}}]$$

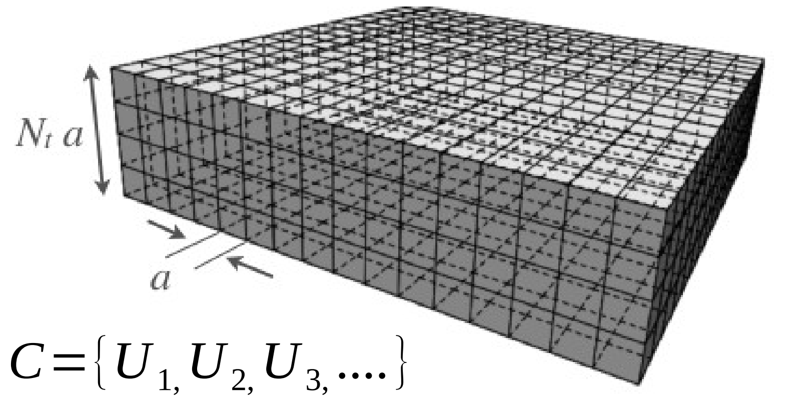
Monte Carlo

$10\cdot 10\cdot 10\cdot 10 = 10^4$ lattice

$10^4\cdot 4\cdot 8 = 320{,}000 \quad dxdydz\ldots$ integrals

10 points/dimension $\Rightarrow 10^{320000}$ terms!

age of universe $\sim 10^{27}$ nanoseconds



$C = \{U_1, U_2, U_3, \ldots\}$

Statistical Methods:

$$\langle O\rangle = \frac{\int \mathcal{D}U\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\, e^{-S} O}{\int \mathcal{D}U\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\, e^{-S}} \Rightarrow \langle O\rangle = \frac{\sum_C e^{-S(C)} O[C]}{\sum_C e^{-S(C)}}$$

clusters

Metropolis algorithm:

generate new configuration C randomly and accept it with probability P(C)

$$C_1 \rightarrow C_2 \rightarrow C_3 \rightarrow \cdots \qquad P(C) \sim e^{-S(C)}$$

Boltzmann factor

temperature	Time extension
T	$\tau=\frac{1}{T}=\beta$

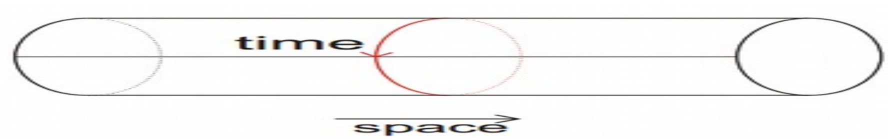
Finite temperature

- Compact time

$$\text{Time}=\frac{1}{\text{Temperature}}$$

restrition of euclidean time $\in[0,\beta]$ and periodic boundary condition

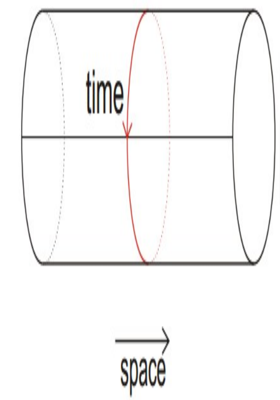
$$\psi(t+\beta)=-\psi(t)$$
$$A_\mu(t+\beta)=A_\mu(t)$$



"High T"

$$\tau\approx\frac{1}{T}$$

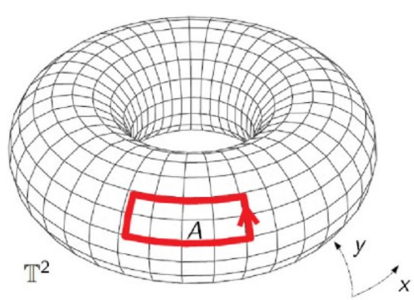
"Low T"



- Time lattice extension:

$N_t\cdot N_s^3=$ lattice grid

$$\tau=N_t a \qquad T=\frac{1}{N_t a}$$



$R^3\, x\, S^1$

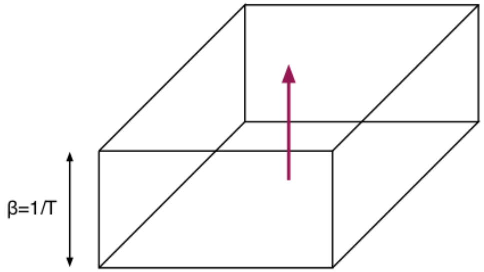
$N_t \ll N_s \rightarrow$ large T

$a\approx 0\rightarrow$ large T

Finite temperature (II)

- Partition function as functional integral:

$$Z = Tr[e^{-H/T}] = \sum_n \langle n | e^{-\tau \cdot H} | n \rangle_{\tau = 1/T = \beta} = \int_{\varphi(0)=\varphi(\beta)} \mathcal{D}\varphi \, e^{-S[\varphi]}$$



$$\tau = \beta = \frac{1}{T} = N_t a$$

$$T = \frac{1}{N_t a}$$

- RG results

$$a = \frac{1}{\Lambda_L} (b_0 g^2)^{\frac{b_1}{2 b_0^2}} e^{-\frac{1}{2 b_0 g^2}}$$

$b_{0,} b_{1,} \ldots = \text{RG constants}$

$\Lambda_L \approx 200 \; MeV$

To vary T:

- Fixed Nt approach

for a given N_t ,vary lattice spacing using the function $a(g)$

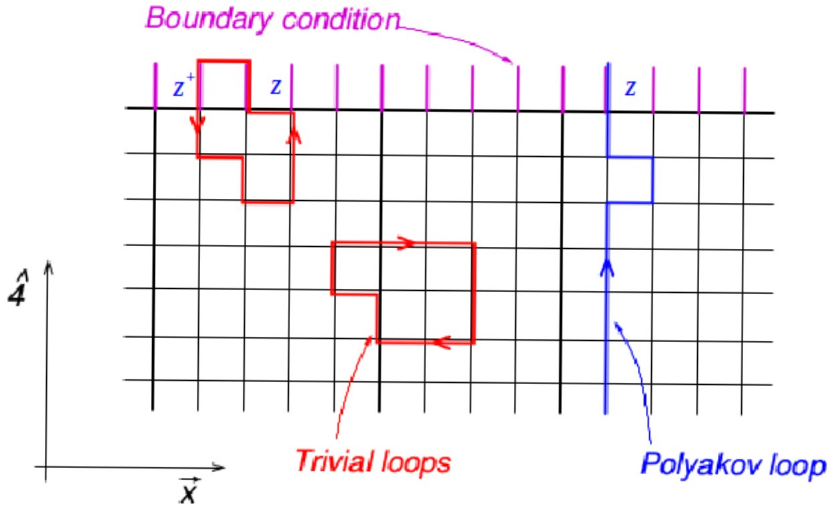
Inverse coupling: $\beta_{lat} = 2 \frac{N}{g^2}$

$a \to \infty$	$g \to \infty$	$\beta_{lat} \to 0$	$T \to 0$	confinement
$a \to 0$	$g \to 0$	$\beta_{lat} \to \infty$	$T \to \infty$	de-confinement

- Fixed scale approach

$N_t \ll N_s \to \text{large T}$

Polyakov and Wilson loops



- Wilson loop

Prototype of gauge-invariant object made from only gauge fields.

$$W(C)=Tr\,P e^{ig\oint_C A_\mu(x)dx^\mu}$$

- Polyakov loop

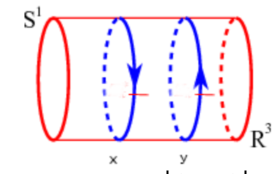
Wilson line: world-line of a **massive** static quark at fixed spatial position x, propagating through the periodic time direction.

$$P(\vec{x})=Tr\,P e^{ig\int_0^\beta A_4(\vec{x})d\tau}$$

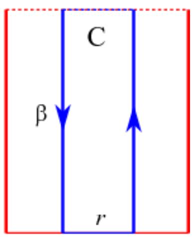
- Polyakov loop correlations

$$\tau\equiv\beta=\frac{1}{T}$$

Two Polyakov loops, having opposite orientations, and spatial distance r=x-y.



$$r=|\vec{x}-\vec{y}|$$



Space correlator between quarks propagators:

$$D(r)\equiv\langle P(x)P^+(y)\rangle_{r=|x-y|}$$

- Large loop behaviour

$\beta\to\infty$

$$\langle P(\vec{x})\rangle\sim e^{-\beta\cdot F_q}$$

F_q = Gibbs energy to create a static quark q at $\vec{x}$

$r\to\infty$

$$\langle P(x)P^+(y)\rangle_{r=|x-y|}\underbrace{\to}_{r\to\infty} e^{-\beta\cdot V_{\bar{q}q}(r)}$$

$V_{\bar{q}q}(r)$= static $q\bar{q}$ potential

Polyakov and Wilson loops (II)

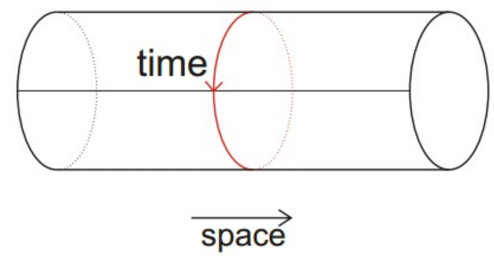
appendix

Physical interpretation

- Polyakov loop and Free Energy

Definition of F (Gibbs function)

$$e^{-\beta F} = Tr[e^{-H/T}] = \sum_n \langle n | e^{-\tau \cdot H} | n \rangle_{\tau=\beta=1/T} =$$
$$= \sum_n e^{-\beta E_n} =$$
$$= \sum_\psi \sum_U e^{-S_{FG}} Tr\, \psi_\tau^+ U_\tau U \ldots U_0 \psi_0 =$$
$$= \sum_U e^{-S_G} Tr[UU \ldots U]_{0\tau}$$



$S_{o\tau} = \psi_\tau^+ U_\tau U \ldots U_0 \psi_0$: quark propagator $0 \rightarrow \tau$

… but $$\sum_U e^{-S_G} \underbrace{Tr[UU \ldots U]_{0\tau}}_{\langle Tr\, P e^{ig\int_0^\beta A_0(\vec{x}) d\tau} \rangle_G} \rightarrow \langle P(\vec{x}) \rangle$$ q.e.d

- Polyakov loop correlator and V(r)

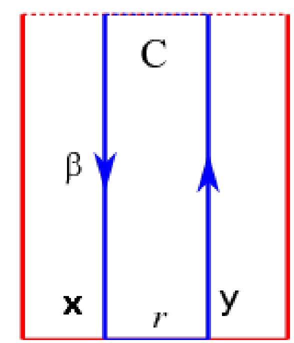
Guassian integration over quarks

$$\langle Tr[\psi_x S_{xy}(0) \psi_y^+]_0 [\psi_y S_{xy}^+(\tau) \psi_x^+]_\tau \rangle_{FG} \rightarrow \langle Tr\, S_{xy}(0) S_{xy}^+(\tau) \rangle_G$$

Time propagation and Trace

$$\langle Tr\, S_{xy}(0) S_{xy}^+(\tau) \rangle_G =$$
$$= \sum_n \langle 0 | S_{xy}(0) | n \rangle \langle n | S_{xy}^+(\tau) | 0 \rangle =$$
$$= \sum_n \langle n | S_{xy}^+(0) | 0 \rangle^2 e^{-\tau E_n} =$$
$$\approx e^{-\tau V(r)} \text{ if } \tau \rightarrow \infty$$ q.e.d

$|n\rangle = q\bar{q}$ states



N.B. Fixing the gauge, this is the Wilson loop

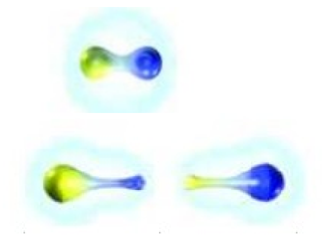
$$\langle Tr\, S_{xy}(0) S_{xy}^+(\tau) \rangle_G \underbrace{\rightarrow}_{\text{temporal gauge } A_0=0} \langle W_C \rangle$$

$S_{xy}(t) = \psi_x^+ U_x U_{x-1} \ldots U_y \psi_y$: quark propagator $x \rightarrow y$

Confinement criteria

$\langle P \rangle = 0$ confinement

$\langle P \rangle \neq 0$ deconfinement



$$\langle P \rangle = \frac{1}{N_s^3} \sum_x P(\vec{x})$$

Spatial invariance

- Hints:

$$\langle P(x) P^+(y) \rangle_{r=|x-y|} \rightarrow e^{-\beta \cdot V_{\bar{q}q}(r)}$$

Cluster limit $r \rightarrow \infty$

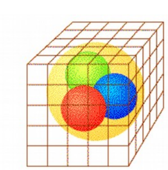
$$\langle P(x) P^+(y) \rangle_{r=|(x-y)|} \rightarrow |\langle P \rangle|^2$$

$D(\infty) \neq 0$
→ big correlation length
→ order at big distances
→ deconfinement

Order and confinement

So, if $\langle P \rangle = 0$ then $D(r)_{r \rightarrow \infty} \approx 0$ ⇒ $V(r)_{\bar{q}q} \approx_{r \rightarrow \infty} \infty$

Confinement!



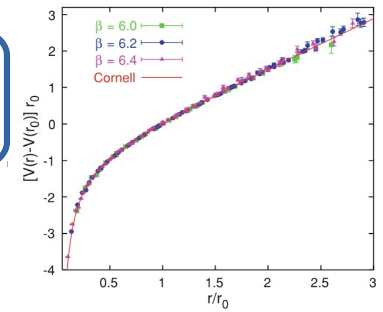
Static Quark-AntiQuark potential

appendix

It is generally believed that quark confinement is a consequence of the non-abelian nature of the gauge interaction in QCD

$$V_{\bar{q}q}(r) \approx a - \frac{b}{r} + \sigma r$$

Interpolation formula
"Cornell potential"



$$\beta = \frac{2N_c}{g^2}$$

- Coulomb part evidences $V(r)_{r\to 0} \simeq -\frac{b}{r}$

The non-linear, self-interacting term, in the F tensor is multiplied by the coupling costant g. So, for small g, F reduces to its abelian counterpart (QED). This suggests a Coulumbian-type interaction.

$$g \to 0$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

Stockes Theorem:

$$\langle W_C \rangle = \langle e^{ig\oint_C A\,dx} \rangle \sim \langle e^{g\int_{area} F_{\mu\nu} d^2x} \rangle \sim e^{c(\frac{g^2}{r^2})\cdot r\tau}$$

$$\to \quad c(\frac{g^2}{r^2})\cdot r\tau = -V\tau \;,\; V = -c\frac{g^2}{r}$$

c: constant depending from group structure

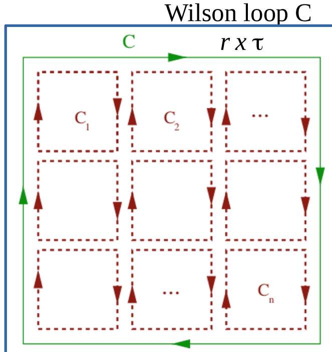
- Linear part evidences $V(r)_{r\to\infty} \simeq \sigma r$

The leading term in the Taylor expansion of the Boltzmann factor in power of the inverse coupling constant exibits an "area law". This suggests a linear term.

$$\langle e^{ig\oint_C A dx} \rangle = e^{-\sigma A(C)} = e^{-\sigma r\tau} = e^{-\tau V}$$
$$\to \sigma r\tau = \tau V(r)$$
$$\to V(r) \approx \sigma r$$

$$\beta_{lat} \to 0$$

Wilson loop C



Group Integration rules: (Haar)

$$\int DU\, U_i U_j^+ = \frac{1}{N_c}\delta_{ij} \qquad \int dU\, U_i U_j U_k \ldots = 0$$

$$U_i = \text{link along path i: } x_a \to x_b$$

$$(\text{Hints: } U_i \sim e^{iA} \to \text{phase oscill. indip.} \to \langle U_i \rangle = 0\,)$$

$$\langle W_C \rangle = \int DU\, e^{\sum_p \frac{2}{g^2}\Re\, Tr[U_p]}\, Tr \prod_{i\in C} U_i =$$

$$= \int DU \sum_n c_n (\frac{1}{g^2})^n\, Tr \prod_{Area} U^+\, Tr \prod_{Perim} U =$$

$$= \prod_{i=1}^{\frac{A(C)}{a^2}} \frac{1}{g^2} \langle U U^+ \rangle \simeq (\frac{1}{g^2 N_c})^{\frac{r\tau}{a^2}} \simeq e^{\frac{r\tau}{a^2}\ln(\frac{1}{g^2 N_c})} \simeq e^{-\sigma A(C)}$$

"string tension"

$$\sigma = \frac{-1}{a^2}\ln(\frac{1}{g^2 N_c})$$

"Area law"

Center Group Z(N)

z commute with each SU(N) element

$$Z(N):\quad zU=Uz\,,\ \ \forall\, U\in SU(N)$$

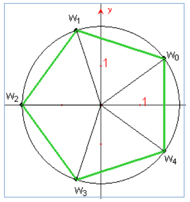
$$Z=\begin{pmatrix} z & 0 & 0\\ 0 & z & 0\\ 0 & 0 & \ddots \end{pmatrix}$$

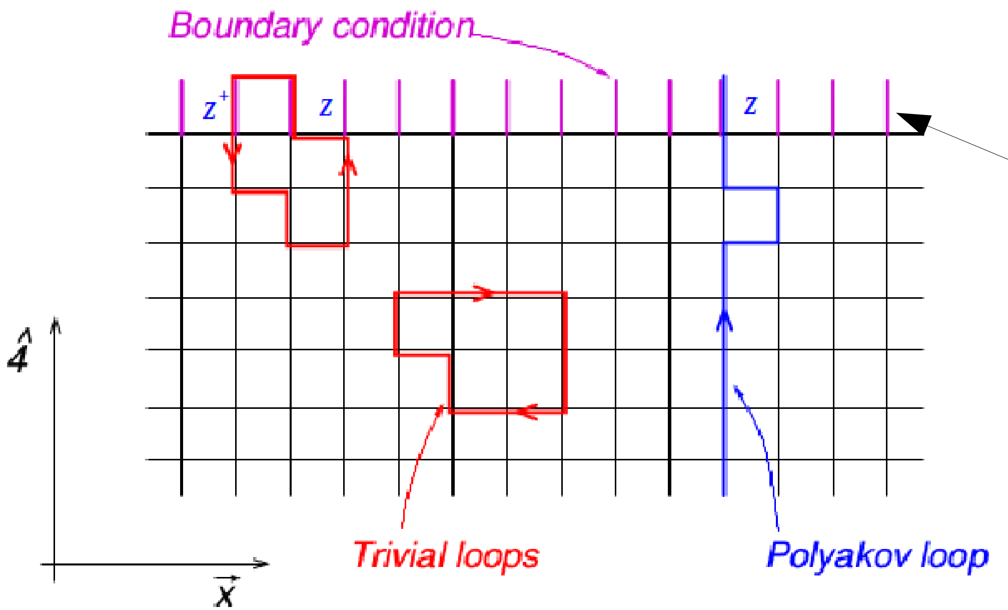
$$Det(z)=1\rightarrow z^{N_c}=1$$

$$z_k=e^{i\frac{2\pi}{N}k}\,,\quad k=0,1,2,\cdots,N-1$$

$$\sum_k^{N-1} z^k=\frac{1+z+z^2+z^3\ldots}{N}=0$$

Polygonal property of unit roots





Global center transformation:

$$U_0(\vec{x},t_0)\rightarrow z\,U_0(\vec{x},t_0)\,,\quad \forall\,\vec{x}$$

Trival loops are invariant: $\langle W'\rangle=\langle W\rangle$

$$Tr[U\ldots zU\ldots z^+U]=zz^+\cdot Tr[UU\ldots U]$$

Polyakov loops are NOT invariant: $\langle P'\rangle=z\langle P\rangle$

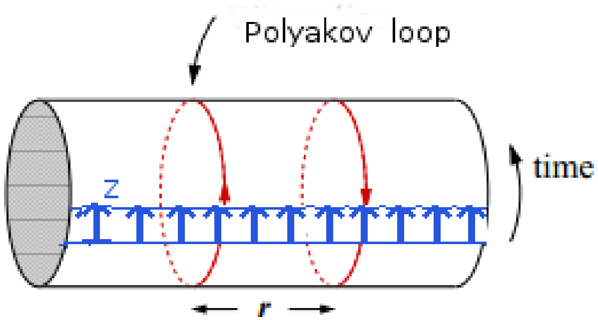
$$Tr[U\ldots zU\ldots U]=z\cdot Tr[UU\ldots U]$$

If a loop winds q times: $\langle P'\rangle=z^q\langle P\rangle$

Polyakov loop $\quad P(\vec{x})=Tr\prod_{j\in C_{[0,\beta]}} U_0(x,j)=Tr[U_0U_0\ldots U_0]_{0\beta}$

Wilson loop $\quad W(C)=Tr\prod_{j\in C} U_j=Tr[U_1U_2\ldots U^+\ldots U_n^+]_C$

Polyakov loops and Z symmetry



$$\langle P \rangle = \frac{1}{N_s^3} \sum_x P(\vec{x})$$

Basic fact: If a loop winds non trivially q times around the compact time direction, then:

$$\langle P' \rangle = z^q \langle P \rangle \text{ under } Z(N)$$

- Pure gauge (no quarks)

With no quarks, the action S contains only plaquettes (i.e. trivial loop, with q=0): S is invariant under Z(N), but P isn't.

$$\begin{cases} \langle P \rangle & \rightarrow & z\langle P \rangle \\ S & \rightarrow & S \end{cases}$$

If Z(N) is a true (unbroken) symmetry, the configurations related by center symmetry will occour with the same probability, and the expectation value <P> must vanish:

$$\frac{1+z+z^2}{3} = 0$$

$$\langle P \rangle = \langle \frac{P_0 + zP_1 + z^2 P_2}{3} \rangle = \frac{1+z+z^2}{3} \langle P_0 \rangle \; = \; 0$$

So, the spontaneous breaking of the Z(N) symmetry signals deconfinement:

$$Z_N \rightarrow \begin{cases} \text{unbroken} & \rightarrow & \langle P \rangle = 0 & \rightarrow & \text{confinement} \\ \text{broken} & \rightarrow & \langle P \rangle \neq 0 & \rightarrow & \text{no confinement} \end{cases}$$

- Gauge + dynamical quarks

The hopping expansion of the quark determinant contains non-trivial loops q=1,2,3,4,….

So, fermionic contribution breaks the center symmetry explicitly and <P> need not vanish in the confined phase.

$$S = S_G + S_F$$

$$S_F[U] \approx \underbrace{\sum_{k=0}^{\infty} \frac{(-1)^k}{k} \frac{Tr\, D^k[U]}{m^k}}_{\text{hopping expansion}}$$

Nevertheless the study of the limit for infinite quark mass provides us with important information for the case where thermal gauge boson gas dominates the free energy (high T)

$$m \rightarrow \infty$$

Chiral condensate

- Partition function for the QCD

$$Z = \langle e^{-\int d^{4x} \bar{\psi}(D+m)\psi} \rangle_{FG} = \\ = \langle Det[D(U)+m] \rangle_G \\ = \langle e^{Tr \ln(D[U]+m)} \rangle_G$$

$\langle \rangle_F$ = average over fermions fields
$\langle \rangle_G$ = average over gluon fields U

Banks-Casher formula:

$$\langle \bar{\psi}\psi \rangle = -\pi \rho(0)$$

$\rho(0)$ = spectral density of the Dirac operator a zero momentum $p \rightarrow 0$

$SU(3)_L \, x \, SU(3)_R$ chiral symmetry

The chiral condensate misure the L-R asymmetry of the QCD vacuum

- Differentiate Z[m] respects -m and expand in power of 1/m

The Z(N) symmetry affects loops that winds non-trivially around compact time → chiral condensate is not Z-symmetric:

$$\langle \bar{\psi}\psi \rangle = \frac{1}{V} \frac{\partial Z}{Z \partial(-m)} = \\ = - \frac{1}{V} \langle Tr[\frac{1}{D[U]+m}] \rangle_G \\ = - \frac{1}{mV} \sum_{k=0}^{\infty} \frac{(-1)^k}{m^k} \langle Tr[D^k[U]] \rangle_G = \frac{a}{m} + \frac{b}{m^2} + \frac{c}{m^3} \dots$$

- Facts

At zero temperature:

$\rightarrow \langle \bar{\psi}\psi \rangle \neq 0$ (chiral symmetry broken)

$\rightarrow$ quark are confined (Z_3 restored)

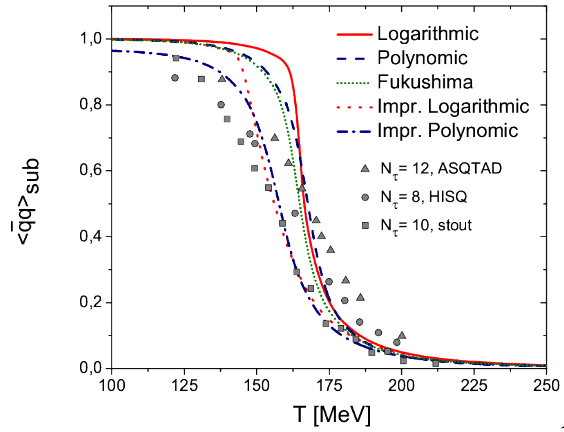
At finite temperature:

$\rightarrow \langle \bar{\psi}\psi \rangle = 0$ (chiral symmetry restored)

$\rightarrow$ quark become deconfined (Z_3 broken)

Is there an underlying mechanism that links the two key features of QCD?

Mass parameter m may be used to relate che chiral condenstate and the conventional Polyakov loop



Ising model analogy

$$H_h = \underbrace{J \cdot \sum_{x,\mu} s_x s_{x+\hat{\mu}}}_{H_0} \overbrace{- h \sum_x s_x}^{H_1}$$

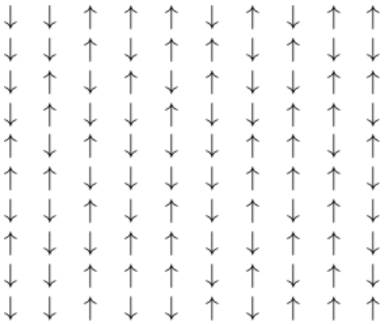
$$\beta = \frac{1}{T}$$

s_x = spin at site x $\in \{-1, 1\}$

$$Z_h = \sum_{[s_x]} e^{-\beta H_h}$$

h = external magnetic field

$$\langle \Omega \rangle = \frac{\sum_{[s_x]} e^{-\beta H_h} \Omega}{\sum_{[s_x]} e^{-\beta H_h}}$$



- The symmetry $\quad Z_2 = \{-1, +1\}$

Global spin flip $\quad \boxed{s_x \rightarrow -s_x}$

$H_0 \rightarrow H_0$ invariant

$H_1 \rightarrow -H_1$ not invariant

- The order parameter

$$\langle s \rangle = \lim_{N_{spin} \rightarrow \infty} \left\langle \frac{\sum_x s_x}{N_{spin}} \right\rangle$$

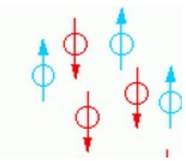
average spin value $\in \{-1, +1\}$

$\langle s \rangle$ changes sign under Z_2

$\rightarrow \langle s \rangle$ is an order parameter $\qquad \langle s \rangle \rightarrow -\langle s \rangle$

- Symmetry unbroken $\quad h = 0$

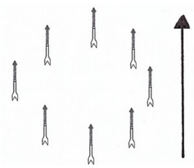
The expectation value of any quantity which is not invariant under the symmetry group, vanish: $\quad \langle s \rangle = 0$



- Symmetry explicitly broken $\quad h \neq 0$

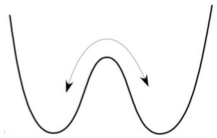
At any temperature T: $\quad \langle s \rangle \neq 0$

($\langle s \rangle = 0$ only for $T \rightarrow \infty$)



- Symmetry spontaneously broken $\quad N_{spin} \rightarrow \infty, h \rightarrow 0$

it is possible that $\langle s \rangle \neq 0$
but $\langle s \rangle = 0$ for $T \rightarrow 0$

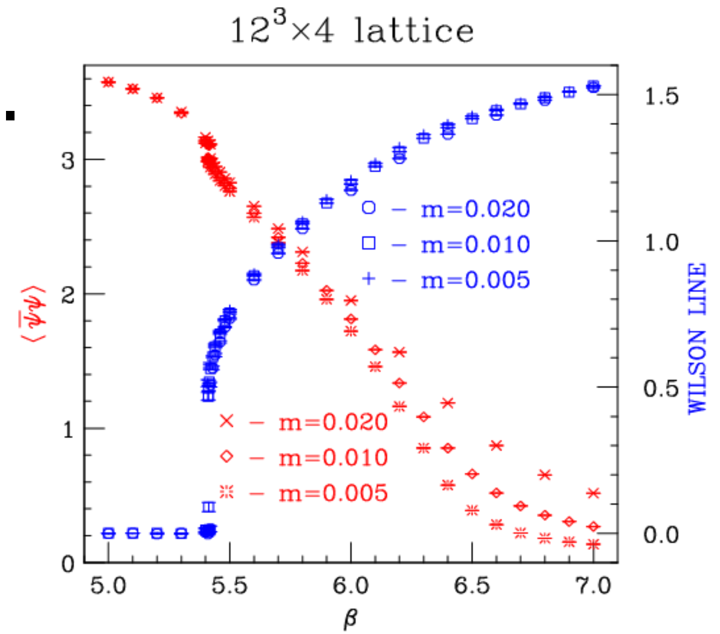


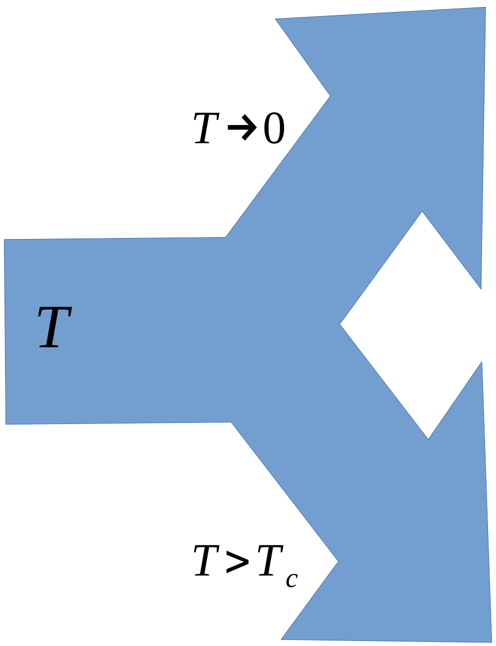
In summing up ...

At temperature T=270 Mev, Yang-Mills theory goes through a “deconfinement” transition: the quark free energy (measured by the Polyakov loop) become finite and hadrons dissolve into their constituents.

$$|\langle P \rangle| \sim e^{\frac{-F}{T}}$$

F= Gibbs energy, static quark creation





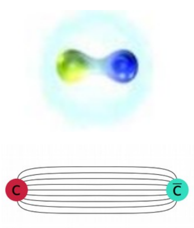
Z_N restored

$$\langle P \rangle = 0 \qquad F_{r\to\infty} = \infty$$

$$\langle \bar{\psi}\psi \rangle \neq 0$$

confinement

“Magnetic” disorder



“At low temperature quark and antiquark are connected by flux tubes → string”

Z_N broken

$$\langle P \rangle \neq 0 \qquad F_{r\to\infty} < \infty$$

$$\langle \bar{\psi}\psi \rangle = 0$$

deconfinement

“Magnetic” order

“At sufficient large temperature, strings breaks and dynamical quarks couples with gauge bosons”

Thank You
For the attention!

Olly

Credits

Slides and articles:

- M. D'Elia, De Forcrand
- J. Greensite
- M. Ogilvie, Z. Fedor, B. Walk (thesis)
- M.Mesiti (thesis)
- K. Holland
- A. Di Giacomo

Books:

- J. Greeensite, LePage
- Creutz, Rothe,
- QCD lectures notes (Meggiolaro)
- C.Gattringer, B. Lang
- Coleman, Makeenko

Prof. M. D'Elia
for discussions & corrections

backup

N-ality dependence

N-ality: # up-down indices in the group representation of matter

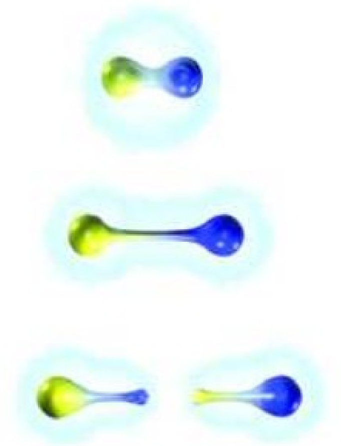
$q \quad k_r = 1$

$\bar{q} \quad k_r = -1$

$\bar{q}\, q \quad k_r = 0$

$\psi^{a_1 a_2 \ldots a_n}_{b_1 b_2 \ldots b_m}(x)$

$k_r = (n - m) mod(N)$



- Gluons (k=0, adjoint representation) can bind to 1 quark (k=1) and 1 anti-quark (k=-1).
- This does'nt change n-ality
- Gluons cannot bind to matter with k=0
- Matter with k not zero → potential V(r) become flat

$$\langle P \rangle = e^{-\sigma_r \cdot RT}$$

$$\text{Energy} = \sigma_r L$$

The fermion part is not invariant under Z(N) because time-like terms $\psi^+_f U^0_{fi} \psi_i$ (linear in U) get a factor z.

String tensions depends on N-ality

$$\sigma_r \simeq \frac{k_r (N - k_r)}{N - 1} \sigma_{casimir}$$

Vortices (I)

In the vortex picture of confinement, the QCD vacuum is considered as a condensate of vortices *with magnetic flux quantized in terms of the center group Z(N)*

- *Vortex = singular gauge transforms*
- *Singular= periodic up Z(N) factor*

Gauge transforms:

$$\psi' = \Omega \psi, \quad U'_{fi} = \Omega_f U_{fi} \Omega_i^+$$

$$\Omega(t+\beta) = z\,\Omega(t), \quad z \in Z_N \;\Rightarrow\; \Omega_f = z \cdot \Omega_i$$

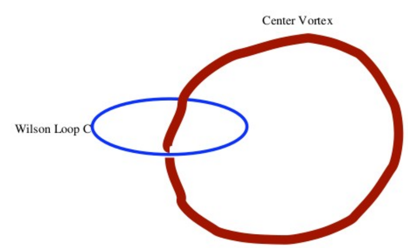
Boundary conditions for fermions and bosons

$$\psi(t+\beta) = -\psi(t)$$
$$A_\mu(t+\beta) = A_\mu(t)$$

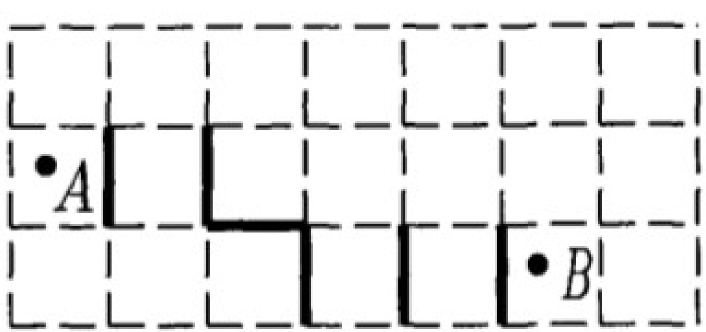
Anyway, Fermions breaks Z-invariance

$$\frac{\psi_f}{\psi_i} = \frac{\psi'_f}{\psi'_i} \rightarrow \frac{\Omega_f}{\Omega_i} = 1 \;\Rightarrow$$

Only trivial z=1 allowed



$$W_n(C) = \langle P e^{-ig \oint A_\mu(\vec{x})\, d\%x} \rangle$$



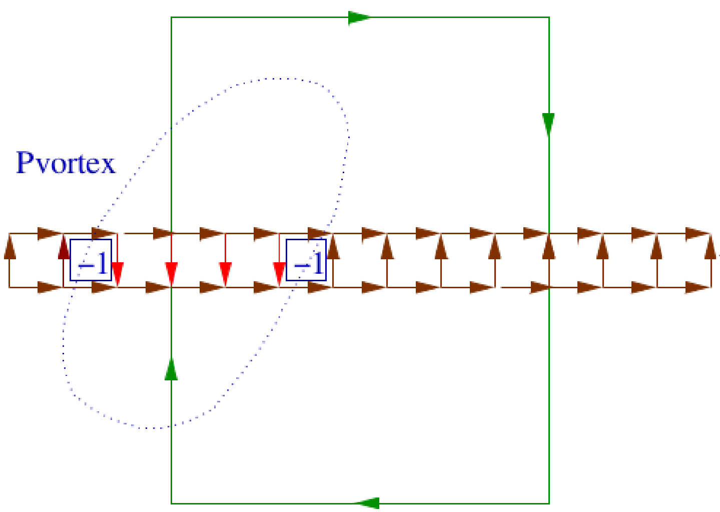
In the figure: vortices in A e B

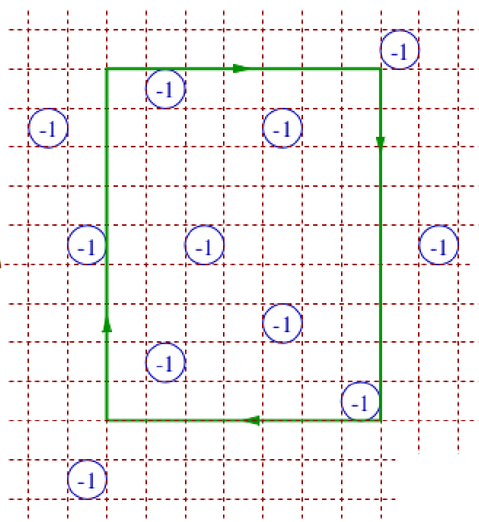
- Wilson loops get a z factor for each vortex linked to it.
- Ex: In the case of Z(2)={-1,1}, this give a (-1)^n with n=number of vortices

Vortices (II)

- *QCD vacuum considered as a condensate of* vortices *with magnetic flux quantized in terms of the center group Z(N)*
- The area low or large Wilson loop followw from fluctuations in the number of vortices linking the loop.

$$W_n(C) = \langle P e^{-ig \oint A_\mu(\vec{x})\, d\%x} \rangle$$





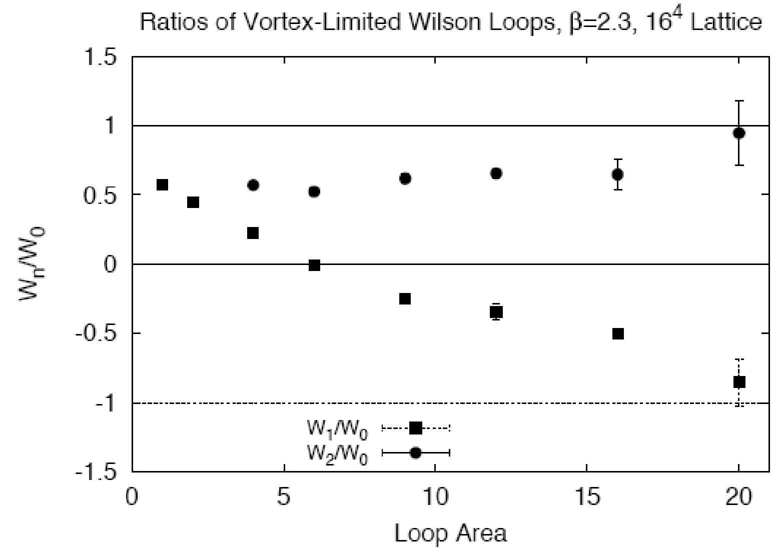
Sub-ensamble of configurations with C pierced by n vortices

Esample: SU(2) → Z(2)={-1,+1}

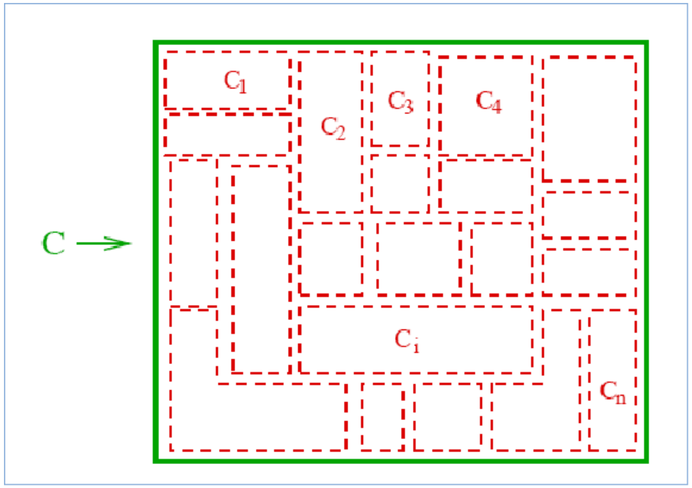
$$W_n(C) = \langle Z(C)\, V(C) \rangle$$

$$W_n(C) \approx W_0 \cdot \langle (-1)^n \rangle$$

If *C is large, products factorizes (indipendence)*



Center Projection



- Center projection is the replacement of links variables by their closest center elements U → Z
- Dominance: the confinement relevant non-perturbative degree of freedom are all in Z(N)
- The claims is that this procedure locate vortices in the original lattice

$$W(C)=Pe^{-ig\oint A_\mu(\vec{x})d\tau}$$

$$U_\mu=Z_\mu V_\mu \quad \text{with } V_\mu \to 1$$

$$\min \sum_{x,\mu} Tr\, U^2_{x,\mu}$$

$$SU(2): \quad Z_\mu = sign[Tr\, U]$$

Using statistical independence for large loop C: $\langle abcd\ldots\rangle=\langle a\rangle\langle b\rangle\langle c\rangle\ldots$
for Wilson loop we have:

$$\langle W(C)\rangle=\prod_{C_i}\langle Z\rangle\langle V\rangle\approx c\prod\langle Z\rangle=\prod_i^{\frac{A}{a^2}}\langle z^{k_r(i)}\rangle\approx e^{-\sigma_r A} \Rightarrow \langle W\rangle=e^{-\sigma_r A(C)}$$

Esample: SU(2) → Z(2)={-1,+1}

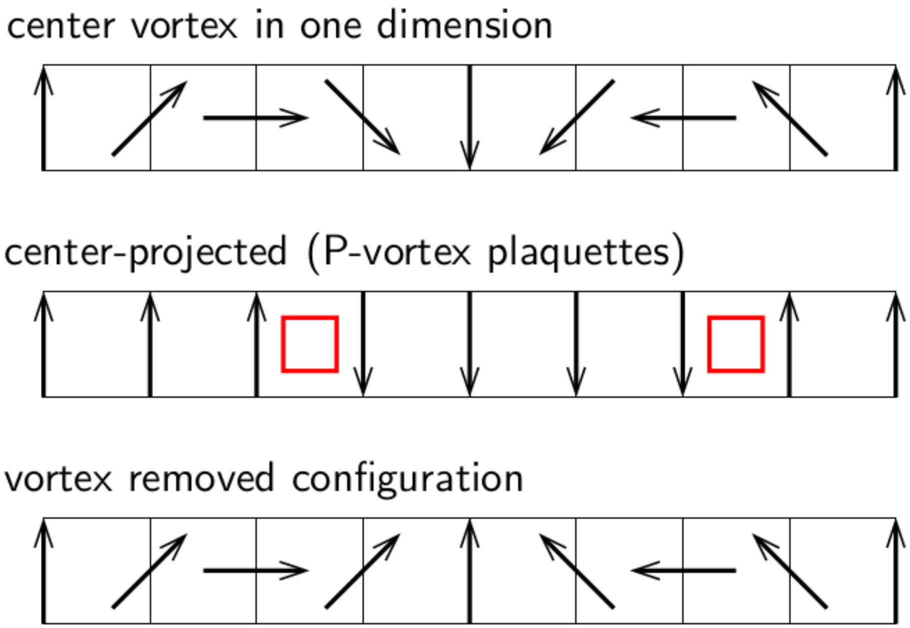
$$\prod\langle z^{k_r}\rangle\approx[f\cdot(-1)+(1-f)(1)]^{\frac{A}{a^2}}\approx e^{-\sigma A}$$

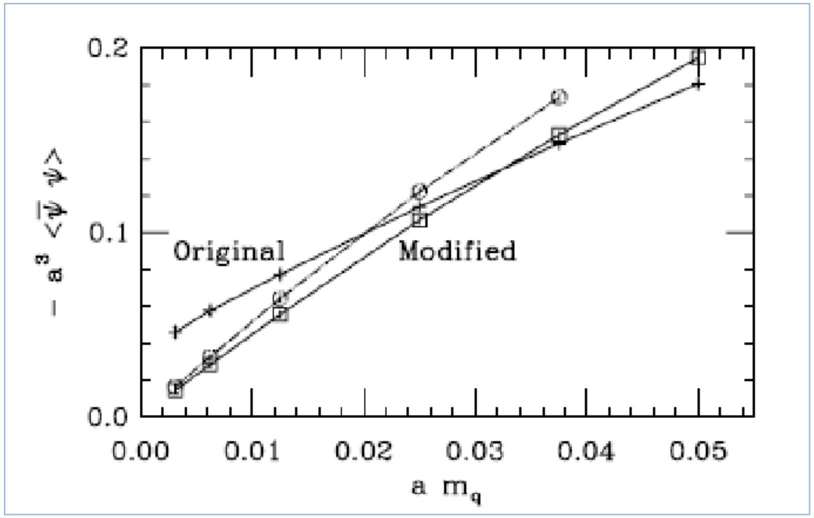
f=probality to have z=-1

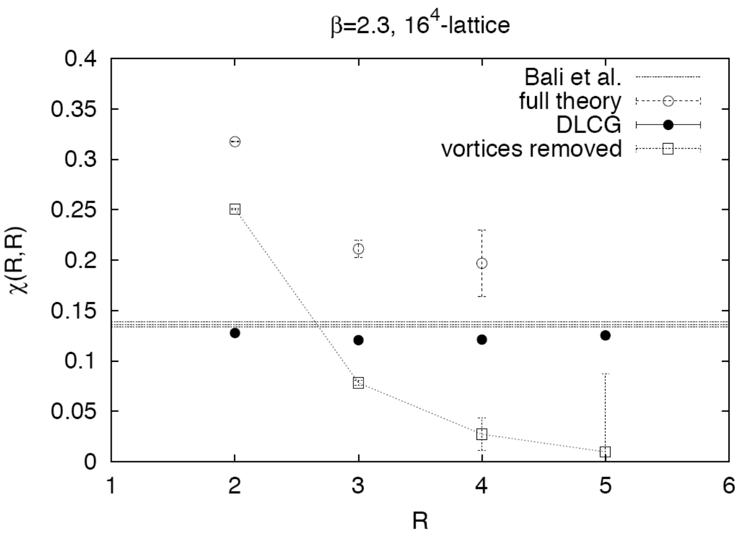
String tensions:

$$\sigma\approx-\ln(1-2f)$$

Vortex removal







Vortex removal
(consistency test)

$$U \rightarrow U' = Z \cdot U \equiv V$$

Removing vortices should remove the asymptotic string tension

Dirac operator Improvements

$$K = 1 - k \overbrace{\sum_{\mu=\pm1,\dots,\pm4} U_\mu \cdot (1-\gamma_\mu) \cdot E^{a\mu}}^{H}$$

Wilson-Dirac

$$(\psi\bar{\psi})_{xy} \simeq \left(\frac{1}{K}\right)_{xy} = \left(\frac{1}{1-kH}\right)_{xy} = (1 + kH + k^2H^2 \dots)_{xy}$$

Hopping expansion

$$\begin{cases} (E^{a\mu})_{xy} = \delta_{x+a\mu,y} \\ k = \dfrac{1}{2a\cdot m + 8} \end{cases}$$

doublers

Kinetic fermionic operator

$$K_{x,y} = M \cdot \delta_{x,y} + \frac{1}{2}[\gamma_\mu U_\mu \delta_{x+a\hat{\mu},y} - \gamma_\mu U_{-\mu} \delta_{x-a\hat{\mu},y}]$$

$$D_W = \frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) + r \cdot a \cdot \nabla_\mu \cdot \nabla^*_\mu$$

better $p \to 0$

$$D_{tw} = D_W + i\mu\gamma_5\tau^3$$

Twisted mass
(*two flavor*)

$$D_{DWF} = \sum_{\mu=1}^{5} \gamma_\mu \partial_\mu - M(x^5)$$

Domain wall fermions

chirality

$$D = \frac{1}{a}(1 + \gamma_5 \cdot sign(H))$$

Overlap operator

solve for D:
$$D\gamma_5 + \gamma_5 D = a D \gamma_5 D$$

Ginsparg-Wilson

doublers

$$\psi_x = \prod_\mu \gamma_\mu^{\frac{x_\mu}{a}} \varphi_x$$

staggered

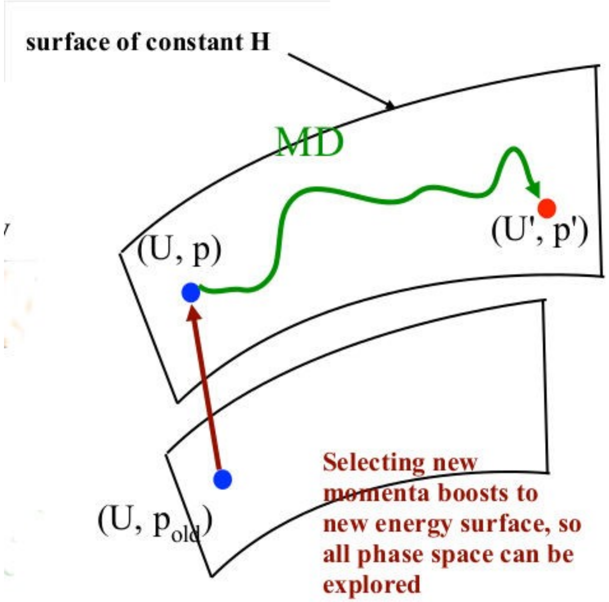
HMC: pseudo-fermions

Using the results:

$$\int \mathcal{D}\varphi\,\mathcal{D}\varphi^{+}\, e^{\varphi^{+}\frac{1}{K^{+}K}\varphi} = |K^{+}K|$$

φ
bosonic auxiliary field

$$S_{eff}[U,\varphi] = S_G - \varphi^{+}\frac{1}{K^{+}K}\varphi = S_G - \left(\frac{\varphi}{K}\right)^{+}\left(\frac{\varphi}{K}\right) = S_G - \chi^{+}\chi$$



1) generate χ gaussian: $P(\chi) = e^{-\chi^{+}\chi}$
2) compute $\varphi = K[U]^{+}\chi$ at fixed U
3) update U using hamiltonian equations at fixed φ

$$C_1 \rightarrow C_2 \rightarrow C_3 \rightarrow \cdots$$

$$P(C) \sim e^{-H(C)}$$

Molecular dynamics
(hamiltonian phace space motion)