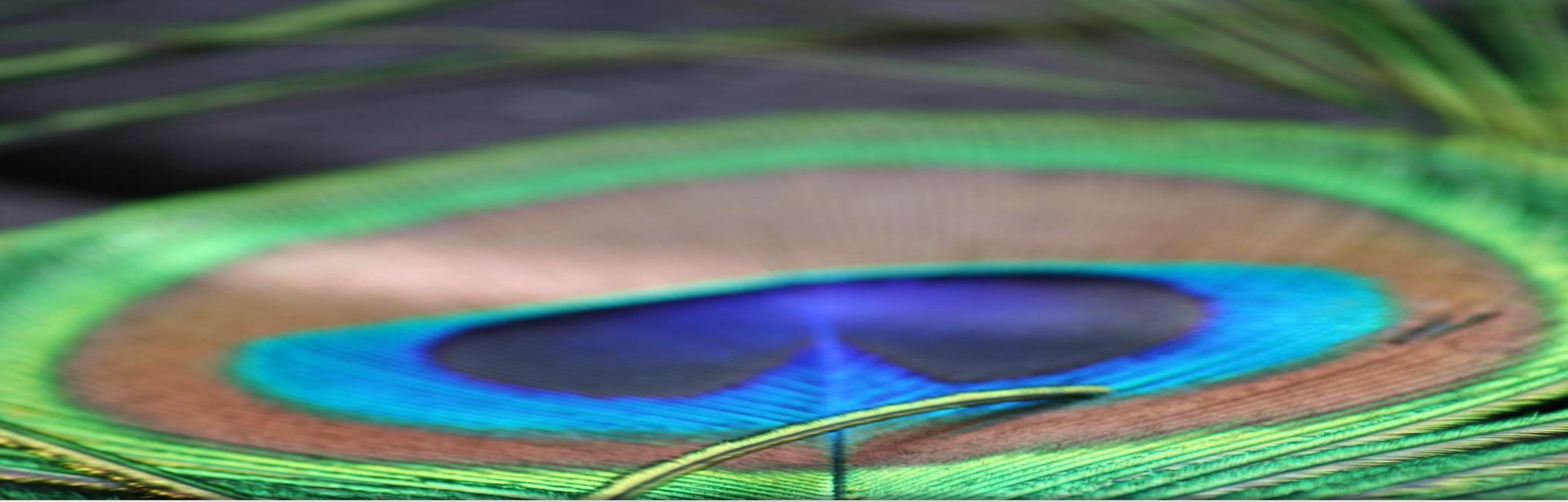

PHOTONIC CRYSTALS

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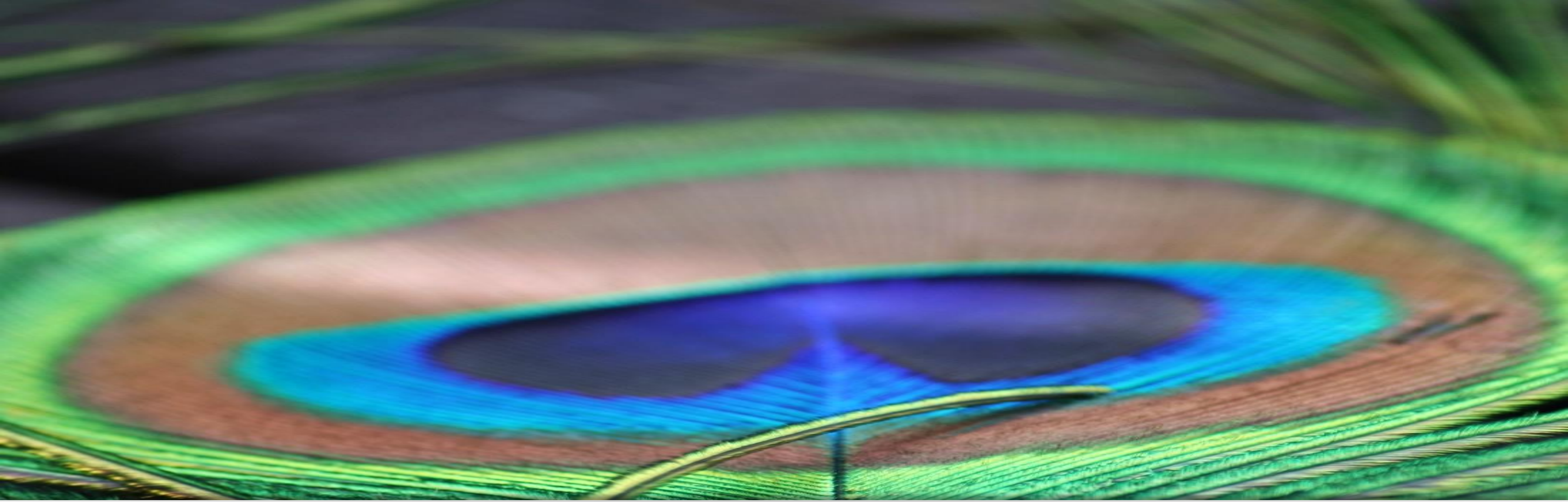
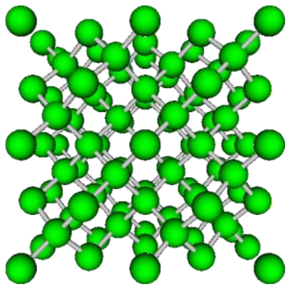
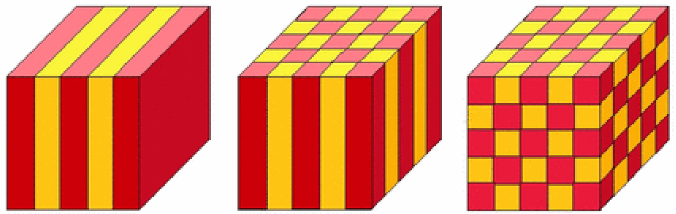


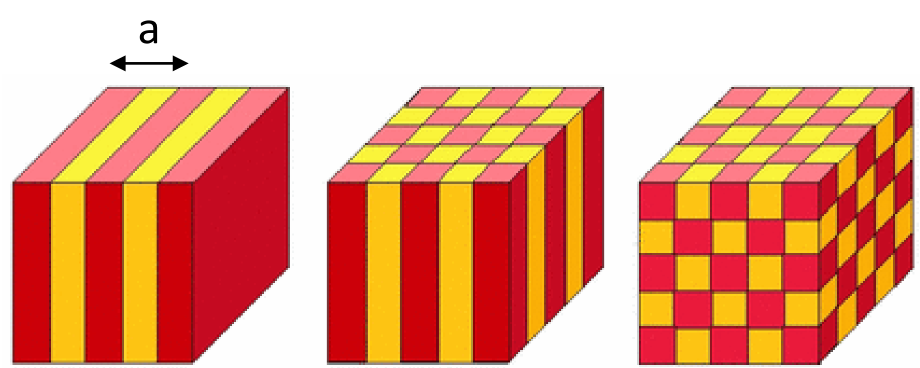
Outline

- Introduction to photonic crystals
- The master equation
- Photonic Bands
- Defects
- Applications

Photonic crystals are periodic dielectric structures which are to photon waves as crystalline solids are to electron waves

Material	Description	Waves
Crystalline solid	Crystalline atomic lattice 	Electron Waves $\Psi(\vec{r})$
Photonic crystal	Periodic dielectric structures 	Photonic Waves $\vec{H}(\vec{r}), \vec{E}(\vec{r})$

Periodicity appears in the refractive index



1D 2D 3D

Classical photonics: electromagnetic waves that propagate according to Maxwell equations

Wavelength of light $\lambda \sim a$

$$n(\vec{r}+\vec{R}) = n(\vec{r}) \qquad \forall \vec{R} \epsilon \text{ direct lattice}$$

$n = \sqrt{\varepsilon_r \mu_r}$ Optical regime $\mu_r(\omega) \sim 1$

Periodic permittivity $\varepsilon(\vec{r})$

Periodicity leads to constructive and destructive **interference**

Periodic permittivity and Maxwell equations are the only ingredients needed to describe the system

Transversality conditions

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{D} = 0 \qquad \vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$$\vec{D}(\vec{r}) = \varepsilon(\vec{r})\vec{E}(\vec{r})$$

$$\vec{B}(\vec{r}) = \vec{H}(\vec{r})$$

<u>Monochromatic fields</u>

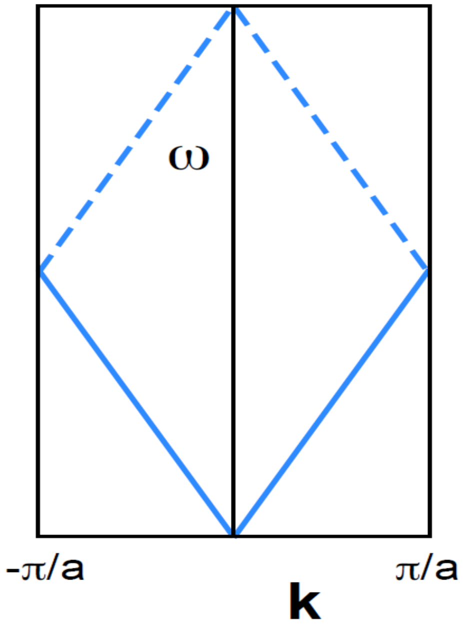
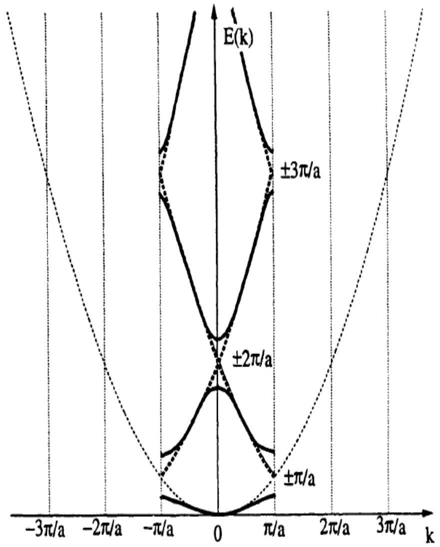
$$\vec{E}(\vec{r},t) = \vec{E}(\vec{r})e^{-i\omega t}$$

$$\vec{H}(\vec{r},t) = \vec{H}(\vec{r})e^{-i\omega t}$$

Helmholtz Equation

$$\vec{\nabla}^2\vec{H}(\vec{r}) + \left(\frac{\omega}{c}\right)^2 \varepsilon(\vec{r})\vec{H}(\vec{r}) = 0$$

There is a strong analogy between the Helmholtz Equation and the Schroedinger equation

Helmholtz Equation	Schroedinger Equation
	
$\vec{\nabla}^2\vec{H}(\vec{r})+\left(\frac{\omega}{c}\right)^2\varepsilon(\vec{r})\vec{H}(\vec{r})=0$	$\frac{-\hbar^2}{2m}\vec{\nabla}^2\Psi(\vec{r})+(V(\vec{r})-E)\Psi(\vec{r})=0$

The generalized eigenvalue problem can be recast in the form of a regular eigenvalue problem

Master Equation

$$\vec{\nabla} \times \left(\frac{1}{\varepsilon(\vec{r})} \vec{\nabla} \times \vec{H}(r) \right) = \left(\frac{\omega}{c} \right)^2 \vec{H}(\vec{r})$$

Introducing the **Maxwell operator** $\theta = \vec{\nabla} \times \left(\frac{1}{\varepsilon(\vec{r})} \vec{\nabla} \times \right)$

$$\theta \, \vec{H}(\vec{r}) = \left(\frac{\omega}{c} \right)^2 \vec{H}(\vec{r})$$

The system can be studied in terms of symmetries

$\varepsilon(\vec{r})$ is invariant under translation

Eigenfunctions → Bloch Functions

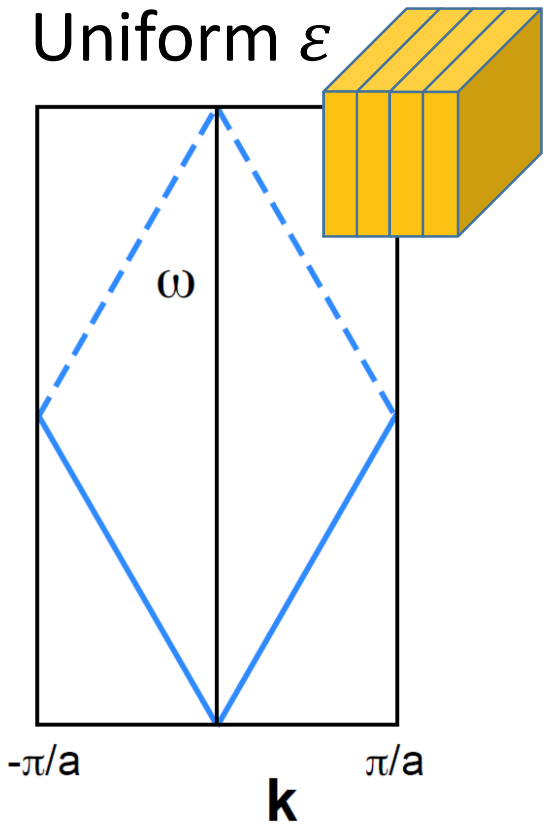
$$\vec{H}(\vec{r}) = e^{i\vec{k}\,\vec{r}}\vec{u}(\vec{r})$$

$$\vec{u}(\vec{r}+\vec{R}) = \vec{u}(\vec{r}) \qquad \forall \vec{R} \epsilon \text{ direct lattice}$$

Solutions can be represented in the reciprocal lattice

Nonreduntant values for the wavevector k → **First Brillouin zone**

Folding in the first Brillouin Zone is a consequence of periodicity



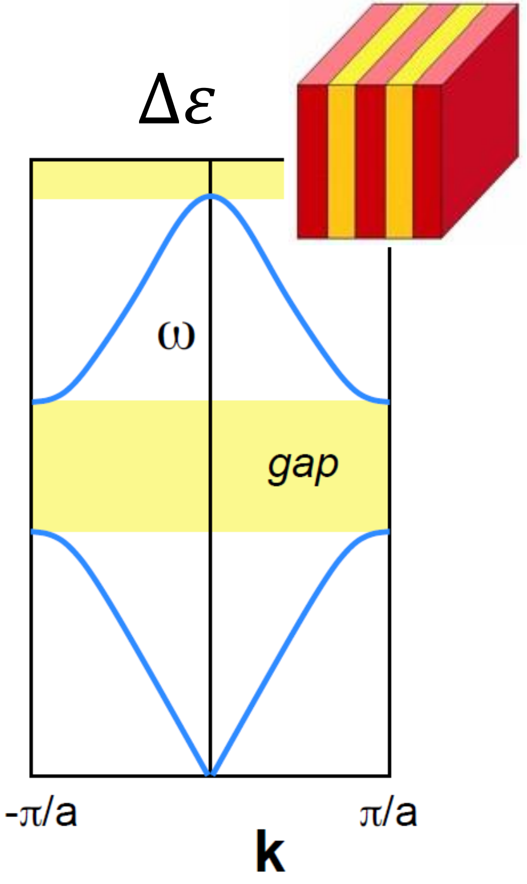
Photon dispersion relation

$$\omega = ck$$

Air Band: lower refractive index

Photonic bandgap: forbidden frequencies

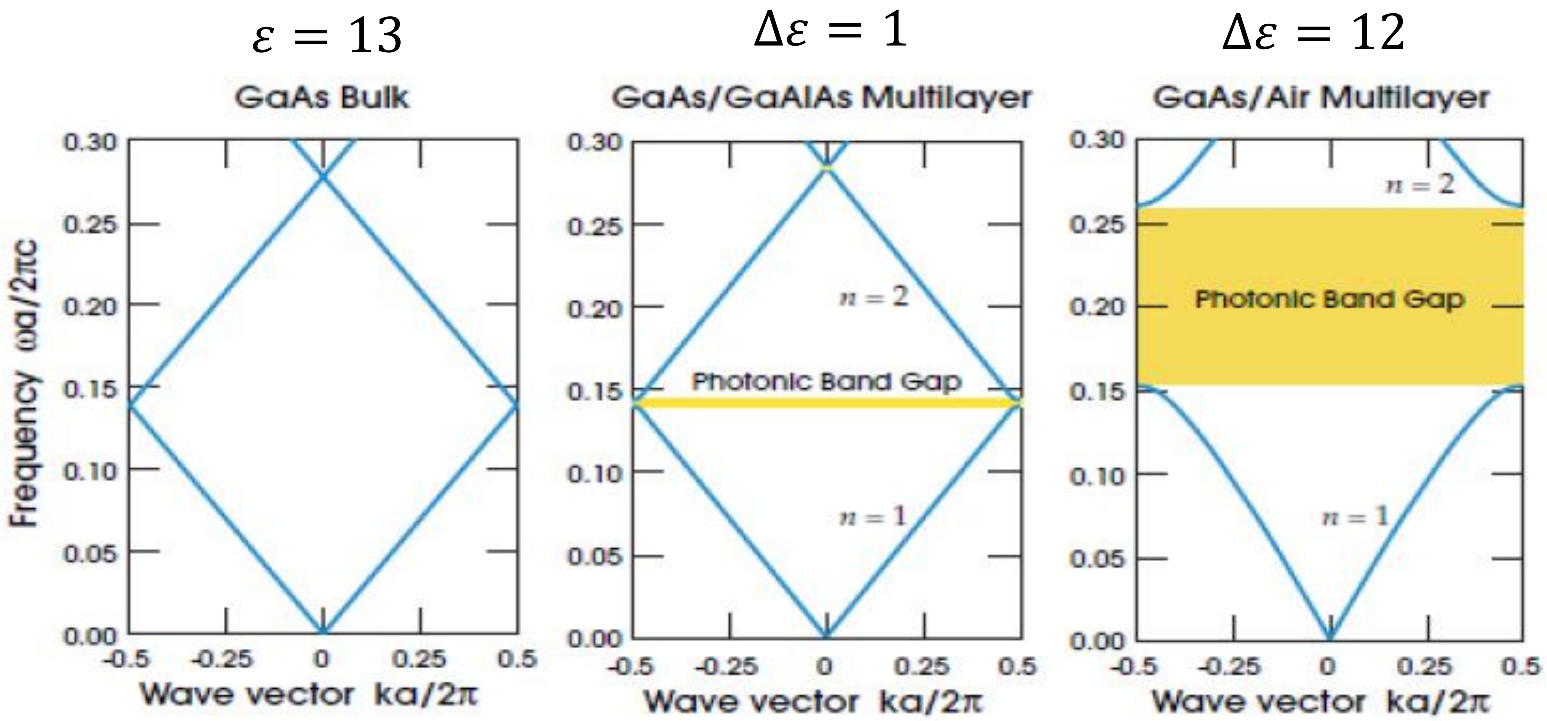
Dielectric band: higher refractive index



Strong analogy to the semiconductor band structure

The amplitude of the photonic bandgap depends on the permittivity differences between the two media

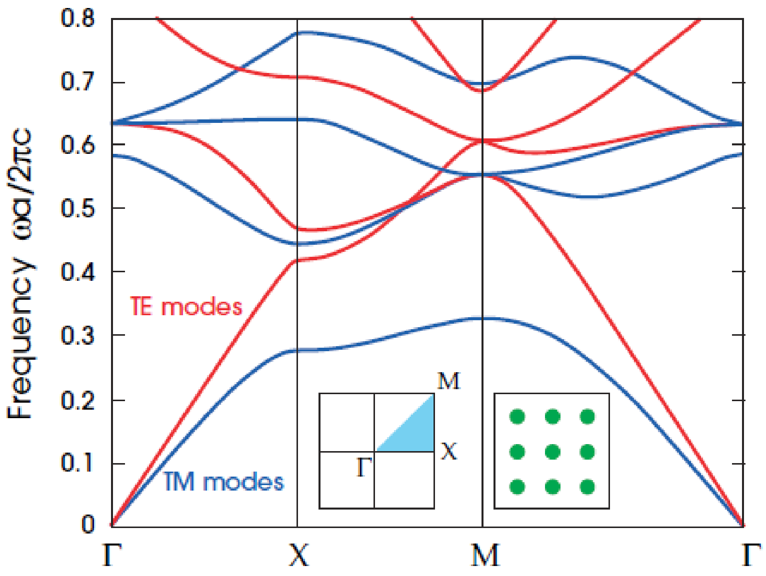
Photonic bandgap: frequencies that cannot propagate into the material



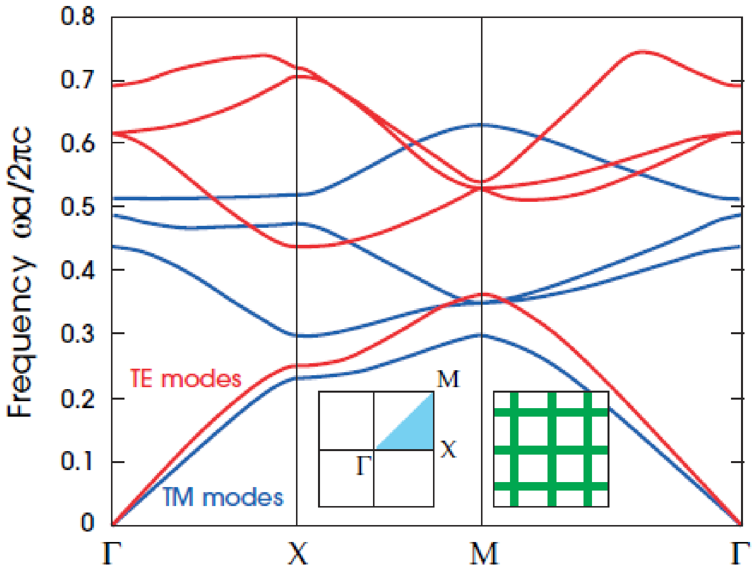
Band structure can be calculated with numerical techniques

The space of all the solution splits into two distinct subspaces

Bloch states characterized by a polarization index

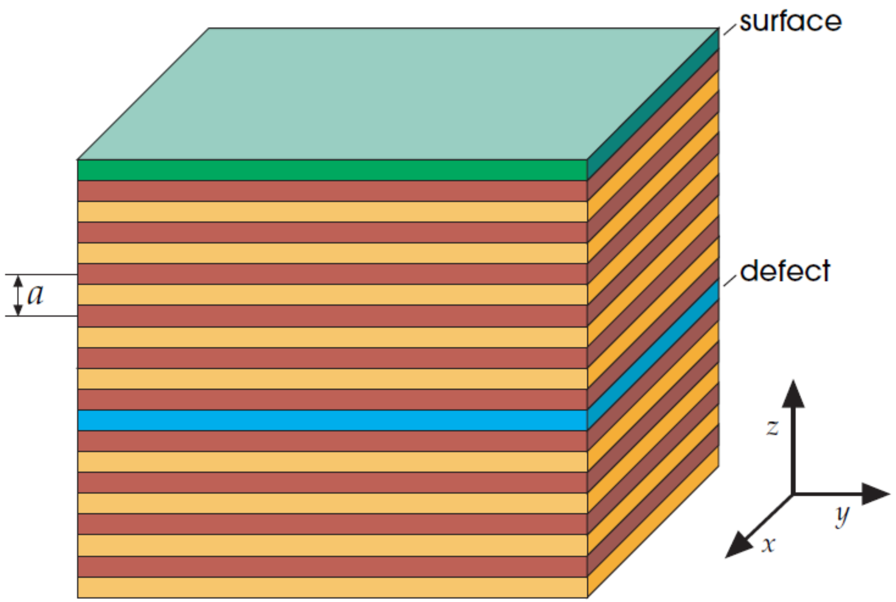


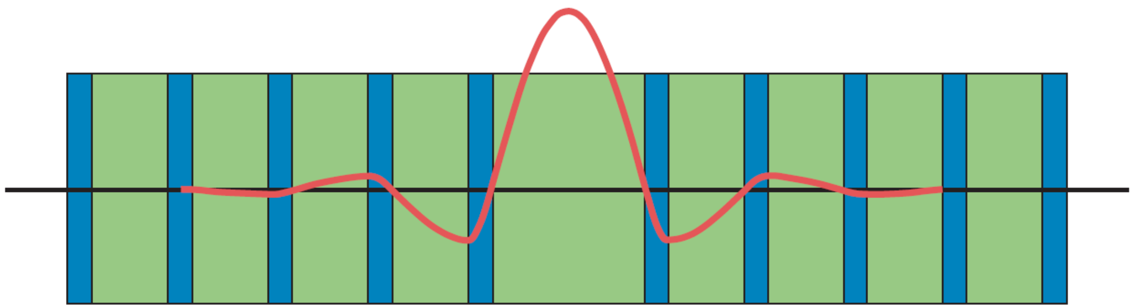
Gap for TM



Gap for TE

As semiconductors photonic crystals can be ‘doped’ inserting defects

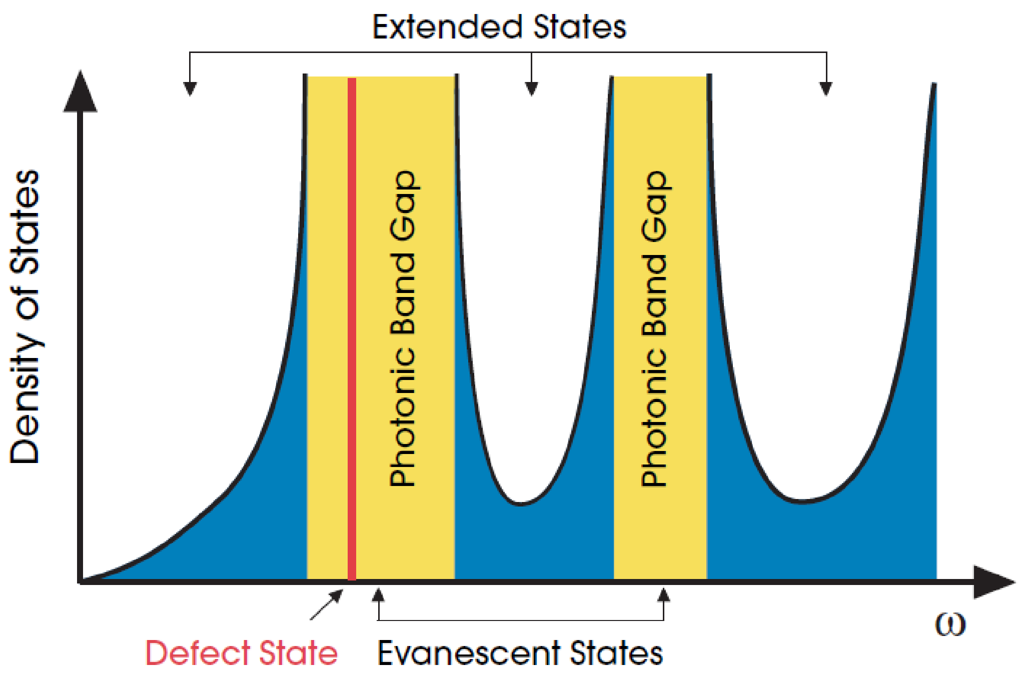




Inserting extra dielectric material
(DONOR ATOMS)

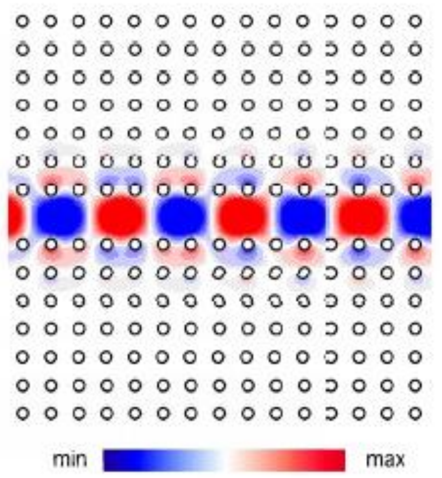
Removing some dielectric material
from the unit cell
(ACCEPTOR ATOMS)

Defects broke the translational symmetry



Many wavelength away from the defect, the modes should look similar to the modes of a perfect crystal

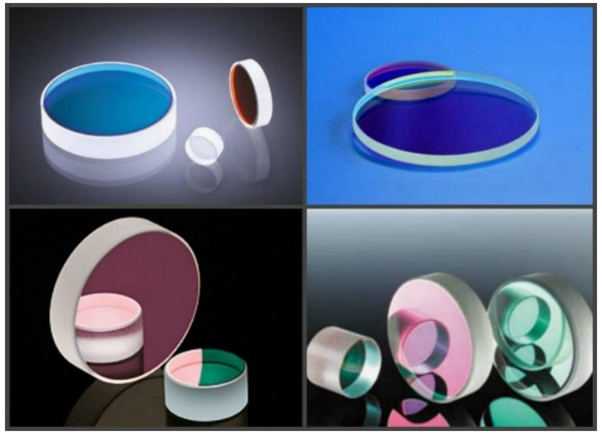
Defects permit **localized modes** to exist, with frequencies inside the photonic band gap

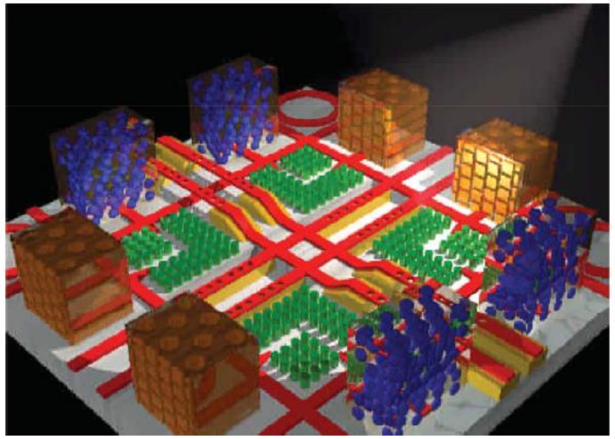


Possibility to guide and trap light with great efficiency

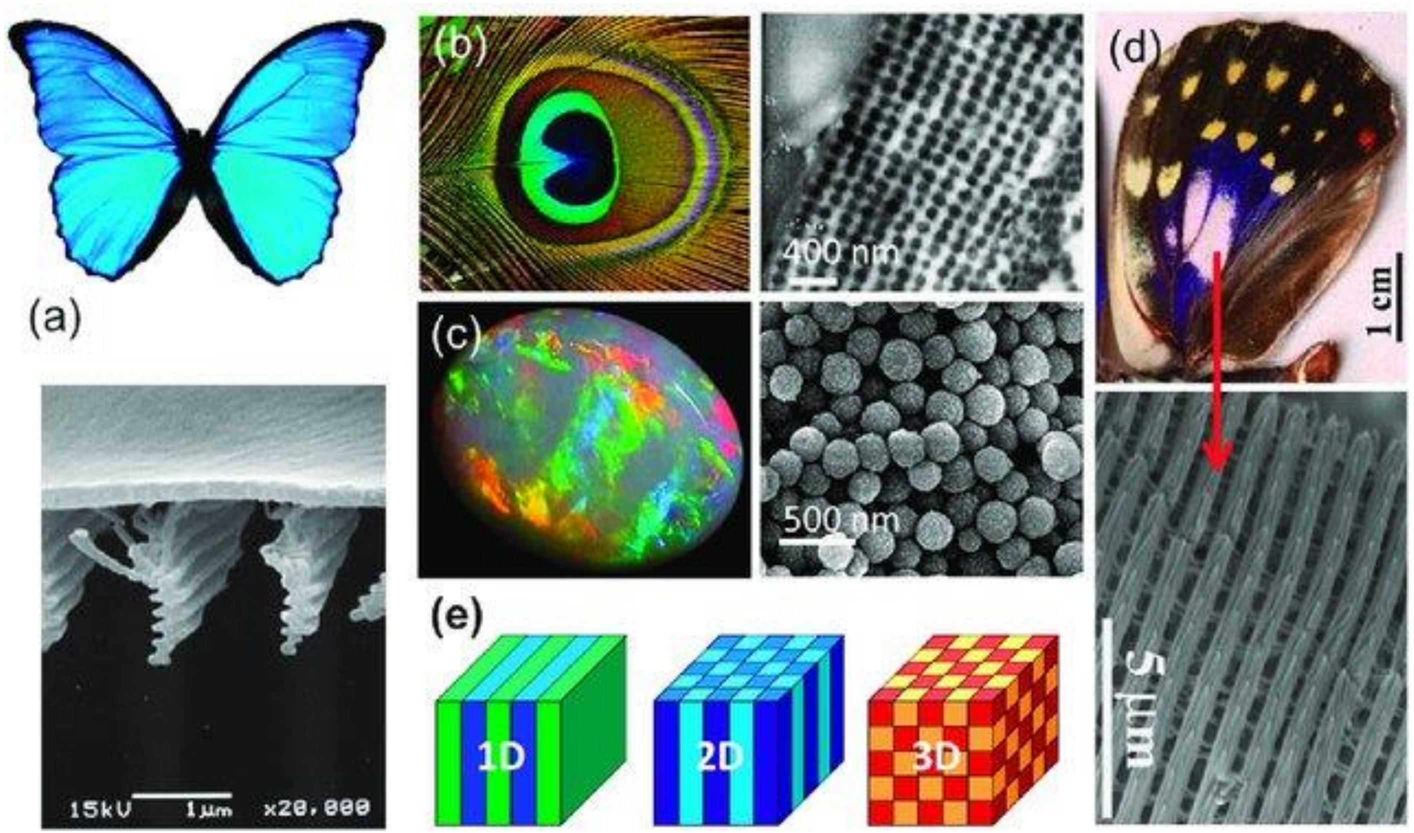
Controlling and manipulating light flow is the main aim of photonic crystals

- Control of spontaneous emission
- Waveguides
- Mirrors
- Microcavities
- Light emitters
- Photonic circuits





Photonic crystals exist also in nature



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