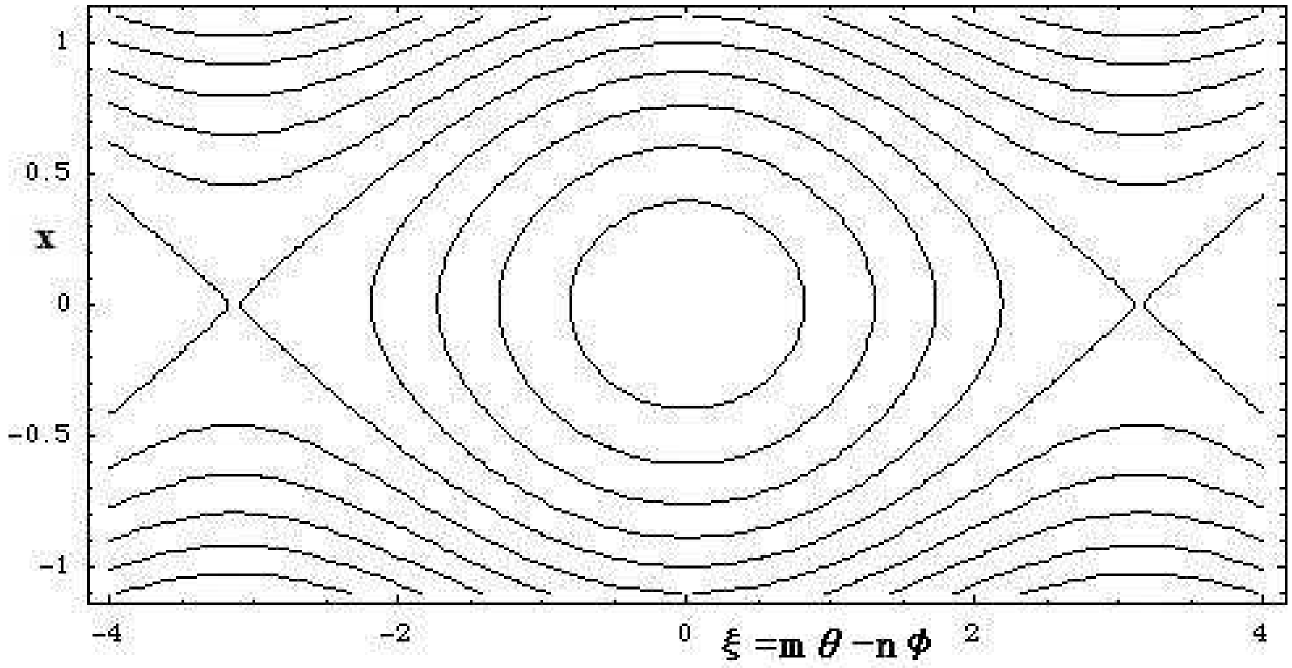
Andrea Casolari

Università di Pisa

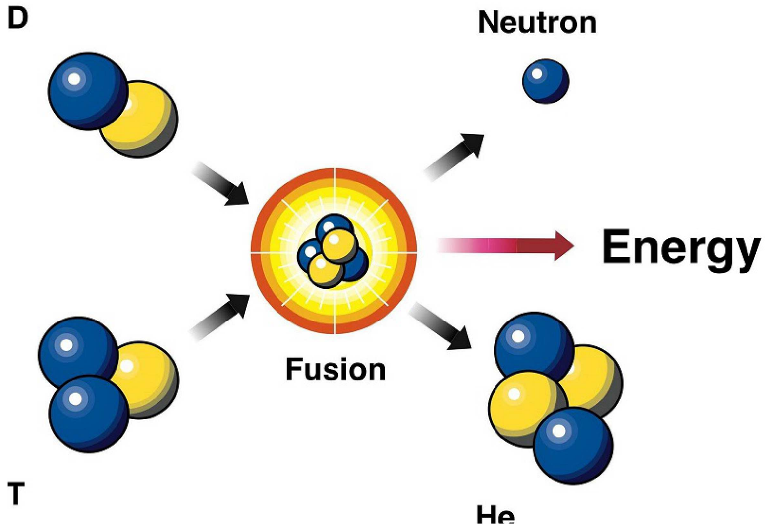


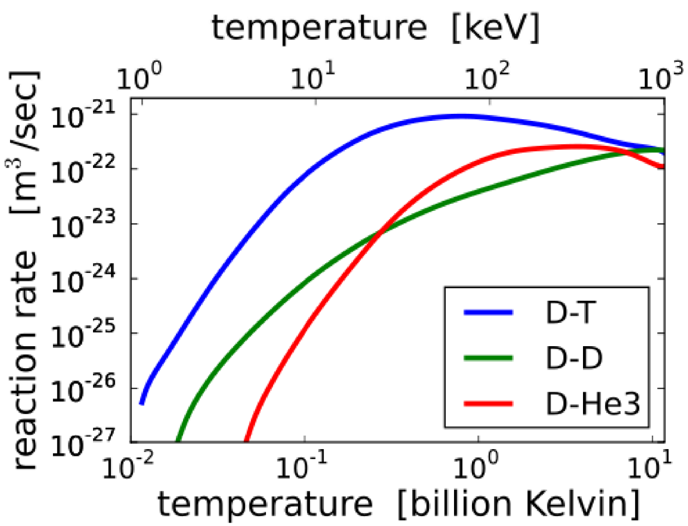
Dinamica delle isole magnetiche

Overview

- Magnetic confinement of plasmas
- Equilibrium and magnetic surfaces
- Onset of magnetic islands
- Magnetic islands' dynamic
- Solution of the equations
- Experimental results

Thermonuclear fusion



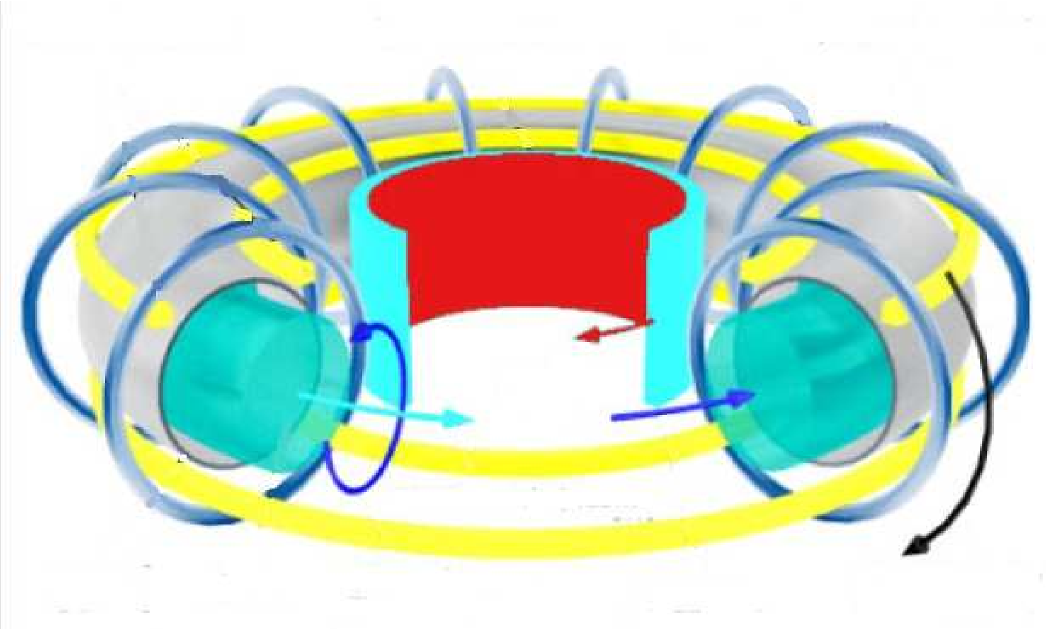


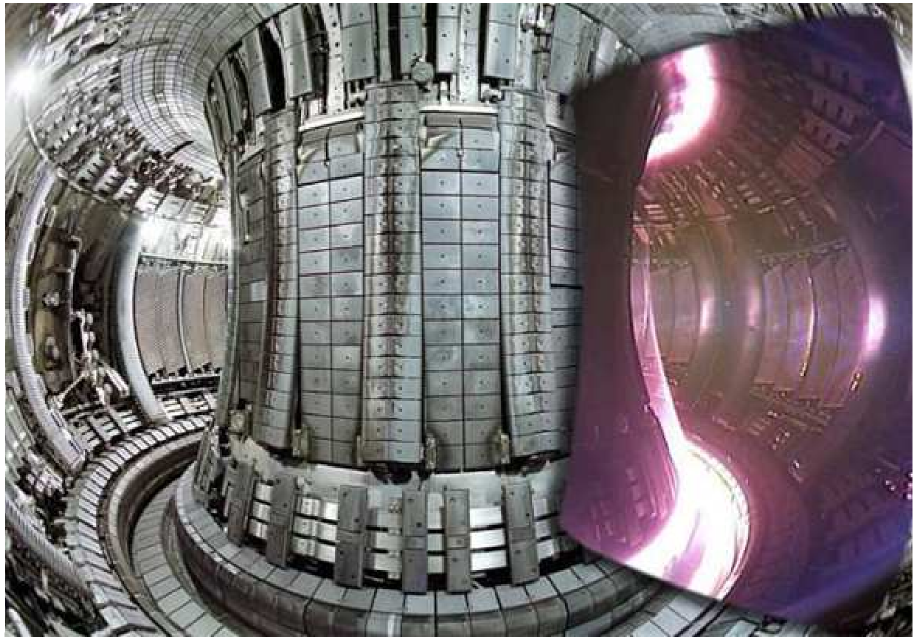
Total energy per reaction: 17.6 MeV

Suitable temperatures range: 10-100 KeV

Potentially self-sustained (burning plasma, ignition)

Magnetic confinement

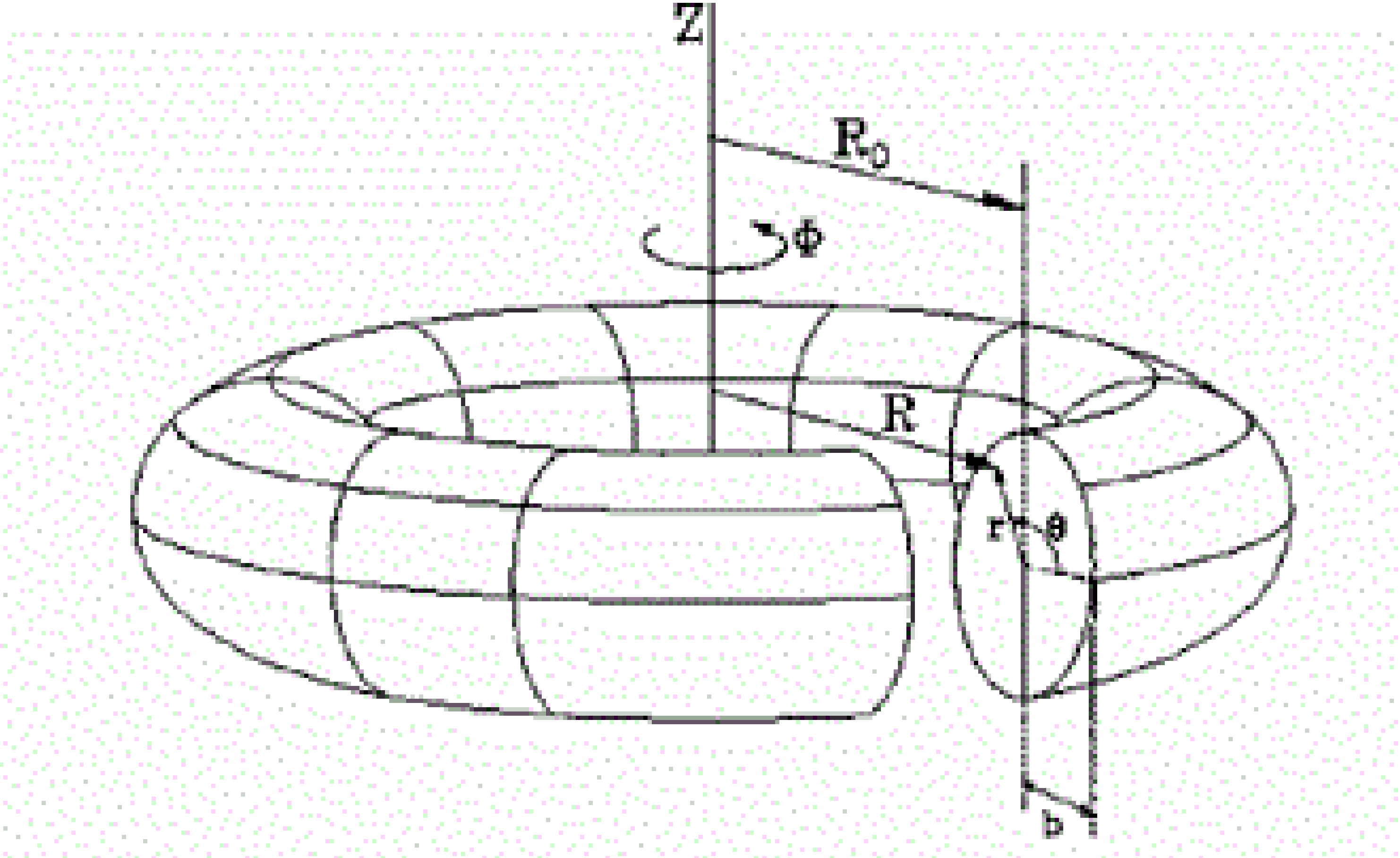




Tokamak structure and view of the JET inside

JET dimensions: $R_0 \approx 3$ m, a=1-2 m (elongated)

Tokamak structure



Magnetic surfaces

Equilibrium between plasma pressure and magnetic pressure:

$$J \wedge B = \nabla P$$

From the Ampere law:

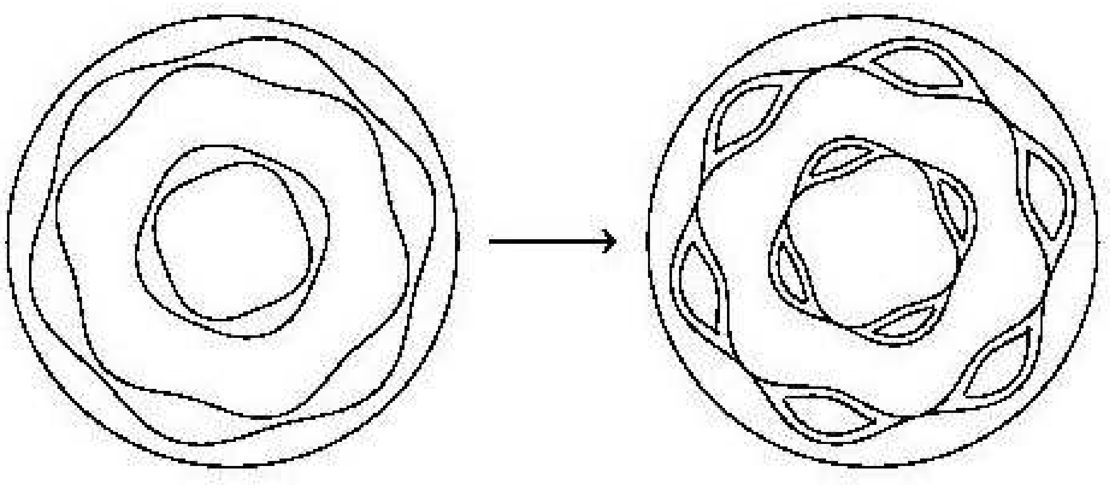
$$J = \nabla \wedge B$$

we obtain the Grad-Shafranov equation

$$\frac{1}{R}\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial \psi}{\partial R}\right) + \frac{\partial^2 \psi}{\partial z^2} = -R^2 \frac{\partial P}{\partial \psi} - F\frac{\partial F}{\partial \psi}$$

ψ magnetic flux function

Onset of magnetic islands



A resonant perturbation causes the magnetic surfaces to break:

- radial transport increases
- confinement decreases

Analogy with Hamiltonian theory

Magnetic surfaces are characterized by the safety factor

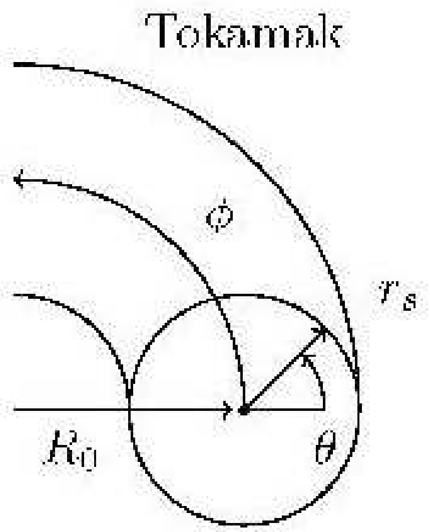
$$q = \frac{rB_{\Phi}}{R_0 B_{\theta}}$$

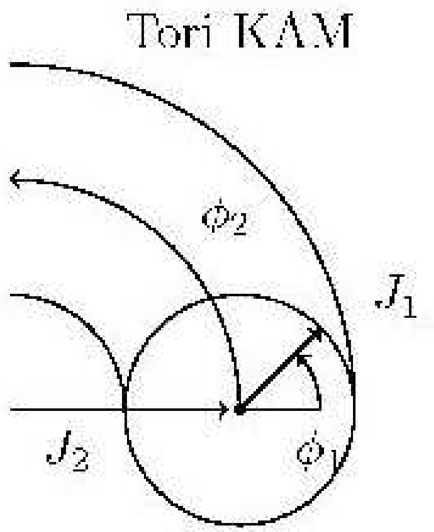
If this parameter takes rational values: $q = \frac{m}{n}$

the surface is said to be rational

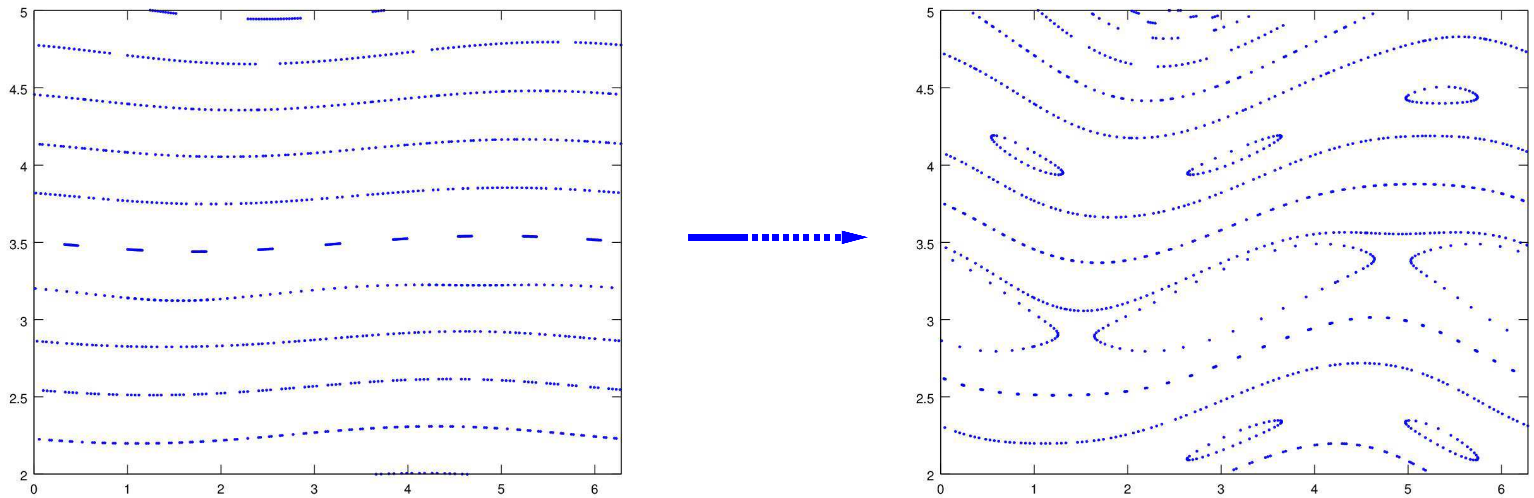
Rational surfaces are destroyed by arbitrarily small resonant perturbations: Poincarè small denominators

$$f(r,\theta,\phi) = \sum_{m,n} \frac{f_{m,n}(r)}{nq(r)-m} e^{i(nq(r)\phi - m\theta)}$$



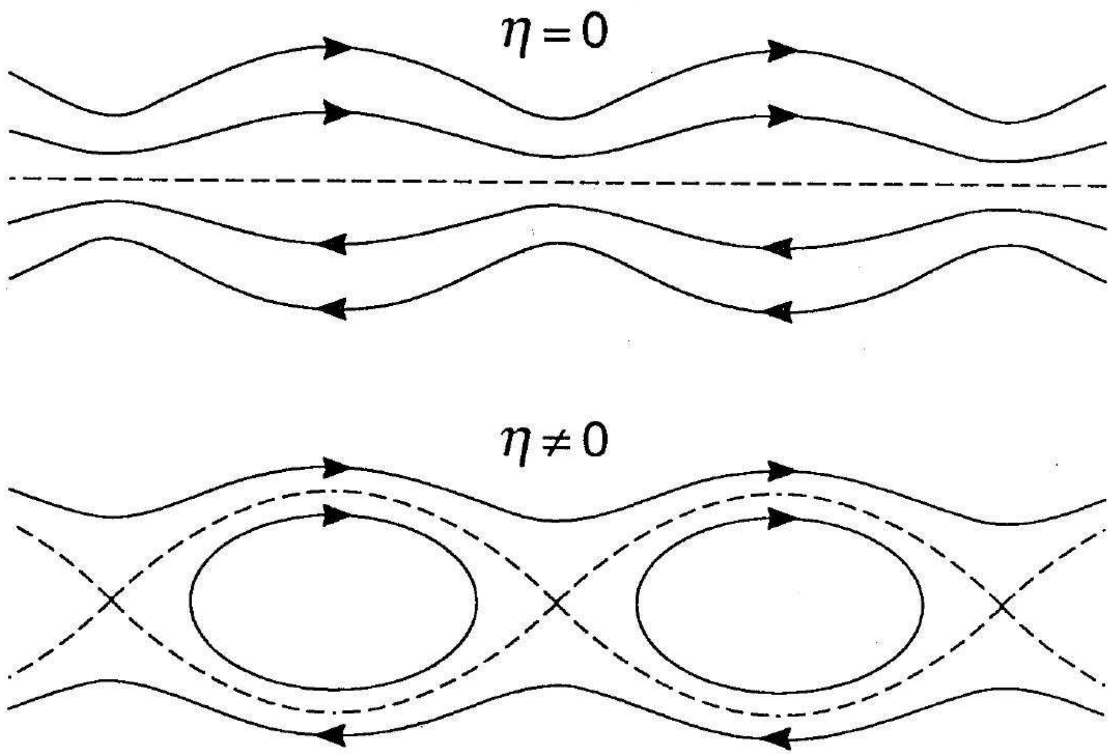


The magnetic surface radius and the major radius correspond to the action variables



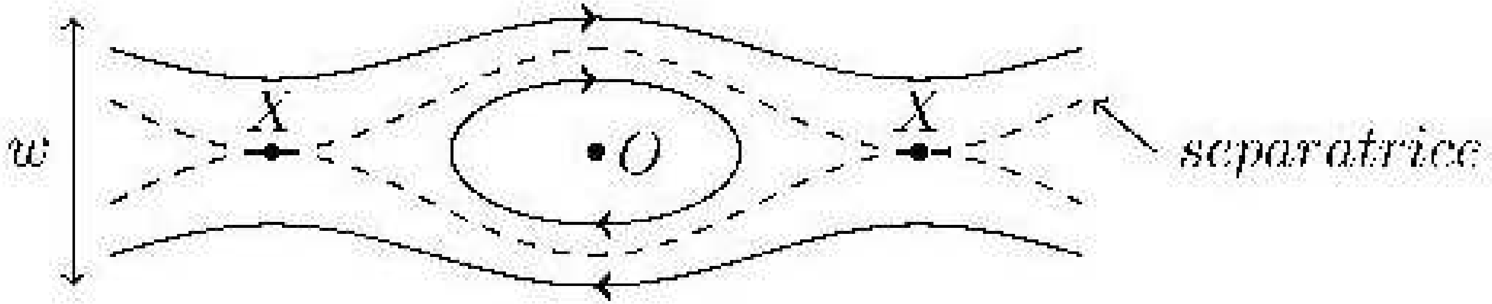
Onset of magnetic islands

$$\frac{\partial B}{\partial t} = \nabla \wedge (v \wedge B) + \frac{c^2}{4\pi} \eta \nabla^2 B$$



Tearing modes

Periodic modulations in the magnetic field appear: these structures are called "magnetic islands"

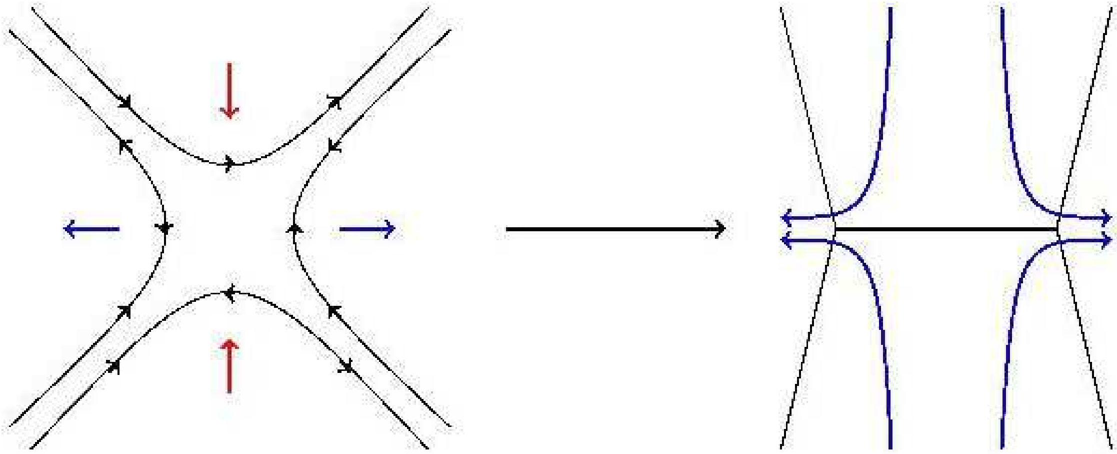


If the islands grow in time, they are called "tearing modes"

Nonlinear growth of tearing modes is described by Rutherford equation ("Nonlinear growth of the tearing mode", P.H. Rutherford, Phys. Fluids 16, 1903 (1973))

Tearing modes are due to magnetic reconnection

Resistive reconnection



Plasma stream near an X point causes the flattening of magnetic field configuration

In a small current sheet, resistivity causes the magnetic field lines to break and reconnect (Sweet-Parker)

Drift-MHD model

$$\left(\frac{\partial}{\partial t}+V_i\cdot\nabla\right)n+\nabla\cdot V_i n=D\nabla^2 n$$

$$E+V_e\wedge B+\frac{1}{ne}\nabla P=\eta J$$

$$n\,m_i\left[\left(\frac{\partial}{\partial t}+V_i\cdot\nabla\right)V_E+\left(\frac{\partial}{\partial t}+V\cdot\nabla\right)V_{\parallel}\right]=J\wedge B-\nabla P-\nabla\cdot\Pi_i+\mu\nabla^2 V_i$$

Half-way between MHD and two-fluids models

Keeps into account the drift velocities

$$V_i=V+V_E \qquad V=V_{\parallel}+V_{*i} \qquad V_e=V_i-\frac{J}{ne}$$

$$V_E=c\frac{E\wedge B}{B^2} \qquad V_{*i}=-T_i\frac{B\wedge\nabla n}{enB^2}$$

Four-field model

$$[\varphi - n, \psi] + \delta_e \eta J = 0$$

$$[\varphi, n] + [\frac{L_n}{L_s}\frac{q}{\epsilon} V + \rho^2 J, \psi] + D \partial_x^2 n = 0$$

$$[\varphi, V] + \frac{L_s}{L_n}\frac{\epsilon}{q}\alpha^2 [n, \psi] + \mu \partial_x^2 V - \nu_\theta \left(\frac{\epsilon}{q}\right)^2 \{V - \partial_x(\varphi + \tau n(1 - c_\theta^{NC}))\} = 0$$

$$\partial_x [\varphi + \tau n, \psi] + [J, \psi] + \mu \partial_x^4 (\varphi + \tau n) + \nu_\theta \partial_x \{V - \partial_x(\varphi + \tau n(1 - c_\theta^{NC}))\}$$
$$+ \nu_\perp \partial_x \{-\partial_x(\varphi + \tau n(1 - c_\perp^{NC}))\} = 0$$

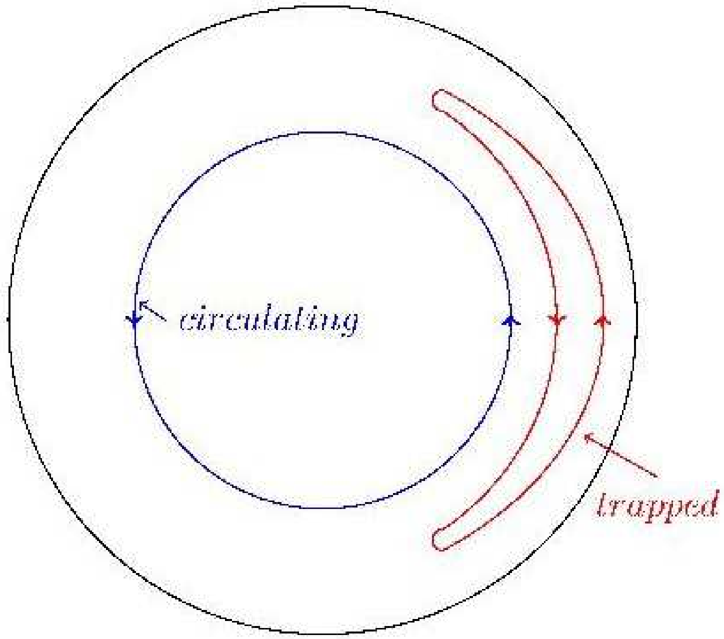
$$\partial_x^2 \psi = -1 + \delta_e J$$

Poisson parenthesis formalism: $[A, B] = \hat{e_\phi} \cdot \nabla A \wedge \nabla B$

Four fields: ψ, V, n, φ

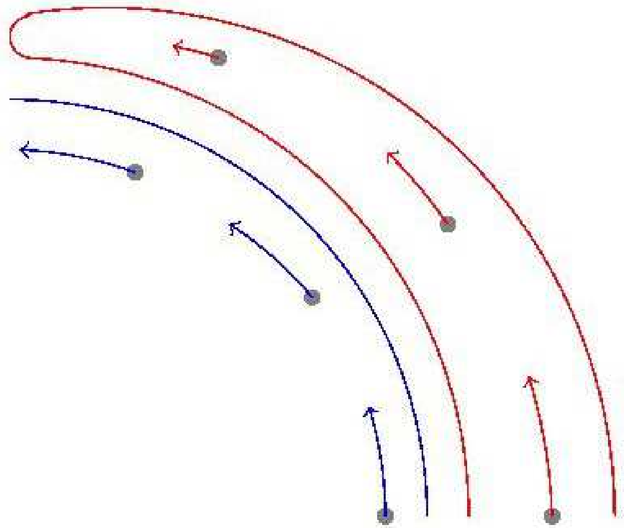
Variables: $x; \xi = m\theta - n\phi$

Neoclassical effects



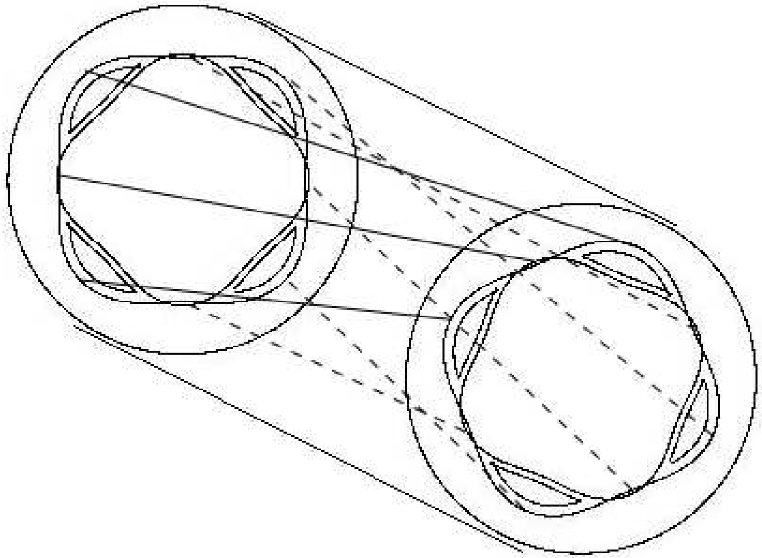
Caused by non-homogeneous magnetic field and low collisionality

Particles with $\sin^2\theta = \frac{v_\perp^2}{v_\perp^2 + v_\parallel^2} > \frac{B_{min}}{B_{max}}$ are trapped in the banana orbits



Momentum exchange between circulating and trapped particles causes poloidal viscosity

The broken axisymmetry of the tokamak due to the magnetic island causes toroidal viscosity



Neoclassical flux-friction relation

Hirshman-Sigmar moment approach: the neoclassical fluxes are proportional to the parallel friction forces (and vice-versa)

"Neoclassical transport of impurities in Tokamak plasmas", S.P. Hirshman & D.J. Sigmar, NUCLEAR FUSION, Vol.21, No.9 (1981)

"Neoclassical Flows and Transport in NonaxisymmetricToroidal Plasmas", K.Shaing & J.D. Callen, Phys. Fluids 26, 3315 (1983)

Classical parallel viscosity comes from parallel flux

Neoclassical viscosity comes from perpendicular flux

Solving the equations

The equations contain several parameters:

$$\epsilon \quad \beta \quad \hat{\nu}_{\theta i} \quad \hat{\nu}_{\perp i} \quad D \quad \eta \quad \mu \quad \alpha \quad \tau$$

A reasonable ordering: $\alpha, \tau \approx 1 \; ; \; 1 \gg \epsilon, \beta \gg \hat{\nu}_{\theta i}, \hat{\nu}_{\perp i}, D, \eta, \mu$

In low-collisionality regimes, the neoclassical damping is by far the most important effect:

$$\hat{\nu}_{\theta i} \gg \hat{\nu}_{\perp i}, D, \eta, \mu$$

"Effect of flow damping on drift-tearing magnetic islands in tokamak plasmas", R.Fitzpatrick and F.L.Waelbroeck, Phys. Plasmas 16, (2009) The system of partial differential equations can be reduced to a ordinary differential equation

$$\frac{d}{d\psi}\left(\langle X^2\rangle \frac{dV}{d\psi}\right) - \frac{\nu_\theta}{\mu}\left(\frac{\epsilon}{q}\right)^2 [\langle 1\rangle (V+V_p) + M + \tau L (1-c_\theta^{NC})] = 0$$

$$M = d\varphi/d\psi \,;\, L = dn/d\psi$$

$\langle X^2\rangle$ and $\langle 1\rangle$ can be written in terms of elliptic integrals

$$K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1-k^2\sin^2\theta}} \qquad E(k) = \int_0^{\pi/2} \sqrt{1-k^2\sin^2\theta}\, d\theta$$

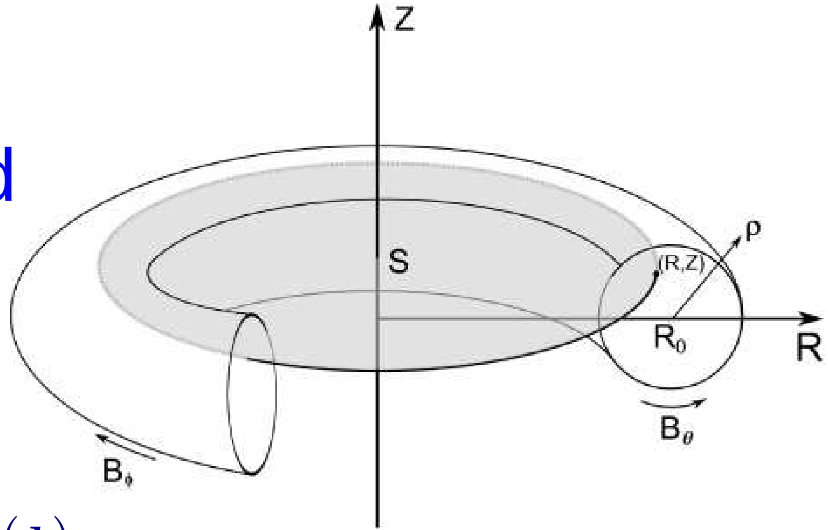
because of the equilibrium flux function:

$$\psi(x,\xi) = \frac{x^2}{2} + \sum_{k=0}^{\infty} \psi_k(x) e^{i(k\xi)} \approx \frac{x^2}{2} + \cos\xi$$

Flux surface average

Flux function: $B = \hat{e}_\phi B_\phi + \hat{e}_\phi \wedge \nabla \psi$

Flux of the poloidal magnetic field across a disk-like surface



Flux surface average: $\langle A \rangle = \oint \frac{dl}{B} A(l)$

Most noteworthy property: $\langle [A, \psi] \rangle = \langle B \cdot \nabla A \rangle = 0$

Final equations

Changing variables and using the force-balance:

$$\frac{\nu_\theta+\nu_\perp}{4\nu_\theta}\frac{d}{dk}\left(A\frac{dQ}{dk}\right)-\frac{\nu_\theta}{\mu}\left(\frac{\epsilon}{q}\right)^2\left(B-\frac{1}{A}\right)Q-\frac{\nu_\perp}{\mu}\left(\frac{\epsilon}{q}\right)^2 B(Q-1)=0$$

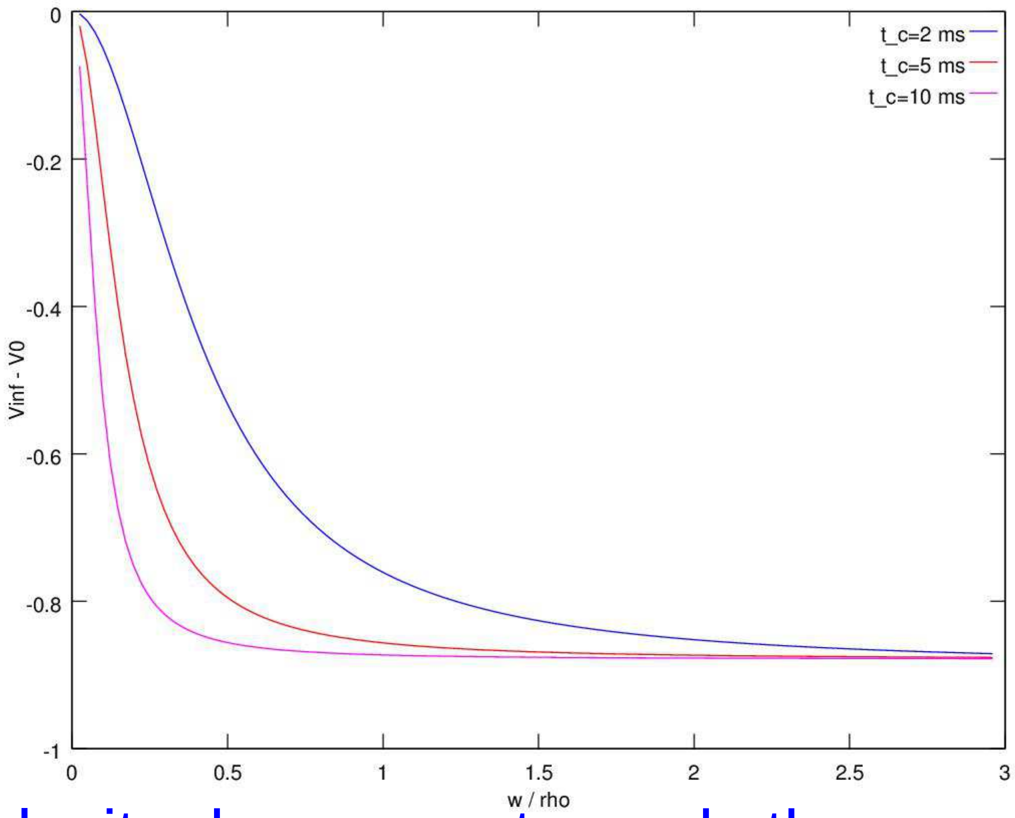
$$V_p+V_\infty=\frac{I_1+I_3}{I_1+I_2}\tau\left(c_\perp^{NC}-c_\theta^{NC}\right)$$

$$I_1=\frac{2}{\pi}\int_0^1 kK(k)\,dk\ ;\ I_2=\frac{\nu_\theta+\nu_\perp}{\nu_\perp}\int_1^\infty\left(A-\frac{1}{B}\right)Q\,dk\ ;\ I_3=\int_1^\infty\left(A-\frac{1}{B}\right)dk$$

Solving the equations with the appropriate boundary conditions gives the fields' radial profiles and the fluid rotation velocity

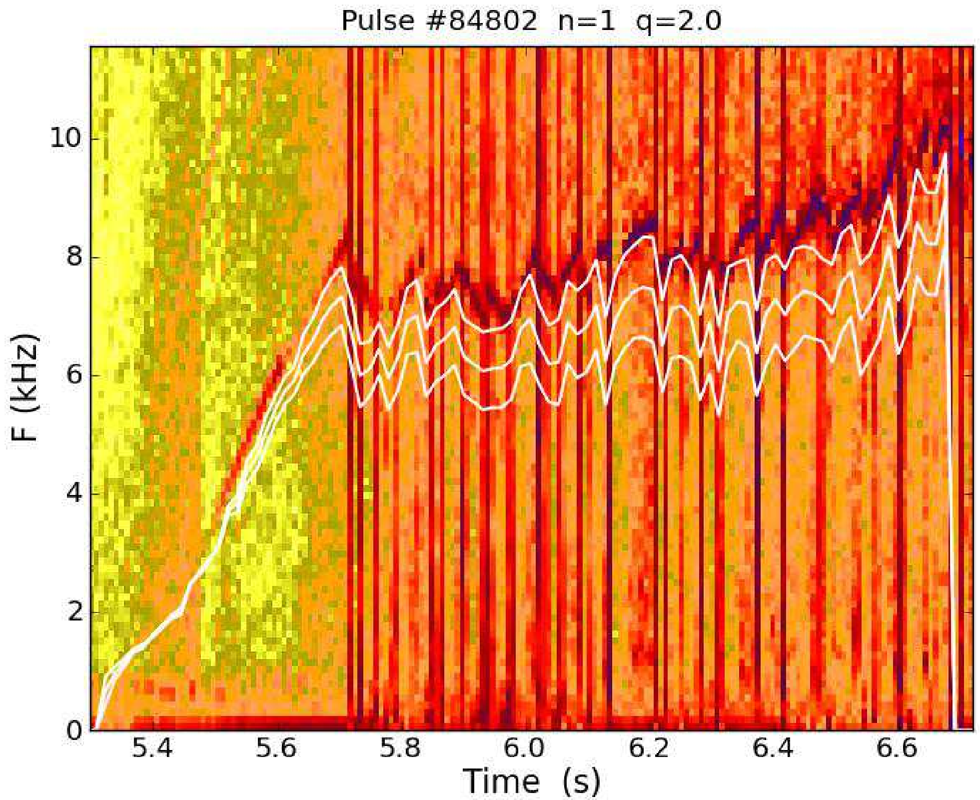
Results

We could ask how rapidly does the velocity decrease outside the separatrix



The ion fluid velocity decreases towards the neoclassical value for large enough islands

Experimental observations



Signal from a magnetic coil inside JET

The upper white line is the ion fluid rotation frequency

Empirical relation: $\omega = n\Omega_D(R)$, $\Omega_D(R) = \Omega_{cX} - \frac{5}{6}\frac{\partial T_i}{\partial \psi}$

cX = carbon ions

From the lowest order poloidal force balance: $V_{\parallel} = -V_p$

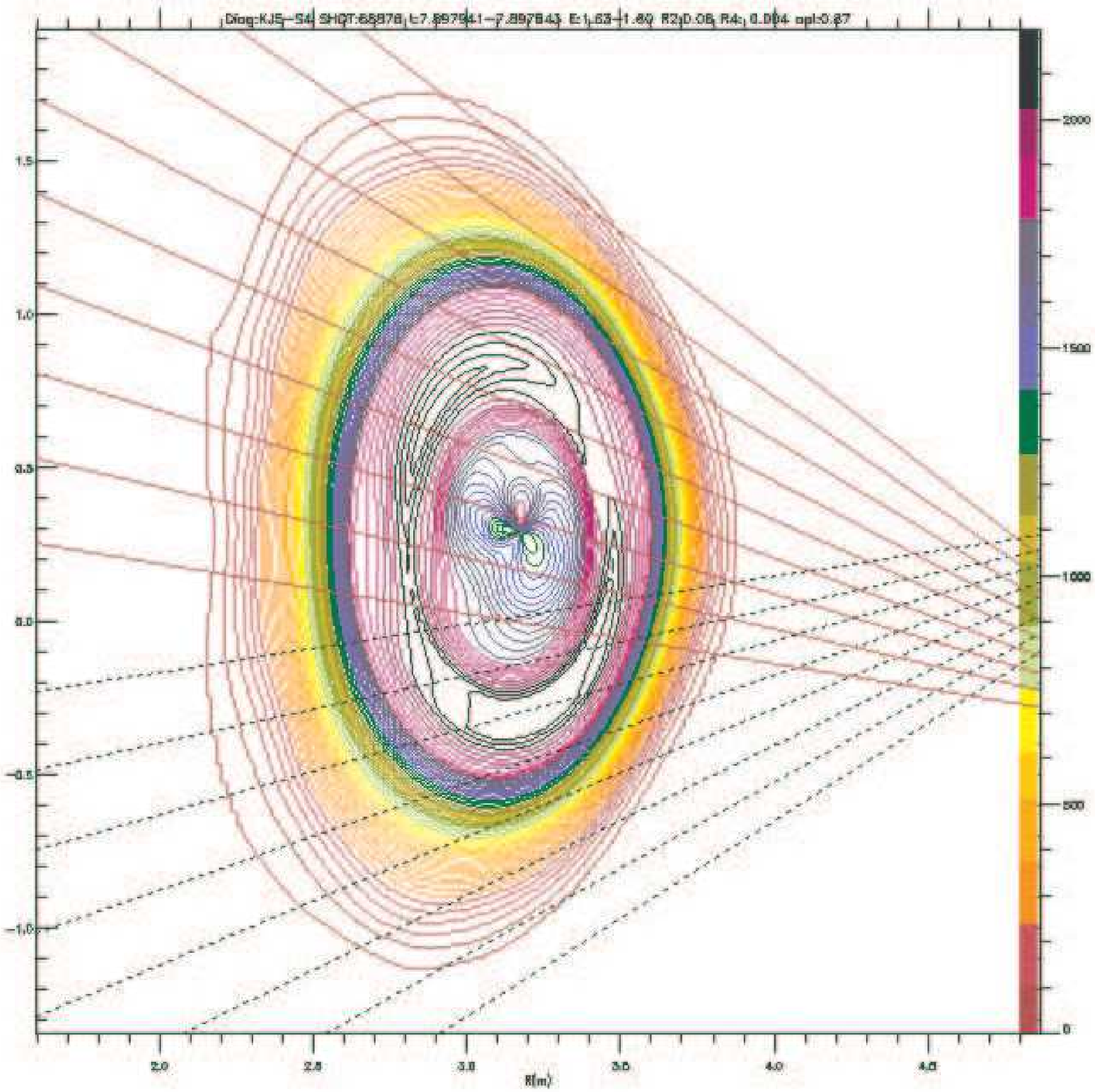
and from the definition of the phase velocity $\omega = k V_p$:

$$\omega = \frac{n}{R} V_{\parallel} = n\Omega_{\varphi i}$$

where $\Omega_{\varphi i}$ is the toroidal ion rotation frequency.

For the case n=1, we find the experimental results

X-ray imaging



Conclusions

- Resonant perturbations cause the magnetic surfaces to break
- Magnetic islands are deleterious for the confinement
- Magnetic islands' dynamic can be studied by using the drift-MHD system of equations
- The four-field model highlights its mathematical properties
- The solution of the equations gives the fields radial profiles and the island's rotation velocity
- There's agreement between the theoretical predictions of the rotation fequency and the experimental results