Characterization and mitigation of non-stationary noise

in Advanced gravitational wave detectors

SEMINARIO DI PRE-TESI PER IL XXXII CICLO DI DOTTORATO
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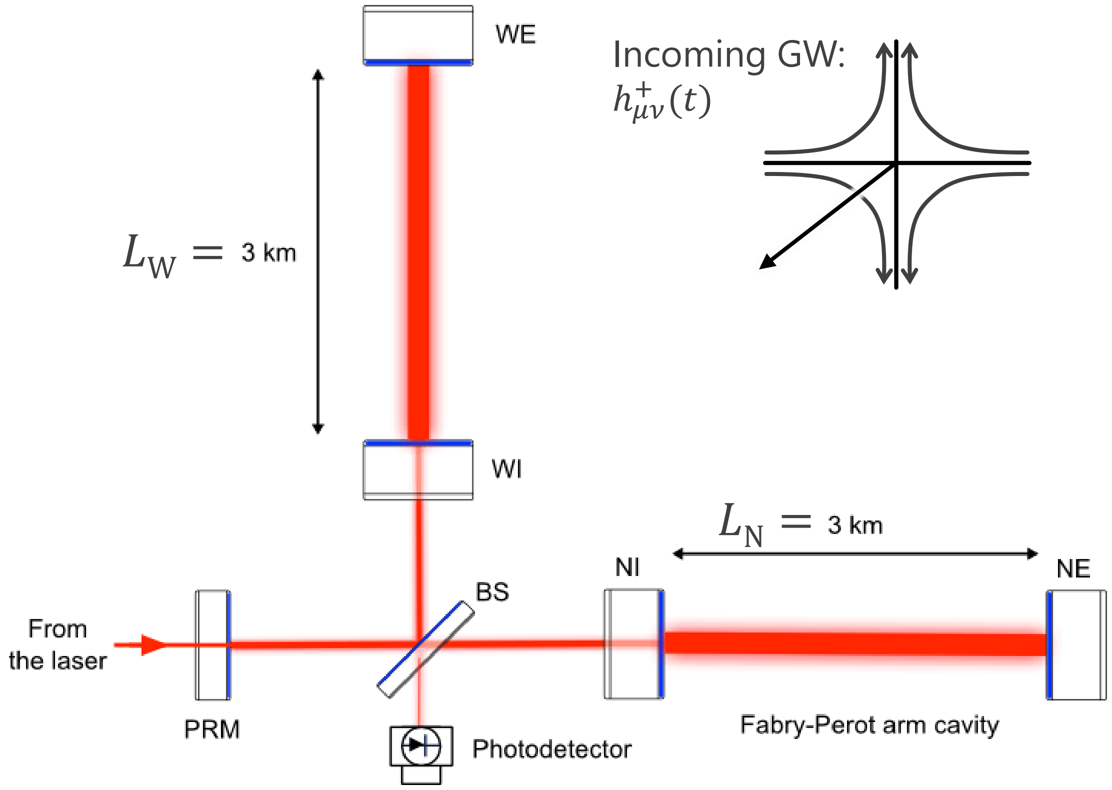
Plan of the presentation

- Introduction on GW detectors
  - notation and key quantities
- Motivations for studying non-stationary noise
  - GW events validation and parameter estimation
- Developed tools
  - Line tracker, cross-correlation and regression tools
- Example results
- Ongoing projects and perspectives

Introduction:
Advanced gravitational wave detectors



Advanced gravitational wave (GW) detectors are **modified Michelson interferometers**.

DARM: **d**ifferential change in the interferometer **arm**-length produced by the passage of a GW:

$$\Delta(L_{\mathrm{W}} - L_{\mathrm{N}}) \propto h(t)$$

measured as a laser power variation from the dark-port photodetector and controlled by actuating differentially on the two end mass mirrors.

$\boldsymbol{h_{\mathrm{rec}}(t)}$: reconstructed amplitude of the GW strain, obtained after calibration and by correcting for the detector optical response. [arXiv:1807.03275v2]

Sensitivity benchmarks: strain sensitivity

Additive noise model:

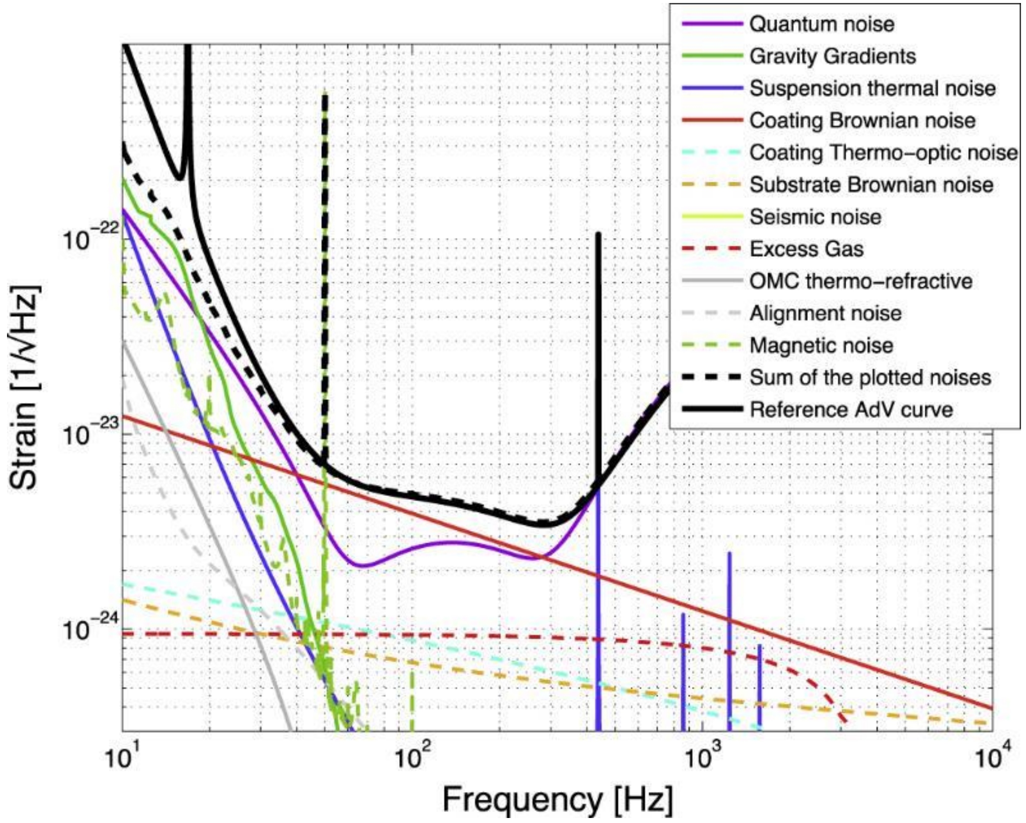
$$h_{\text{rec}}(t) \equiv s(t) = h(t) + n(t)$$

Predicted strain sensitivity:

$$\tilde{h}(f) = \sqrt{S_n(f)}$$

where $S_n(f)$ is the **power spectral density** of the noise, $n(t)$.

Figure: reference sensitivity (solid lines) and the noise budget (dashed lines) for the Advanced Virgo detector. [arXiv:1408.3978v3]



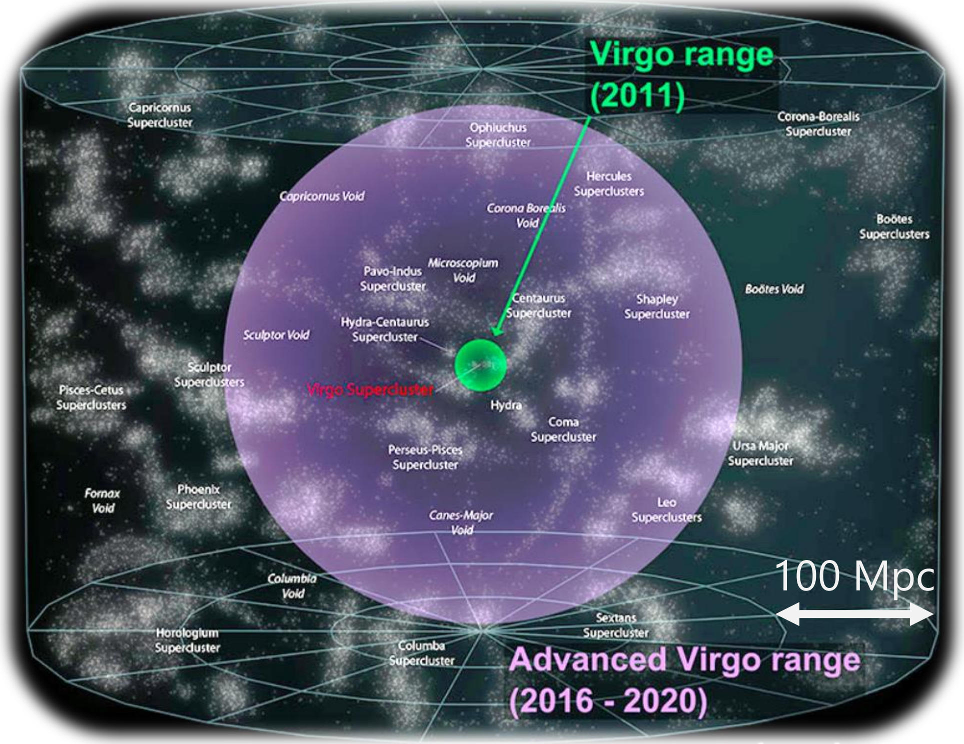
Sensitivity benchmarks: BNS range

Sensitivity range: [Bassan2014]

$$\frac{d_{\text{range}}}{1\ \text{Mpc}} = 0.86 \times 10^{-20} \left(\frac{\mathcal{M}}{M_{\odot}}\right)^{5/6} \sqrt{\int_{f_{\text{min}}}^{f_{\text{ISCO}}} \frac{f^{-7/3}}{S_n(f)} df}$$

Multiplied by the **rate density of sources** of a given type, this gives an estimate of the number of detectable events of that type.

Figure: BNS range for Virgo and Advanced Virgo.



GW detection and parameter estimation
Gaussian, stationary noise assumption

The expected signal-to-noise ratio (SNR) $\boldsymbol{\rho}$ of a GW signal, $h(t)$, can be expressed as:

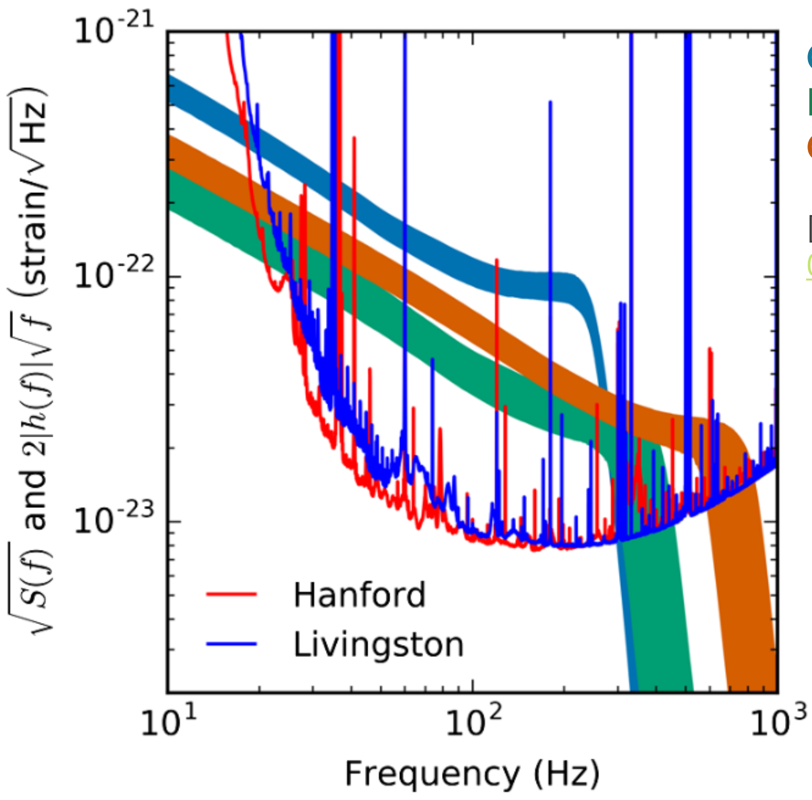
$$\rho^2 = (h|h) = \int_0^\infty \frac{\left|2\tilde{h}(f)\sqrt{f}\right|^2}{S_n(f)} d\ln f$$

The corresponding detection statistic is optimal for **Gaussian, stationary noise**.

Under the same assumptions, parameter estimation is obtained form the posterior probability (Bayes theorem):

$$p(\theta|s) = \mathcal{N} \exp\left[(h|s) - \tfrac{1}{2}(h|h)\right] p(\theta)$$

where $h = h(t;\theta)$ and θ is a (set of) GW parameter(s).

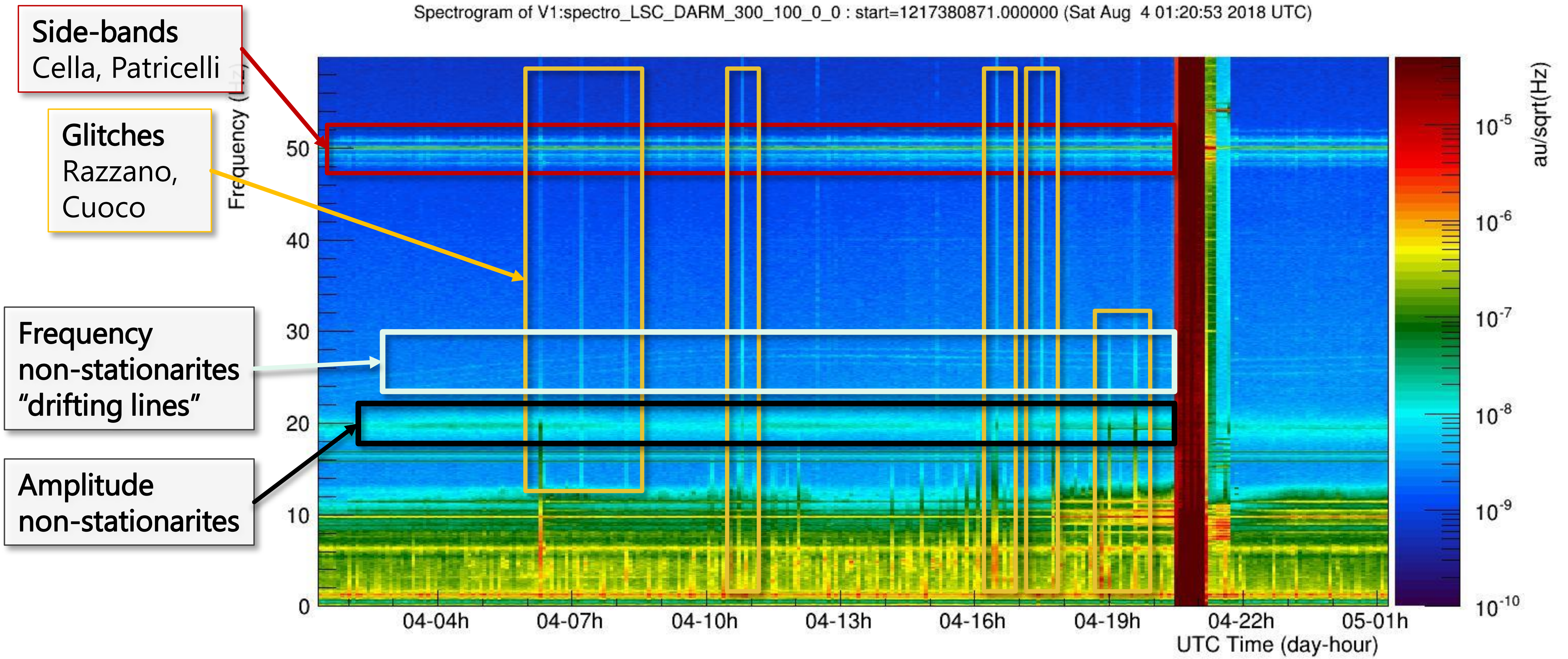


GW150914
LVT151012
GW151226

[Phys. Rev. X 6, 041015]

But noise is often non-stationary and non-Gaussian

especially during the commissioning phase



Non-stationary noise studies

Glitch: "short" duration bursts of excess power. Typical time scales $\lesssim$ 1 second.

Slow non-stationary noise: characteristic time scales longer than 1 second.

- **Amplitude non-stationarities:** variations of the "average power" within a certain frequency band. We can study them by means of the Band-limited Root Mean Square of their power spectral density:

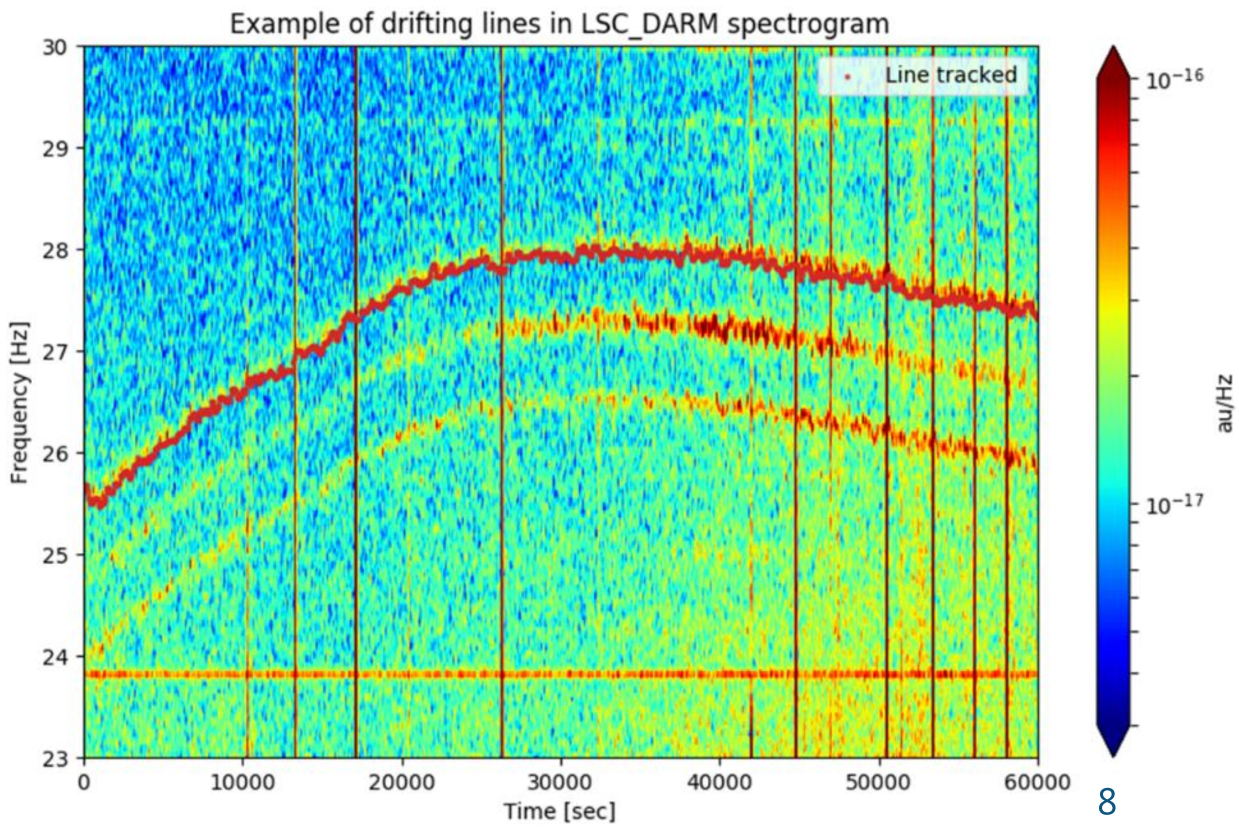
$$BRMS(t) = \sqrt{\int_{f_1}^{f_2} S_n(f;t)\, df}$$

- **Frequency non-stationarities:** the information about a drifting line can be represented by the time series of its maxima, varying in time (*line tracker tool*).

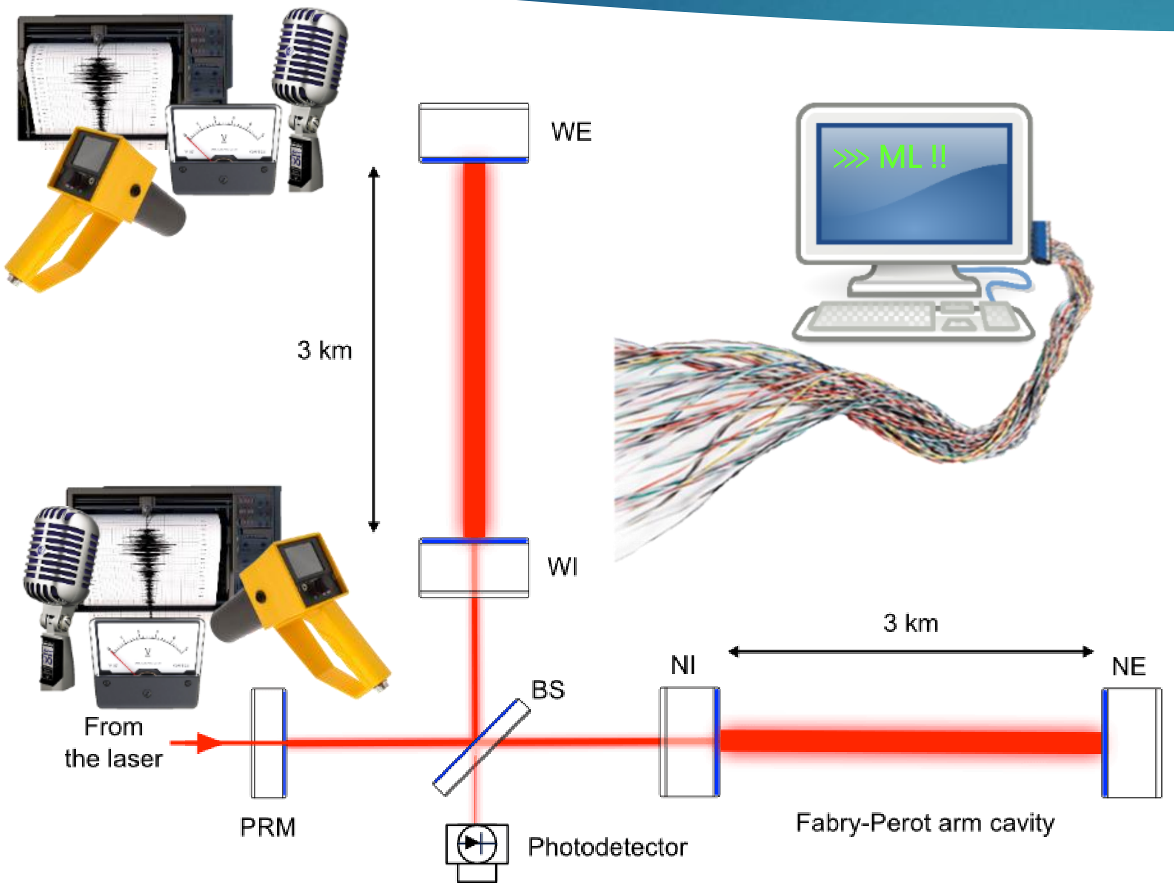
Frequency non-stationarity example:
line tracker tool

Line tracker tool: based on edge detection algorithms (Canny's algorithm): [Canny]

- **Gaussian filter** to smooth the image in order to remove the noise
- Intensity gradient of the image: **Sobel operator** (vertical, to remove glitches)
- Track edge by **hysteresis**: suppress all the other edges that are weak and not connected to strong edges.
- Convert to the time series of the maxima.



Auxiliary sensor channels



The detector and its environment is continuously monitored by $\mathcal{O}(10\mathrm{k})$ auxiliary sensors: seismometers, magnetometers, etc.

➔ Flux of data: ~**40 Mb/s**

The idea is that some of these channels may “**witness**” the noisy behavior of the detector.

Cross-correlation analysis tool

Target channel: y_i, for all times $t_0, t_1, \ldots, t_N$. **E.g.:** BRMS, line, slow sensitivity channel (BNS range).

Auxiliary channels ($\boldsymbol{n}$): $x_{1i}, x_{2i}, \ldots, x_{ni}$. May also include powers and cross-terms.

Pearson's cross-correlation coefficient:

$$r_{xy} = \frac{1}{N-1} \sum_{i=1}^{N} \frac{(x_i - \bar{x})(y_i - \bar{y})}{s_x \, s_y}$$

where $\bar{x} = \frac{1}{N} \sum x_i$ is the sample mean and $s_x = \frac{1}{N-1} \sum (x_i - \bar{x})^2$ the sample variance.

- **Brute force approach:** when no clues are available on the origin of the noise, search over all the available information provided by the auxiliary channels:

Typical set up: $\mathcal{O}$(10k) seconds of data, $\mathcal{O}$ (10k) auxiliary channels (fs 1 Hz)

Execution time: 5÷10 minutes

Multiple linear regression analysis tool:
model construction

We want to *predict* slow time variations of the **target channel** y_i (h_{rec}, BNS range, DARM, etc…) by means of a linear combination of the **auxiliary channels**:

$$\hat{y}_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \cdots + \beta_n x_{ni} \equiv \mathrm{X}_i \boldsymbol{\beta}$$

that is:

$$y_i = \mathrm{X}_i \boldsymbol{\beta} + e_i$$

where e_i is the **residual** difference between the estimate $\hat{y}_i$ and the target y_i.

- Different models differ on the choice of channels to include, powers and cross-terms, or lagged variables.
- Other subtler parameters: output frequency for downsampling, signal transformations (PCA), etc.
- Optimal models will be chosen on the base of the properties of their residuals, e_i's.

Multiple linear regression analysis tool:
OLS coefficients estimators

Under the Classical Linear Model (CLM) assumptions

- $X_i \equiv (x_{1i}, x_{2i}, \dots, x_{ni})$ is full rank } Principal Components Analysis
- $E[e_i] = 0, \quad E[e_i^2] = \sigma^2$ and $E[e_i e_j] = 0$ } Residual diagnostics

the **Gauss-Markov theorem** [Gauss-Markov] says that the **Ordinary Least Squares** (**OLS**) estimator $\hat{\boldsymbol{\beta}}$ of the regression coefficients is **BLUE**: [Robinson]

- **B**est (minimum variance, according to the Cramèr-Rao lower bound [Cramèr])
- **L**inear function of the data
- **U**nbiased ($E[\hat{\boldsymbol{\beta}}] = \boldsymbol{\beta}$)
- **E**stimator of $\boldsymbol{\beta}$

If the e_i's are also normally distributed, $\hat{\boldsymbol{\beta}}$ becomes efficient, and reliable $\boldsymbol{t}$ **and** $\boldsymbol{F}$ **tests** can be carried out to asses models and predictors significances.

Regression tool flowchart

Target pre-processing:

- Quality checks: ITF mode, locking

Amplitude non-stats.:

- Signal BLRMS

Frequency non-stats.:

- Line tracker

Auxiliary pre-processing:

- Quality checks: nans, constant, piecewise const., etc.
- **PCA:** energy cut, collinearity, condition number, etc.
- Line tracker

Computation of the OLS regression coefficients $\hat{\beta}$ through SVD decomposition and target estimation:

$$\hat{y}_i = \mathrm{X}_i \boldsymbol{\beta}$$

Residuals analysis

$$e_i = y_i - \hat{y}_i = y_i - \mathrm{X}_i \boldsymbol{\beta}$$

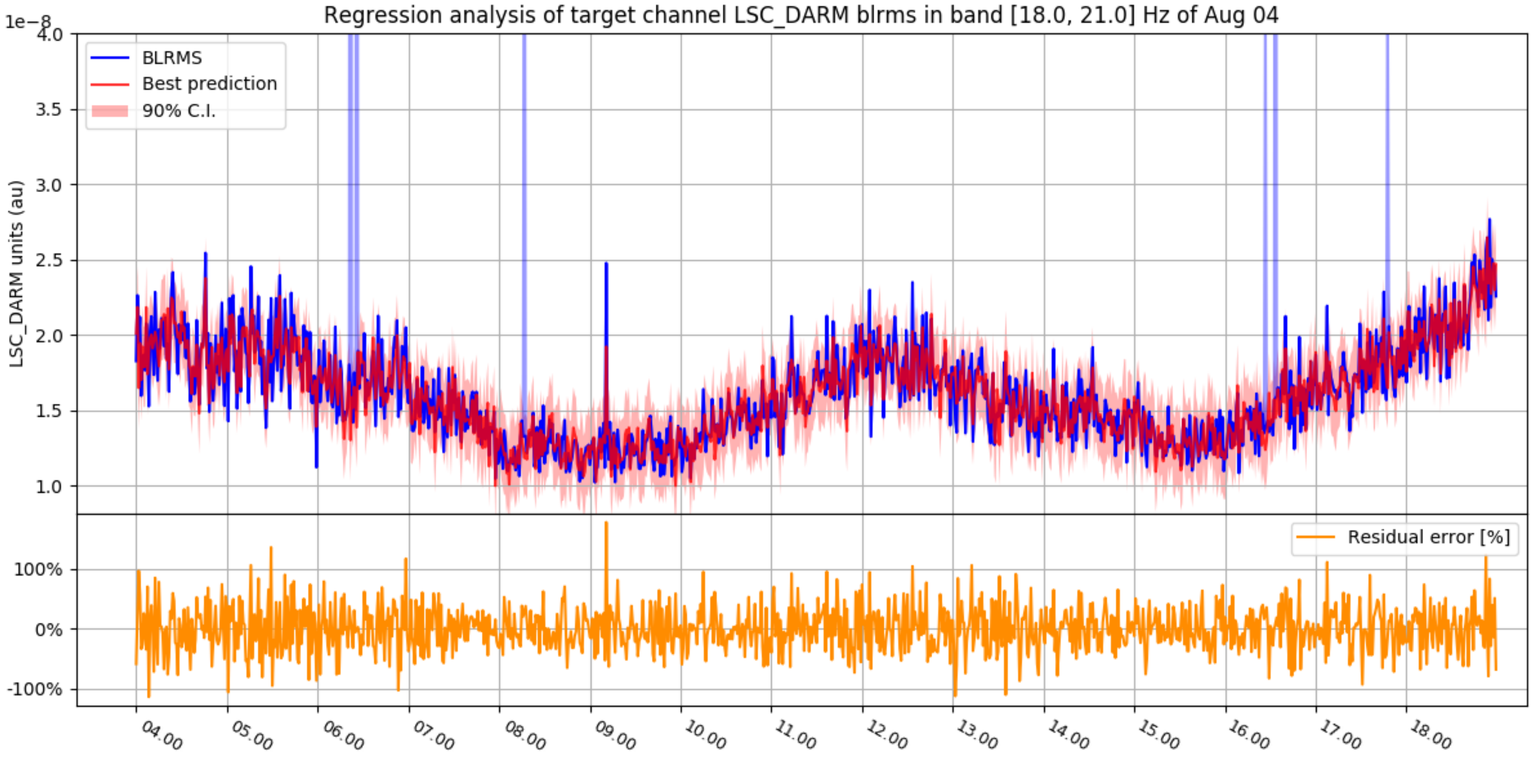
- Gaussianity
- Independence
- Engle's ARCH test

Models and predictors significances

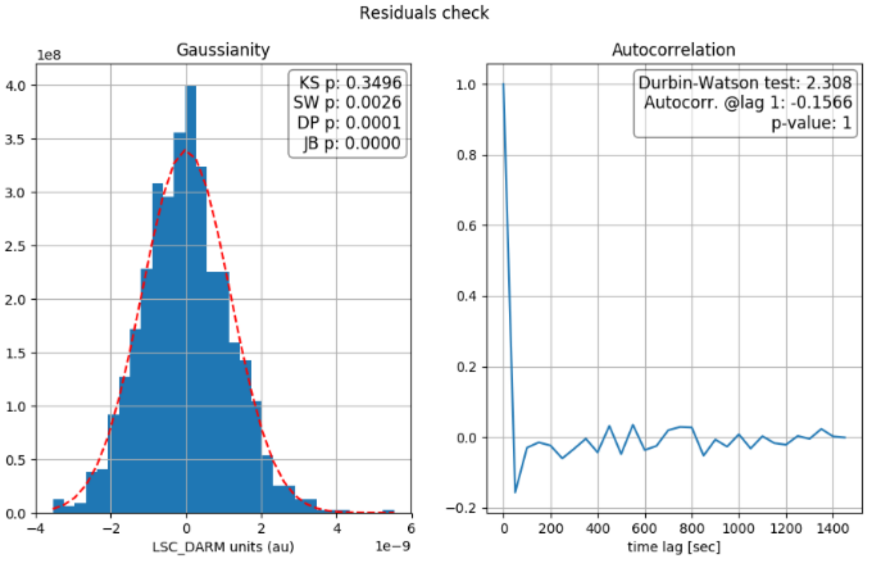
- t test on predictors
- F test on models
- Log-likelihood, AIC, BIC, etc.

Noise prediction

Regression analysis example



Refer to the spectrogram on page 8



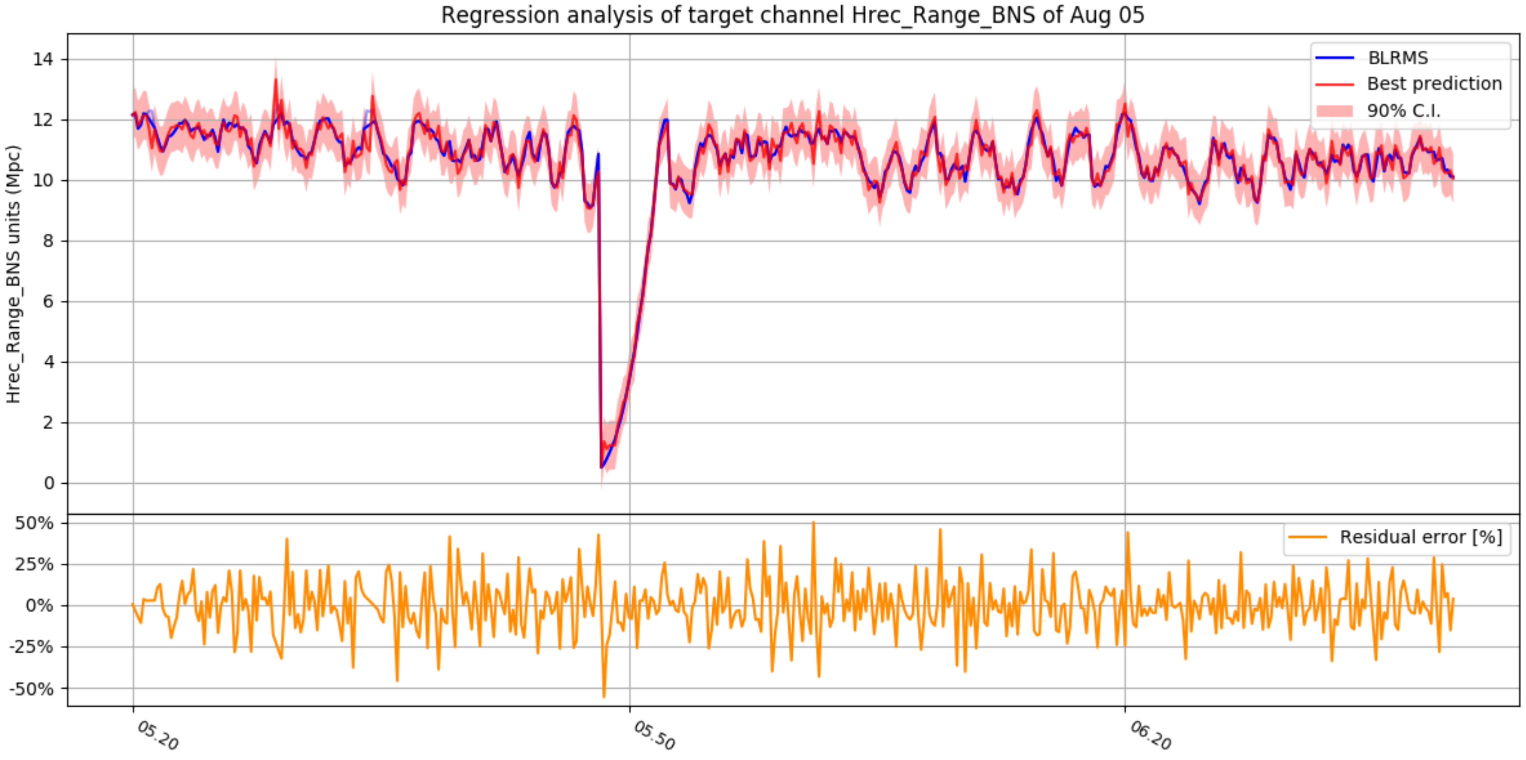
Results:

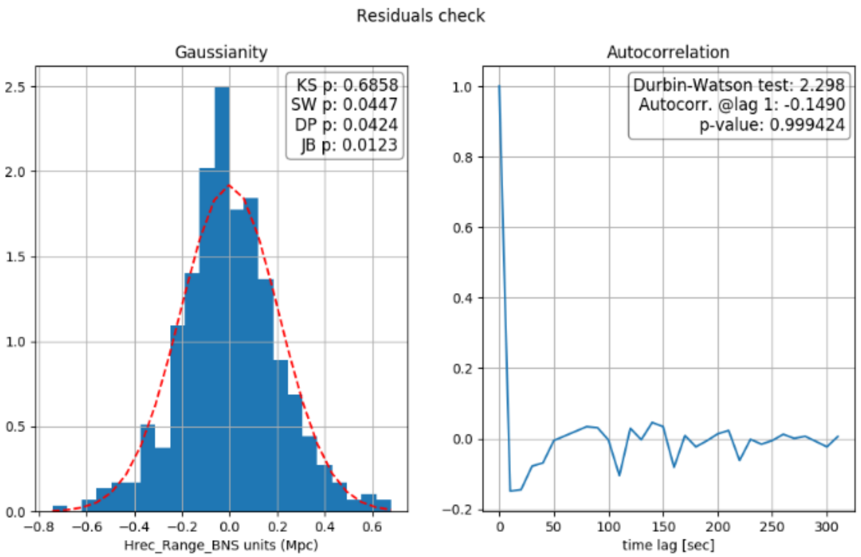
54k seconds of data, 216 model params.

$R^2_{\text{adj}} \simeq 72\%$.

Many channels related with the pre-stabilized laser and the injection subsystem.

Regression analysis example (II)



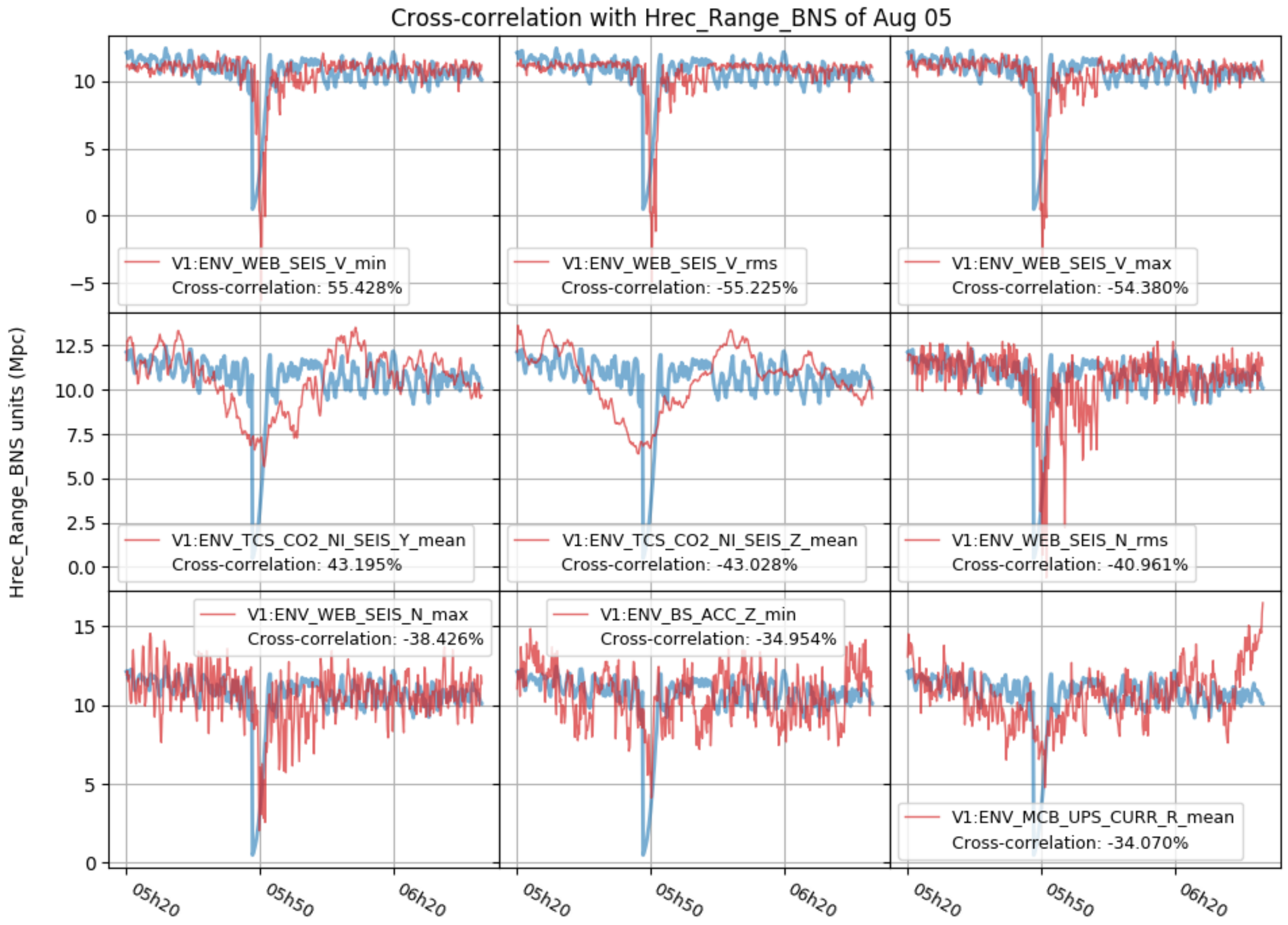


Results:
5k seconds of data, 216 model params.
$R^2_{\text{adj}} \simeq 91\%$.
Seismometers and magnetometers in the West End Building.

Cross-correlation analysis example

Cross-correlation analysis of the previous **BNS range drop**.

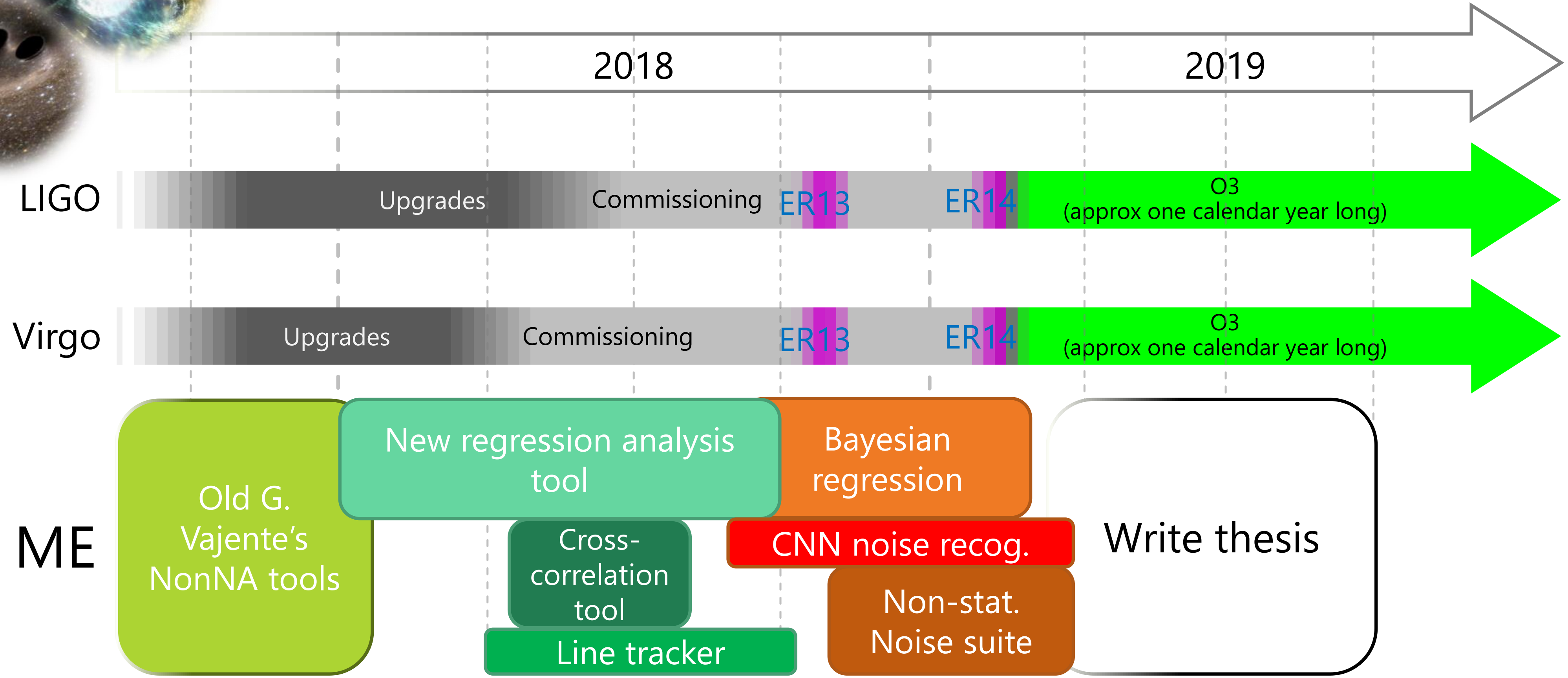
Plots of the **9 most correlated auxiliary channels** with the BNS range channel; again, seismometer sensors in the West End building.



Conclusions and perspectives

- Two tools (plus the line tracker) have been developed for studying slowly varying, non-stationary noise.
- These tools are meant to help commissioning activities involving noise hunting and characterization, and to provide a "more stationary and Gaussian" detector for the next science run (O3, February 2019).
- They have proven to be fast and reliable aids during the last commissioning runs (C9 and C10).
- Improve better modelling for target prediction and noise subtraction: Bayesian regression, principal components classification.
- Automatic noise identification through convolutional neural networks (figure).
- Integration with the other existing tools: comprehensive non-stationary noise suite.

Working schedule and project plans



Joint LIGO-Virgo working schedule for O3 [LIGO-G1800889-v4]

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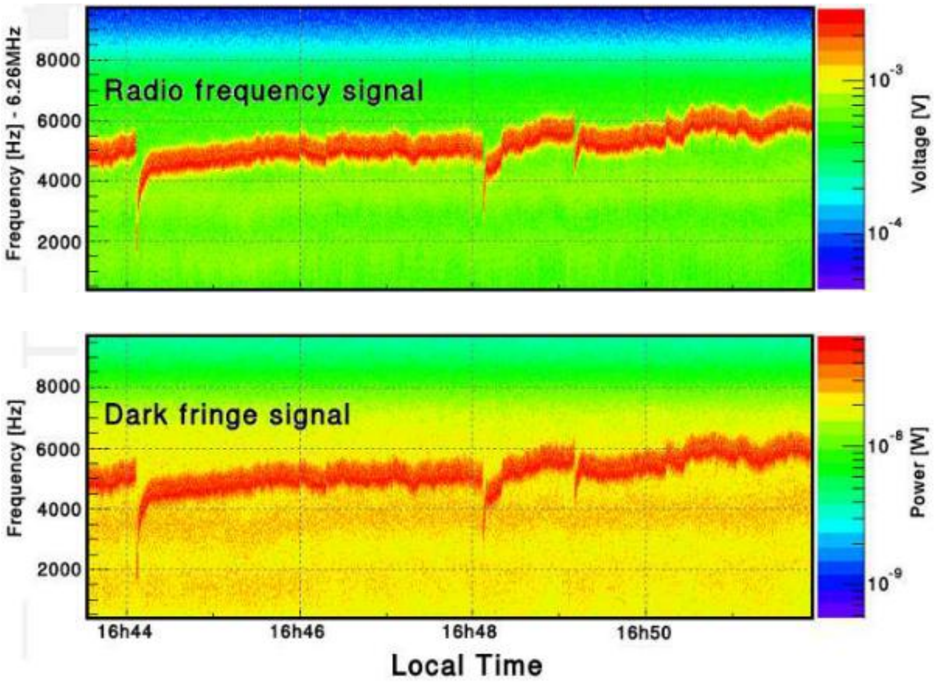
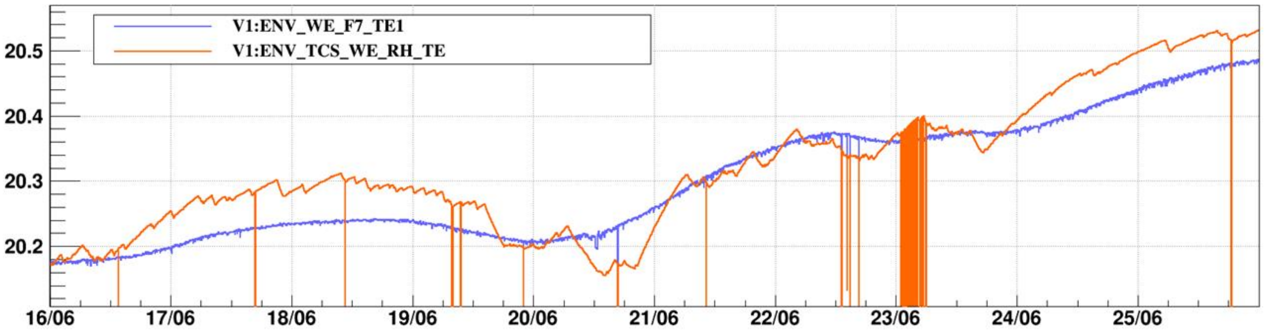
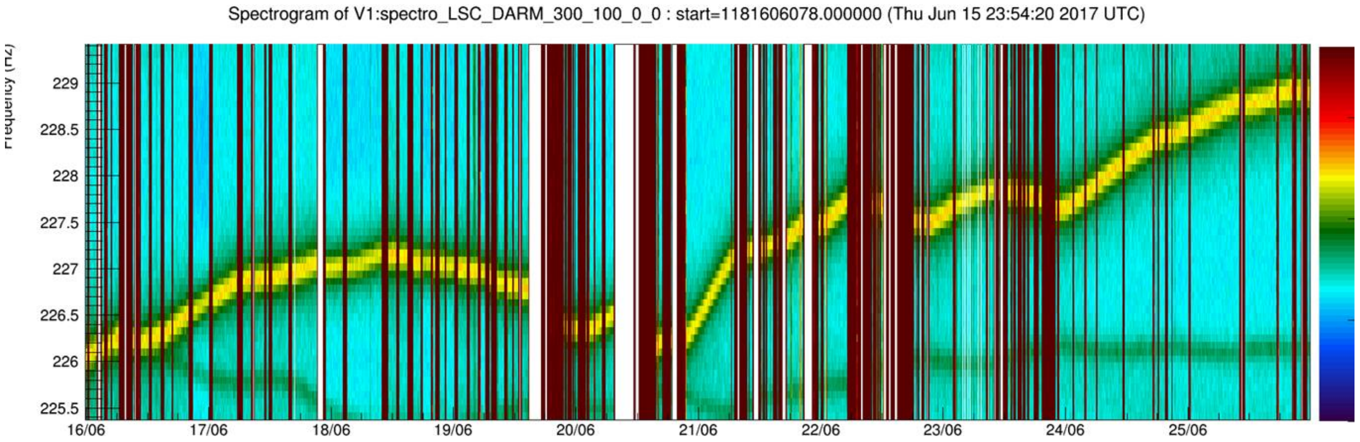
Backup slides

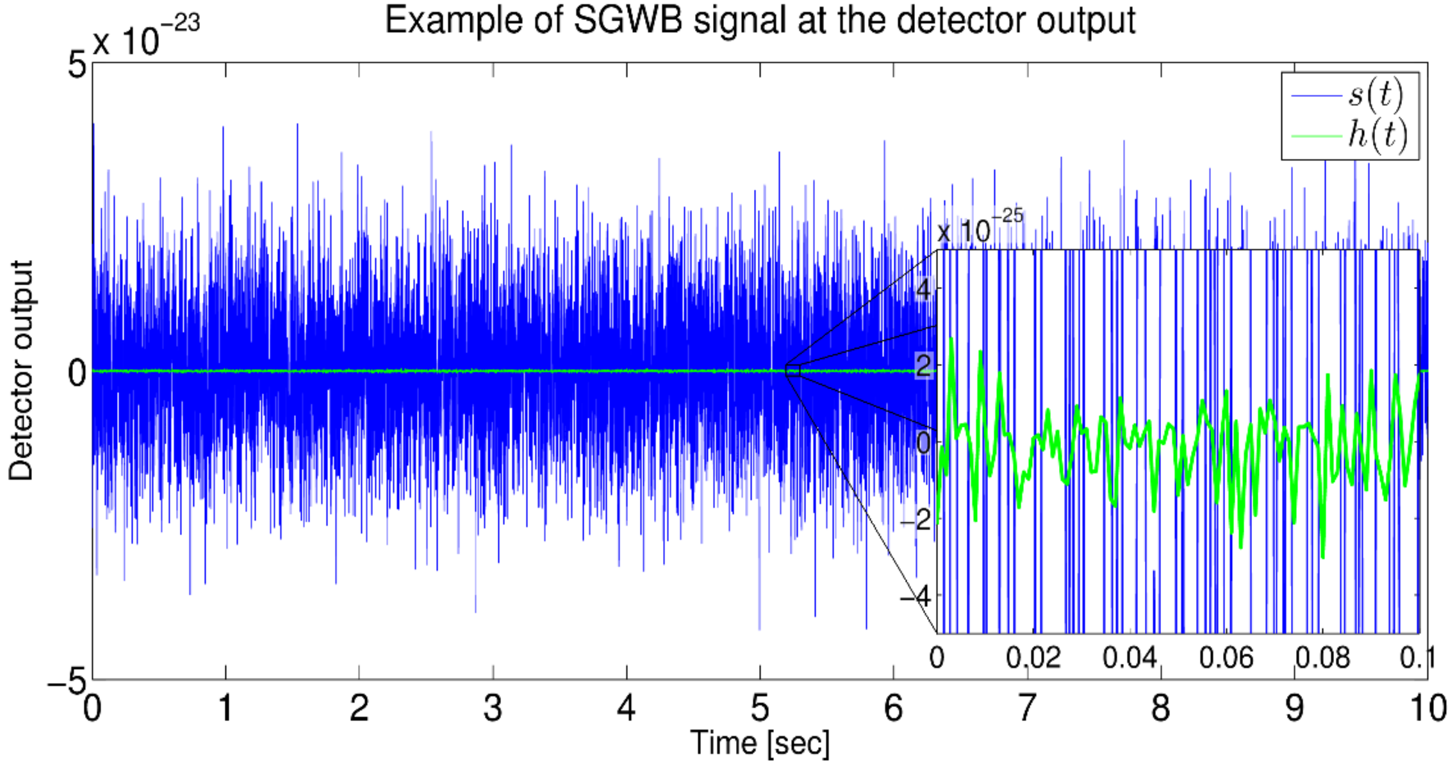
Similar patterns in different channels







A (biased) example: the Stochastic Background of GWs



A stochastic background of GW is the signal produced by the superposition of a large number of independent, unresolvable GW sources.

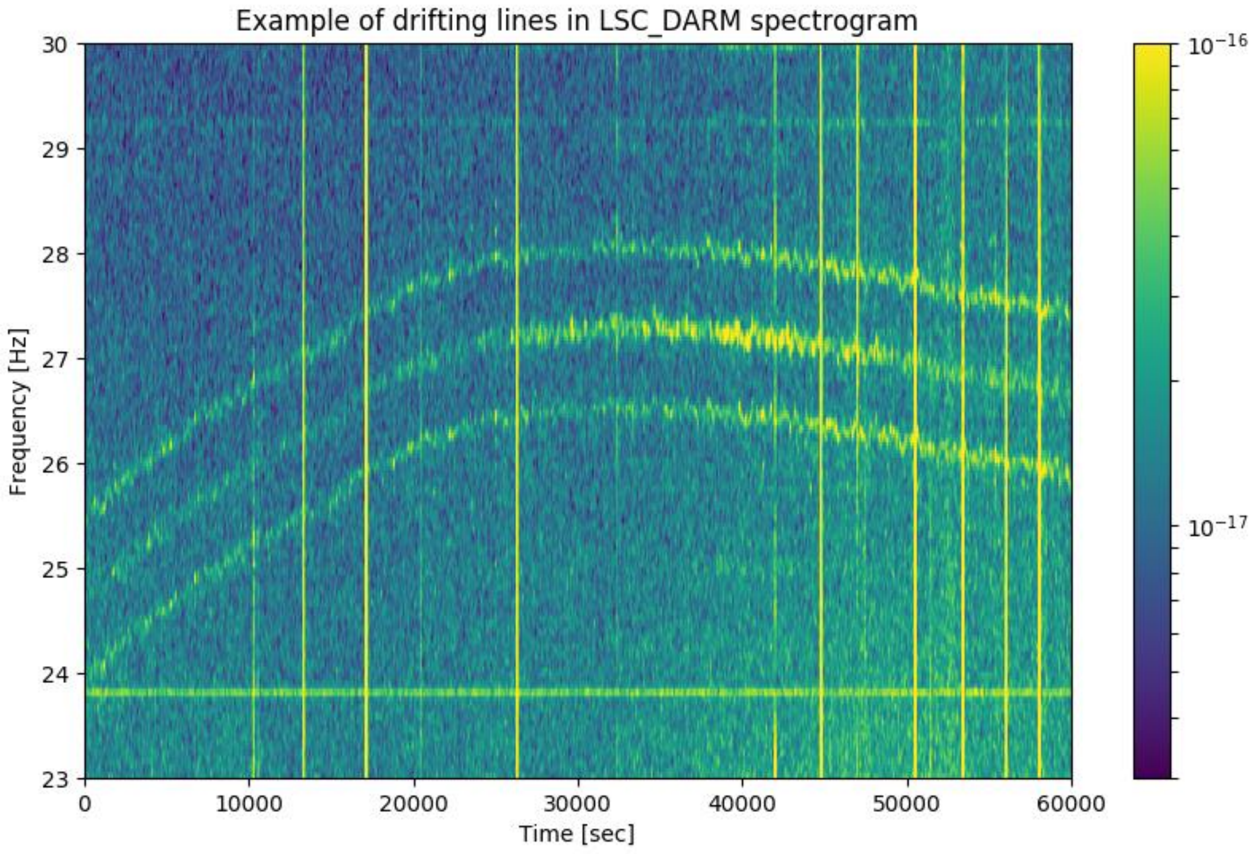
SGWB data analysis is **very demanding:**

- Huge amount of high quality data: few **notches** and **cuts**
- Low (stationary) noise in a broad frequency band

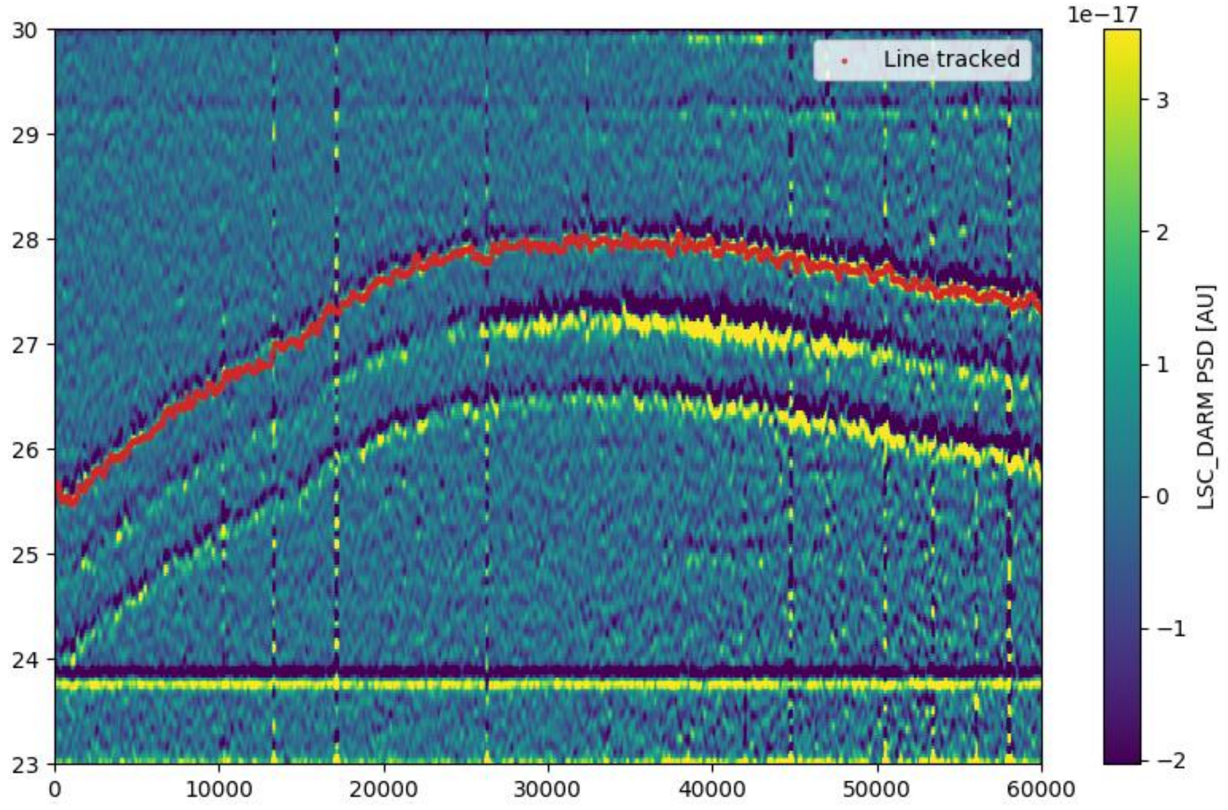
Sensitivity: [Allen]

$$\Omega_{\rm gw} \propto \frac{S_n(f)}{\sqrt{\Delta f\, T}}$$

Example: Line tracker tool



Example: Line tracker tool



Example of non-stationary noise during commission phase (June 2017)