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INFORMATION TRANSFER BETWEEN GROUPS OF DYNAMIC VARIABLES

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Outline of the presentation

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- Concepts of complex systems and information theory
- **Transfer Entropy** as a way to measure the direction of the information flow
- **Cluster Index**: identifying significant groups of dynamic variables
- Examples and simulations
- Conclusions and questions!

Complex systems study

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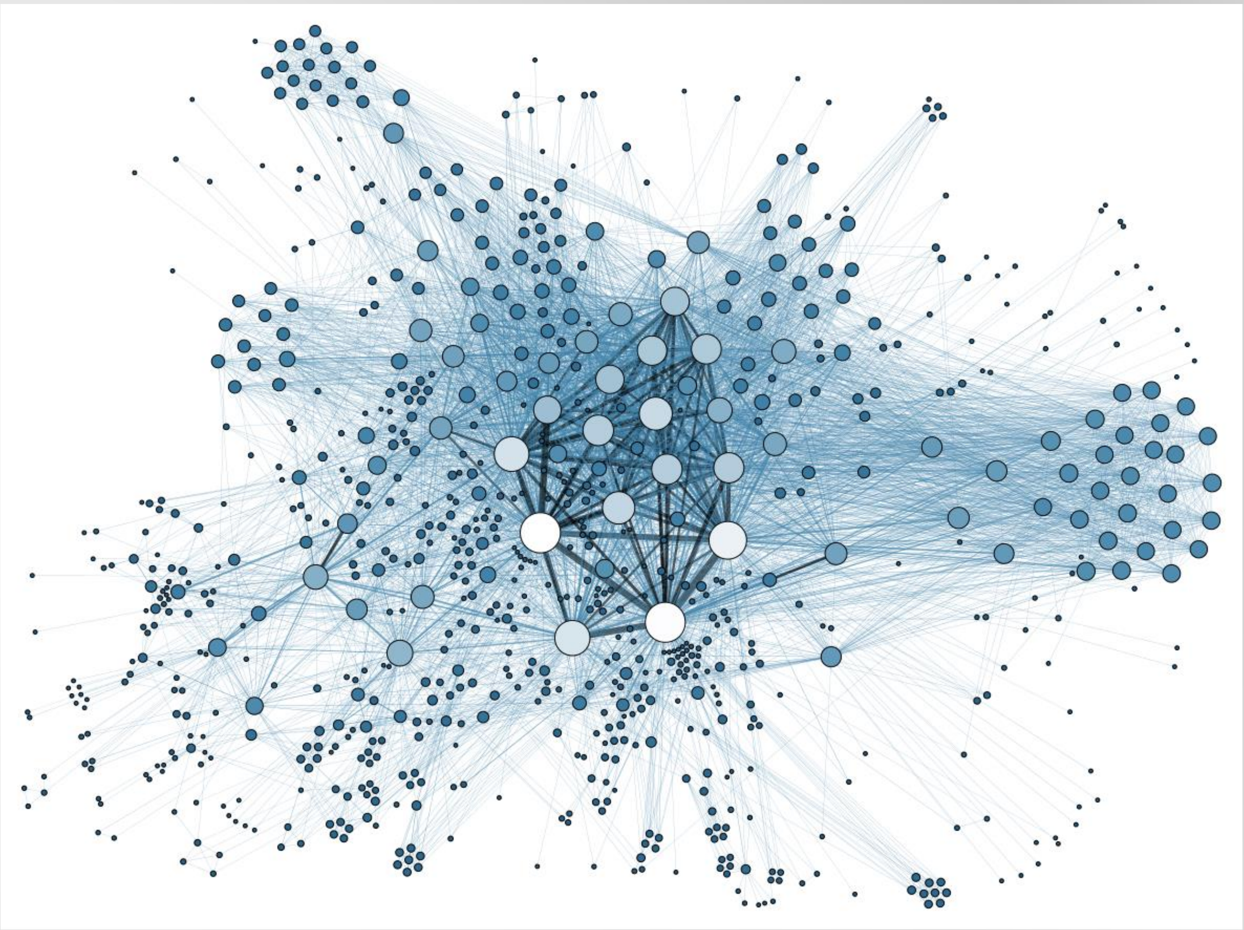
- Dynamical complex system as a collection of variables evolving in time
- We know nothing about the variables
- We can observe how the system evolves
- We want to understand which variables are more important



Information flow: applications

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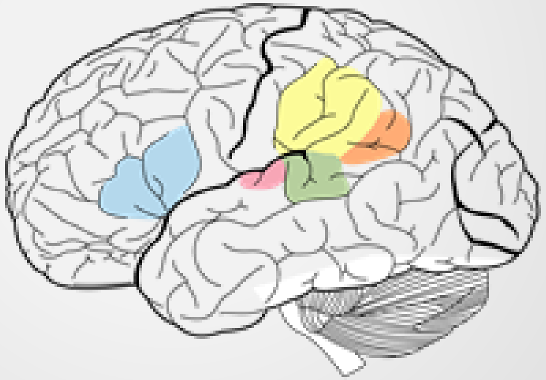
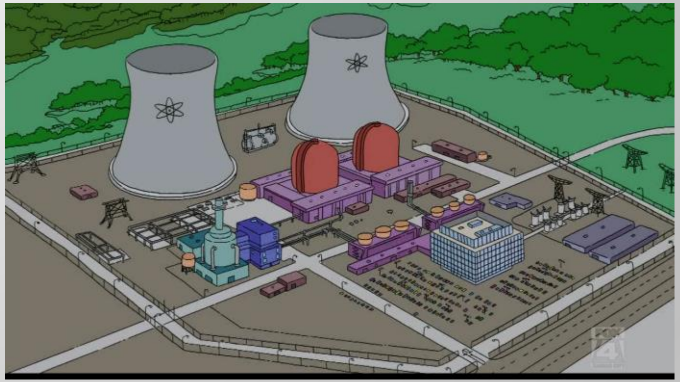
- Studying the directionality of information flow is of great importance in many different fields
  - Economics
  - Genetics
  - Neuroscience
  - Industrial processes





Importance of the directionality

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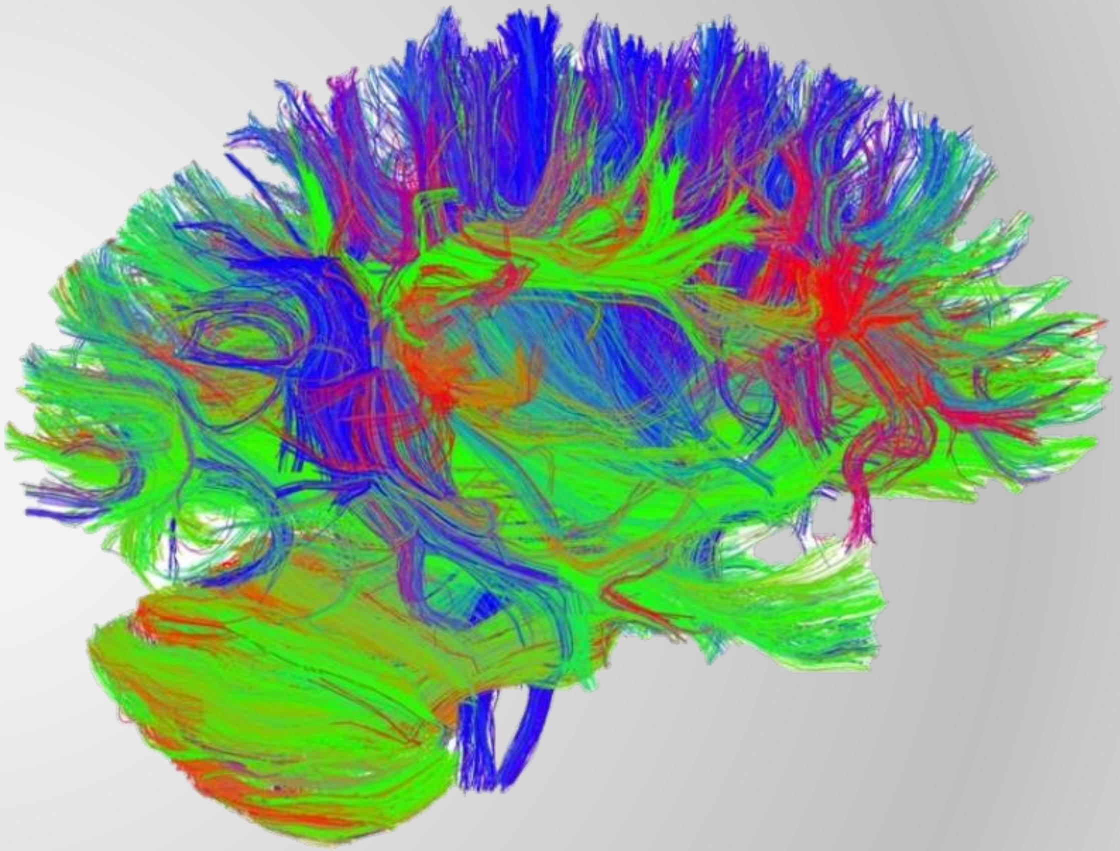
- Correlation and mutual information are symmetric quantities
- Inadequate to study the directionality of the information flow.
- This directionality can be of great importance in neuroscience



Ito, Shinya, et al. "Extending transfer entropy improves identification of effective connectivity in a spiking cortical network model." *PloS one* 6.11 (2011): e27431.

Transfer entropy: definition

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- Shannon entropy:

$$H_I = -\sum_{i\in I} P(i)\log_2\big(P(i)\big) \qquad H_{I|J} = -\sum_{i\in I, j\in J} P(i,j)\log_2\big(P(i|j)\big)$$

- Measures the average number of bits that need to be used to encode independent draws of the discrete variable I, following a probability distribution $P(i)$

Transfer entropy: definition

- Kullback-Leibler divergence:

$$K_I = -\sum_{i \in I} P(i) \log_2 \left(\frac{Q(i)}{P(i)} \right) \qquad K_{I|J} = -\sum_{i \in I, j \in J} P(i,j) \log_2 \left(\frac{Q(i|j)}{P(i|j)} \right)$$

- Measures the loss of information when the distribution Q is used to approximate distribution P

- It is the excess number of bits necessary to encode $P(i)$ with $Q(i)$

Transfer entropy: definition

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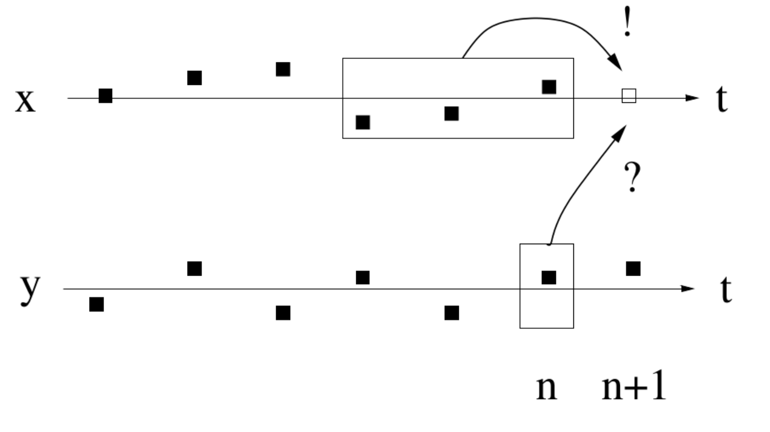
- Transfer entropy:

  $TE(Y \rightarrow X)$ = information lost using $P(x_{n+1}|x_n, x_{n-1}, \dots, x_{n-k+1})$ to approximate $P(x_{n+1}|x_n, x_{n-1}, \dots, x_{n-k+1}, y_n)$



- Transfer entropy:

$$TE(Y \rightarrow X) = -\sum P(x_{n+1}, x_n, x_{n-1}, \dots, x_{n-k+1}, y_n) \log_2 \left(\frac{P(x_{n+1}|x_n, x_{n-1}, \dots, x_{n-k+1})}{P(x_{n+1}|x_n, x_{n-1}, \dots, x_{n-k+1}, y_n)} \right)$$

Schreiber, Thomas. "Measuring information transfer." *Physical review letters* 85.2 (2000): 461.

Characteristics of Transfer Entropy

$$TE(Y \to X) = -\sum P(x_{n+1}, x_n, x_{n-1}, \dots, x_{n-k+1}, y_n)\log_2\left(\frac{P(x_{n+1}|x_n, x_{n-1}, \dots, x_{n-k+1})}{P(x_{n+1}|x_n, x_{n-1}, \dots, x_{n-k+1}, y_n)}\right)$$

- When y_n has no influence over x_{n+1}: $TE(Y \to X) = 0$
- Transfer entropy is asymmetric: $TE(Y \to X) \neq TE(X \to Y)$
- Different from the more widely known mutual information:

$$M_{X,Y} = -\sum_{x,y} P(x,y)\log_2\left(\frac{P(x,y)}{P(x)P(y)}\right)$$

- This allows to capture the directionality of the information flow

Finding clusters: Cluster Index

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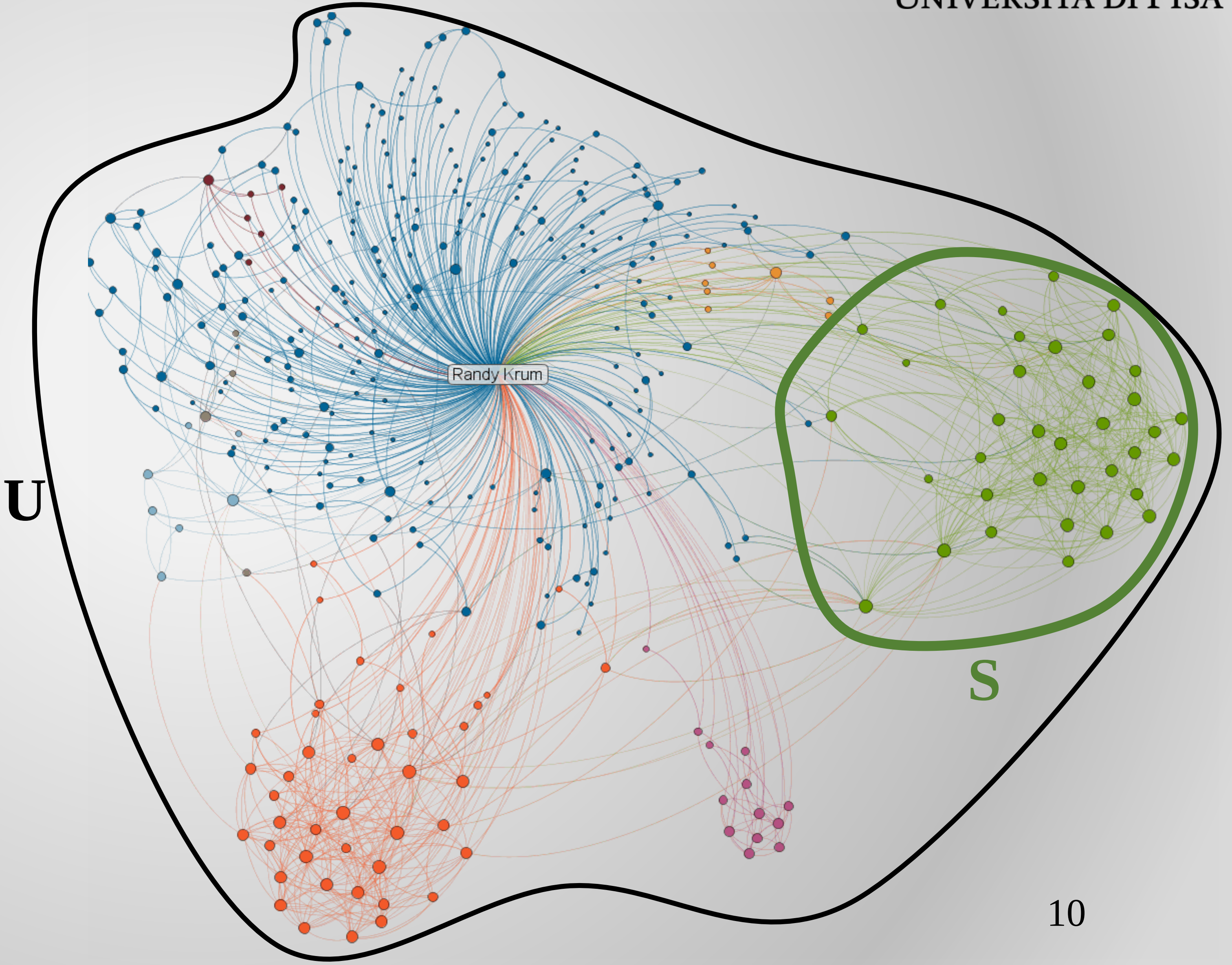
- Integration:

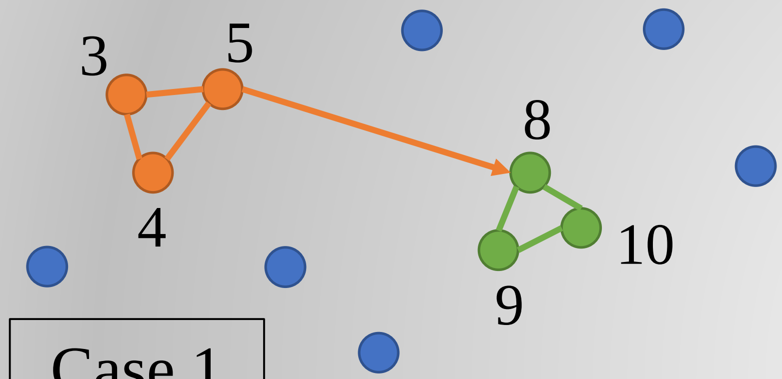
$$I(S) = \left(\sum_{j \in S} H_{x_j}\right) - H_S$$

- Cluster Index:

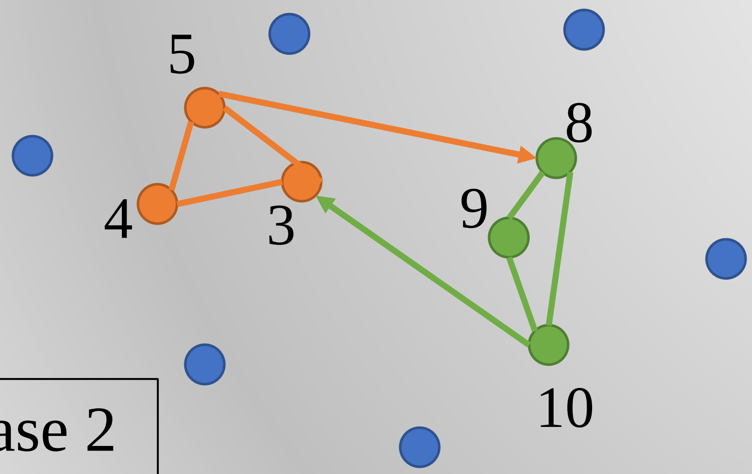
$$CI(S) = \frac{I(S)}{M_{S,(U-S)}}$$



Simulations



Case 1



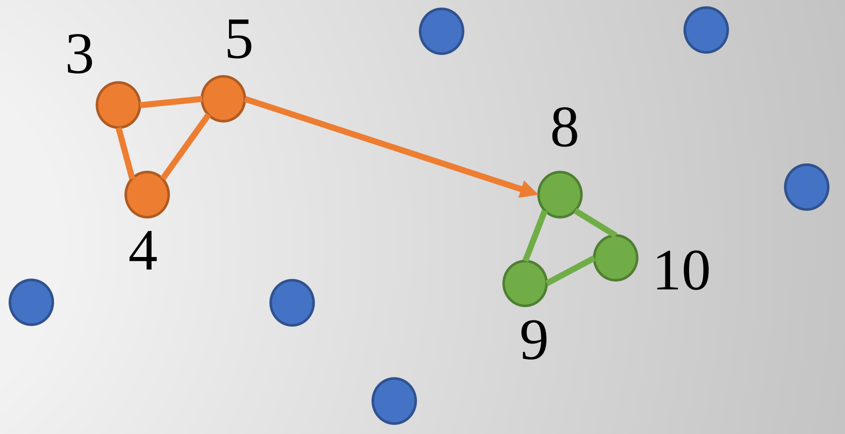
Case 2

$\wedge$: **AND**
$\oplus$: **XOR**

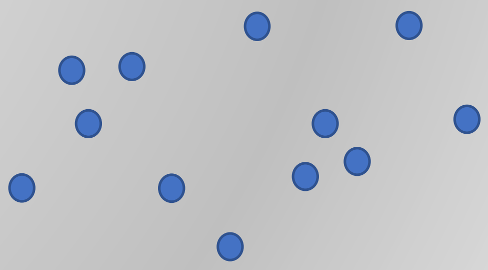
Node(t+1)	Case 1	Case 2
N01	Random	Random
N02	Random	Random
N03	N04 $\oplus$ N05	N10 $\wedge$ (N04 $\oplus$ N05)
N04	N03 $\oplus$ N05	N03 $\oplus$ N05
N05	N03 $\oplus$ N04	N03 $\oplus$ N04
N06	Random	Random
N07	Random	Random
N08	N05 $\wedge$ (N09 $\oplus$ N10)	N05 $\wedge$ (N09 $\oplus$ N10)
N09	N08 $\oplus$ N10	N08 $\oplus$ N10
N10	N08 $\oplus$ N09	N08 $\oplus$ N09
N11	Random	Random
N12	Random	Random

Cluster Index: case 1

Position	Group
1	8 | 9 | 10
2	3 | 4 | 5
3	9 | 10
4	3 | 4 | 5 | 8
5	4 | 5



- CI depends on group dimensions, therefore it must be normalized with reference to a "*homogeneous system*"



- Different kind of normalization yielded the same results

Transfer Entropy: case 1

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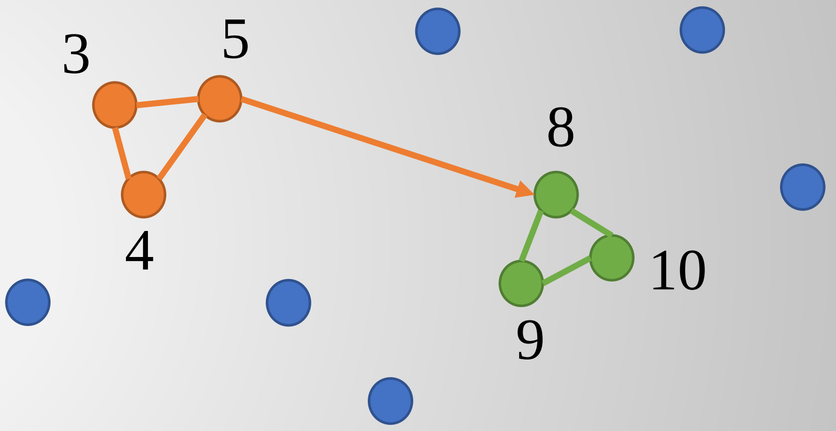
$TE(row \rightarrow column)$

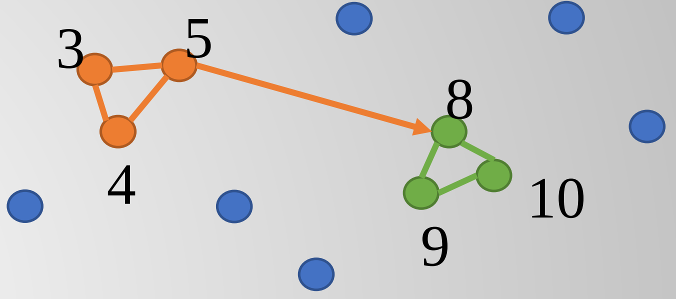
	3 | 4 | 5	8 | 9 | 10	4 | 5	9 | 10
3 | 4 | 5	0.00	0.22	0.12	0.40
8 | 9 | 10	0.00	0.00	0.03	0.69
4 | 5	0.00	0.22	0.00	0.38
9 | 10	0.00	0.00	0.00	0.00

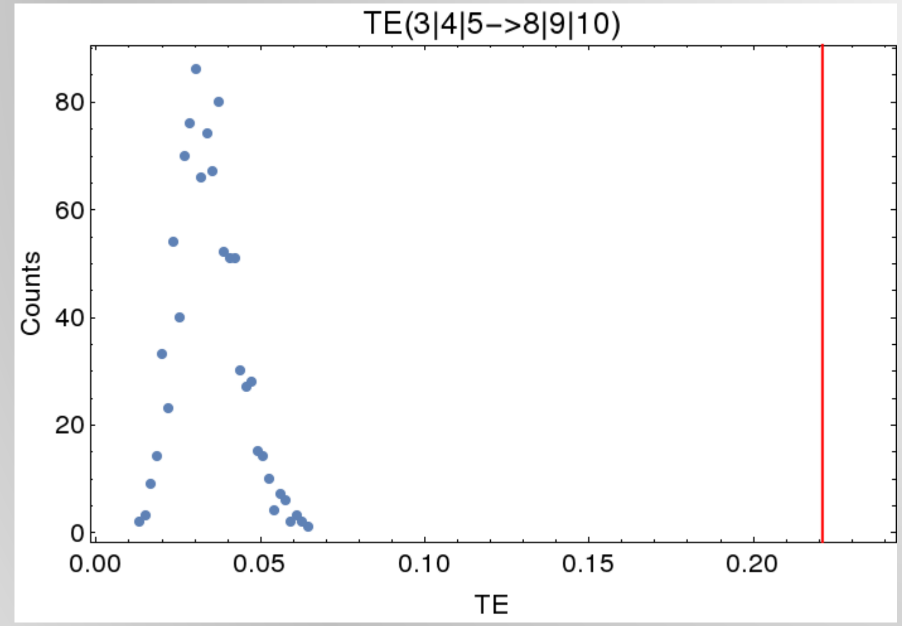


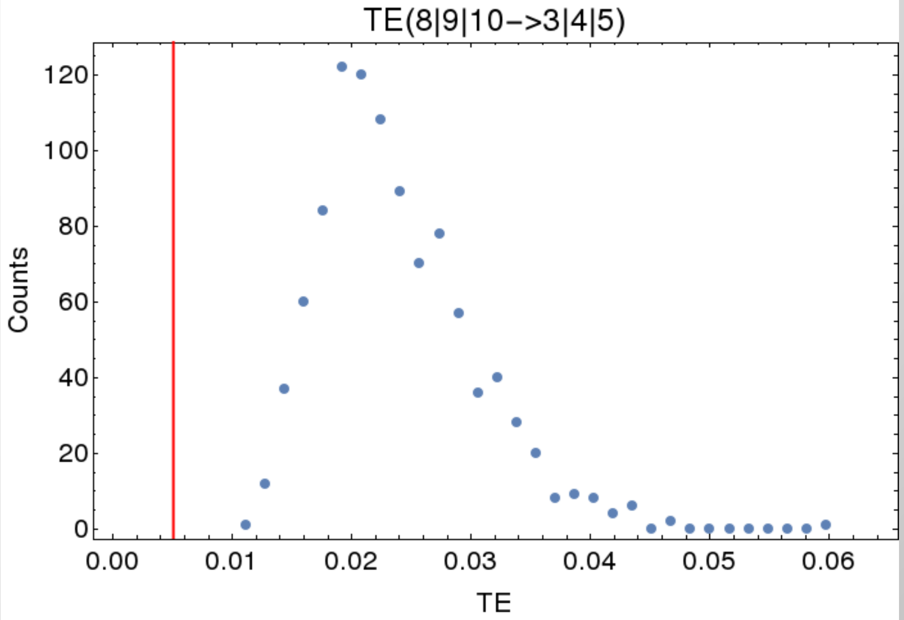
- TE depends on group dimensions too
- More reliable figures may be found with a statistical significance test

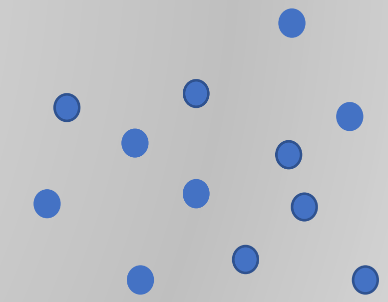
Transfer entropy: significance test

Several homogeneous systems are created and the TEs between the groups of interests are computed on them









Cluster Index: case 2

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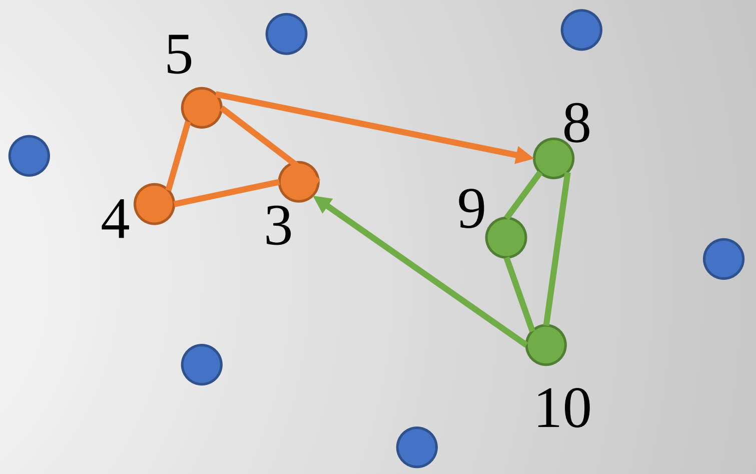
Position	Group
1	9 | 10
2	4 | 5
3	8 | 9 | 10
4	3 | 4 | 5
5	3 | 9 | 10



- The largest groups are identified
- Smaller ones (4 | 5 and 9 | 10) seem to be more tightly bound

Transfer Entropy: case 2

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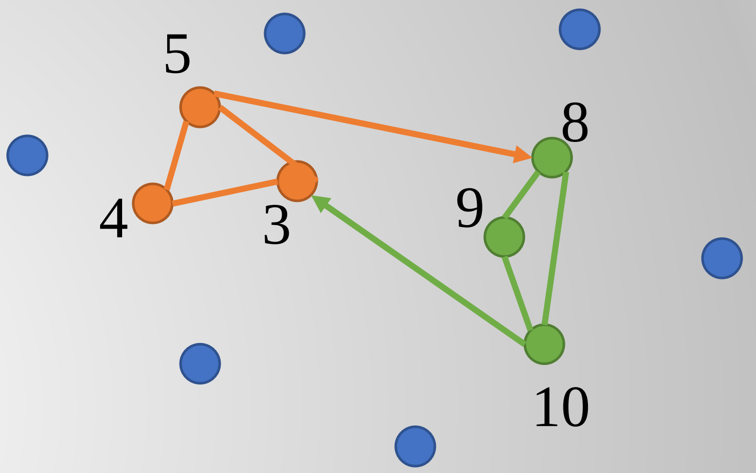
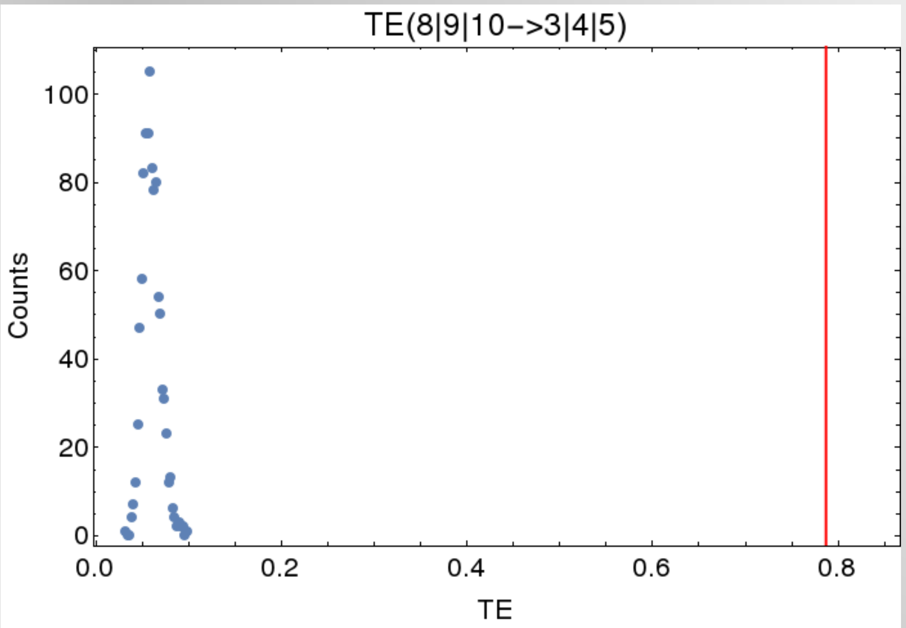
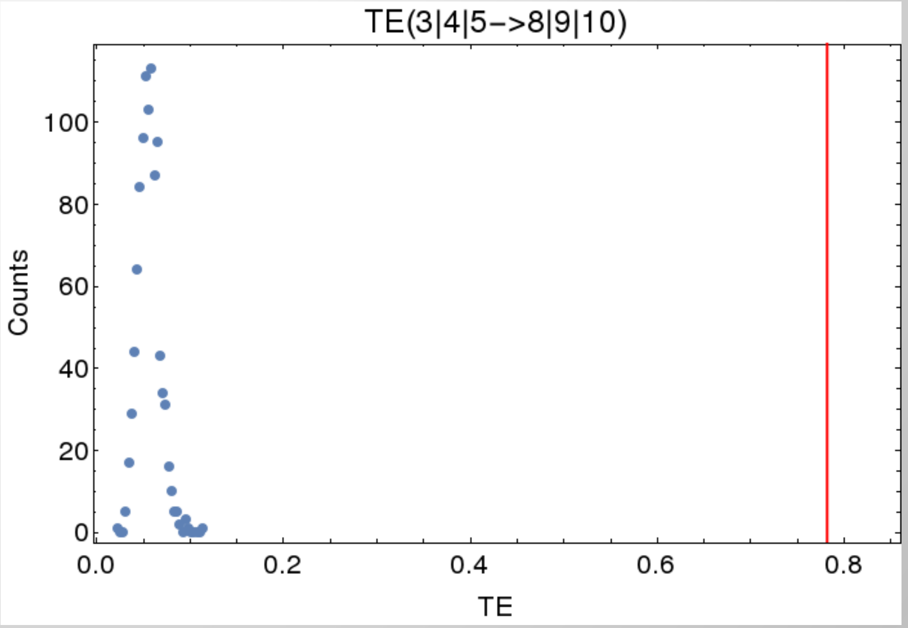
$TE(row \rightarrow column)$

	3 | 4 | 5	8 | 9 | 10	4 | 5	9 | 10
3 | 4 | 5	0.00	0.79	0.85	0.16
8 | 9 | 10	0.79	0.00	0.14	0.84
4 | 5	0.00	0.75	0.00	0.65
9 | 10	0.78	0.00	0.58	0.00







Conclusions

- Cluster Index and Transfer Entropy form a powerful combination that allows to investigate the hierarchy and the dynamics of a complex system

- They are not model-dependent, thus they can be applied to a wide range of fields

- Drawbacks:
  - Not quantitative enough (too dependent on cluster size)
  - Very high computational complexity

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Case studies

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- Now let's see if TE works in real life too!

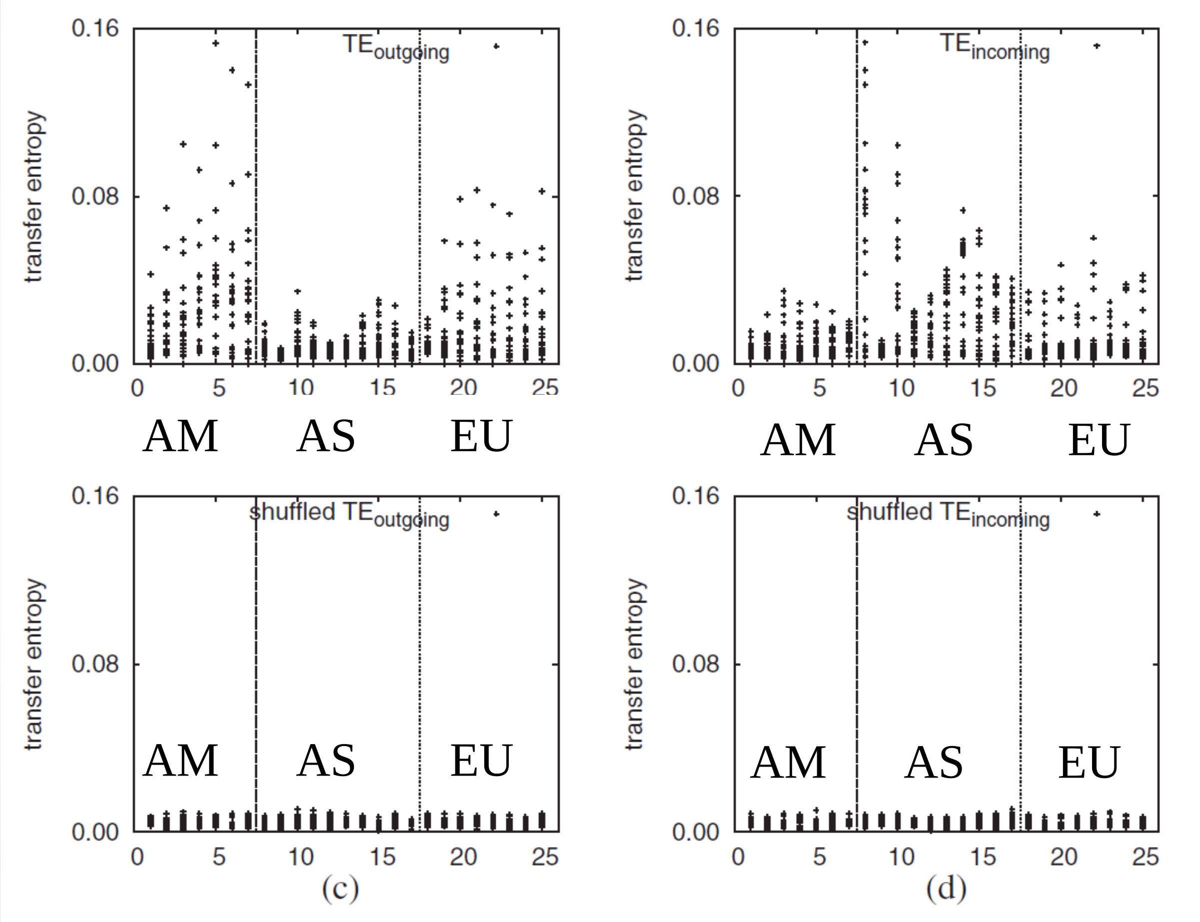
- I will present two studies in two different fields

  - **Finance**: study of information flow between stock indices

  - **Neuroscience**: Transfer Entropy as a measure of connectivity in the brain

Case study: Finance

- The information flow between the biggest stock indices has been analyzed
- Indices were grouped in regions: Europe, Asia and America
- Daily return was used as the variable associated to the indices
- Return values were discretized in three levels

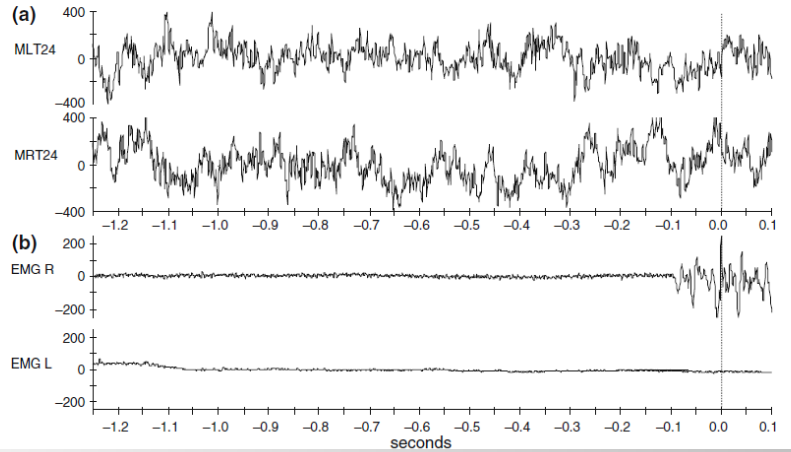


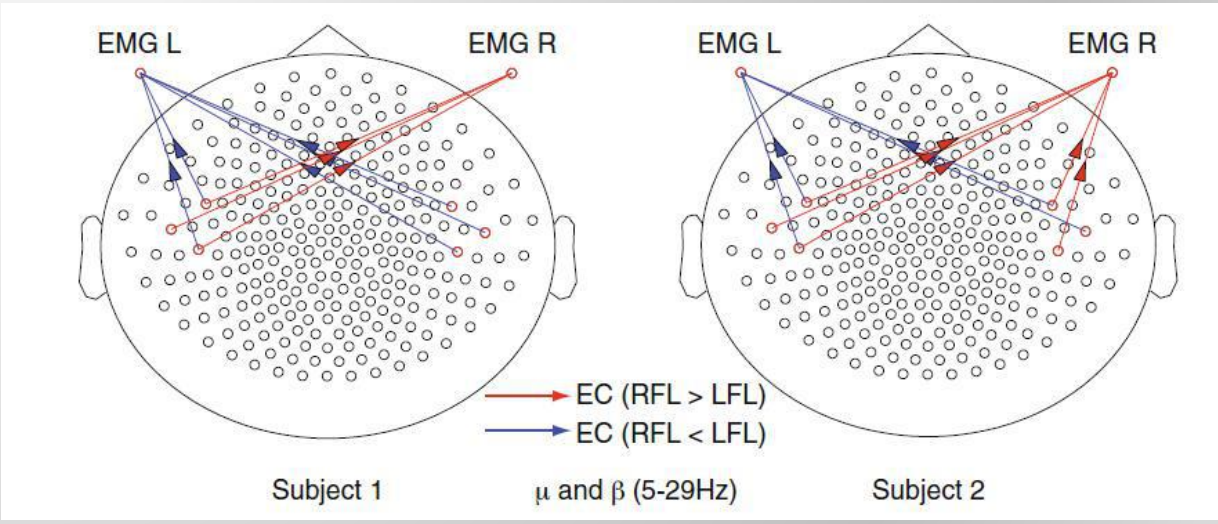
$$TE(Y \to X) = -\sum P(x_{n+1}, x_n, x_{n-1}, \dots, x_{n-k+1}, y_n) \log_2 \left(\frac{P(x_{n+1} | x_n, x_{n-1}, \dots, x_{n-k+1})}{P(x_{n+1} | x_n, x_{n-1}, \dots, x_{n-k+1}, y_n)} \right)$$

Kwon, Okyu, and J-S. Yang. "Information flow between stock indices." *EPL (Europhysics Letters)* 82.6 (2008): 68003.

Case study: Neuroscience

- The study analyzes time series from simulations and EEG and MEG data on simple motor tasks
- Neural data present various difficulties:
  - The same signal can be picked up by different sensors at different times
  - Data are continuous and single signals are spread over time
  - The time delay with which information is transferred can vary significantly (1-100 ms)
- Even in this complex environment, TE can detect connectivity between brain regions, while other methods fail





Vicente, Raul, et al. "Transfer entropy—a model-free measure of effective connectivity for the neurosciences." *Journal of computational neuroscience* 30.1 (2011): 45-67.