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## Performances of ‘G-Pisa’: a middle size gyrolaser

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### Abstract

G-Pisa is an experiment dealing with a high sensitivity laser gyroscope with an area of the order of  $1\text{ m}^2$ . It aims at improving the performances of mirror suspensions of the future-generation gravitational-wave antenna Virgo. The experimental set-up consists in a He–Ne ring laser with a four-mirror square cavity. The device is operational on a stable regime, with the laser operating in both single mode or multi mode. The low-frequency sensitivity, 0.001–1 Hz, is limited by the environmental noise, but it has been checked that the requirements for the inverted pendulum tilt control given for AdVirgo are fulfilled ( $10^{-8}\text{ rad Hz}^{-1/2}$  at 30 MHz).

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(Some figures in this article are in colour only in the electronic version)

### 1. Introduction

Inside the gravitational-wave experimental community there is a large interest for inertial devices sensitive to rotations and tilts, necessary for active controls. AdVirgo plans to implement the tilt control for the inverted pendulum, which is the first stage of the superattenuator suspension, designed to support and orientate the interferometer mirrors. The sensitivity requirement of AdVirgo for the IP control is usually expressed in the power spectrum of the angle, and it is  $10^{-8}\text{ rad Hz}^{-1/2}$  above 30 MHz [1].

Laser gyroscopes are devices sensitive to inertial angular motion, based on the Sagnac effect: in a closed cavity rotating at angular velocity  $\Omega$  the two counter propagating beams complete the path at different times. The main advantage of such devices in comparison to the mechanical based system is that the coupling among different degrees of freedom is very little, since they do not have moving parts. Different kinds of such devices have been developed mainly for navigation. We distinguish between passive (fiber optic gyros) and active (ring lasers) Sagnac interferometers. The passive device measures the phase shift between the two beams, while the active one measures the frequency difference, an inherently more accurate measurement. Small fiber gyros (FOG) are typically used for navigation and so far have a resolution of  $2 \times 10^{-8} \text{ rad s}^{-1} \text{ Hz}^{-1/2}$  [2], while the large ring laser gyros used in geophysics and geodesy reach the level of  $10^{-12} \text{ rad s}^{-1}$ , and are still improving. The requirement of ADVirgo for tilt control expressed in terms of the angular velocity is  $2 \times 10^{-9} \text{ rad s}^{-1} \text{ Hz}^{-1/2}$ , a factor of 10 lower than available FOG devices, which could be likely improved in the near future.

In the following we will focus on an active ring laser (gyro). One application of large gyros is the monitoring of the variations of the Earth angular velocity vector. The orientation with respect to the Earth axis is important since the induced signal is proportional to the scalar product between the normal of the gyro area and the Earth axis, see equation (1). For horizontal gyros the signal is zero at the equator and maximum at the pole; at intermediate latitudes, both horizontal and vertical cavities work fine. Up to now the reached resolution is  $10^{-8}$  of the Earth rotation rate [7–11]. The Sagnac frequency, i.e. the beat signal between the two output beams, is

$$\delta\phi = 4A\mathbf{n} \cdot \boldsymbol{\Omega}/(\lambda P) + \phi_\rho,$$

where  $A$  and  $P$  are the area and the perimeter of the cavity respectively,  $\lambda$  is the wavelength of the laser beam,  $\mathbf{n}$  is the normal vector of the plane of the ring cavity and  $\boldsymbol{\Omega}$  is the induced vector of rotation.  $\phi_\rho$  denotes the additional, usually very small, contributions to the Sagnac frequency due to non-reciprocal effects in the laser cavity, such as Fresnel drag [12] and shot noise. A laser gyro can monitor with high accuracy the orientation of the laboratory reference frame. The sensitivity of G (located in the Laser Ranging Station of Wettzell in Germany) is getting closer and closer to VLBI sensitivity providing an independent measurement for the comprehension of the Earth reference frame; in short we can say that at the moment geodesy is the most successful application of large gyrolaser, and it is becoming a fundamental instrument for geophysics [4] as well. But it can be used as well to improve the performances of future gravitational wave antennas [3, 5, 6, 13]. Application for gravitational-wave antenna detectors needs dedicated apparatus; the emphasis is more on a small device, with a very high duty cycle, working in an environment where large resonances due to the proper frequencies of the metallic structure are present, which in principle could spoil the sensitivity through up-conversion produced by the nonlinear dynamic of the laser. The large ring laser gyros built by the Joint Ring Laser Working Group in New Zealand and Germany have sides of the order of several meters. ‘G’ [14, 15] located at the Geodetic Observatory in Wettzell (Germany) is a monolithic square with 4 m sides; ‘UG2’, a rectangle with 40 m by 20 m sides, is the largest ring [16]. In principle the larger the area the better the sensitivity, but the best gyro, so far, is ‘G’, which is operating very close to the quantum limit, and its measured noise power spectrum is of the order of  $10^{-12} \text{ rad s}^{-1} \text{ Hz}^{-1/2}$ , with a duty cycle close to 100%. The Allan deviation has not shown any systematic effects on a time scale of about 3 h, and the sensor drift is normally below 1.5 parts in  $10^8$  of the Earth rotation ( $1.1 \times 10^{-12} \text{ rad s}^{-1}$ ). Among the different instruments developed by the German NewZealand group, there is a very compact and flexible mechanical design, called GEOSENSOR, expressly designed to

be transportable. Existing GEOSENSORS are used mainly for geophysics, and have sides of 2 m, which is considered too large to be implemented inside a suspension. G-Pisa that is based on the GEOSENSOR design has been built by a INFN, University and CNISM of Pisa and Naples collaboration, financed by INFN commission V in order to learn the technique for future applications in Virgo and for general relativity tests. For G-Pisa, the GEOSENSOR design has been changed in order to be scalable down to 0.9 m; further scaling of the device is in principle feasible, but a new mechanical design is necessary. In the following section the limits in sensitivity due to shot noise are discussed. The third section describes the experimental apparatus. The fourth section discusses the results on sensitivity to be compared with the AdVirgo requirements. At the end we conclude, with considerations about our near future plan.

## 2. Sensitivity limits

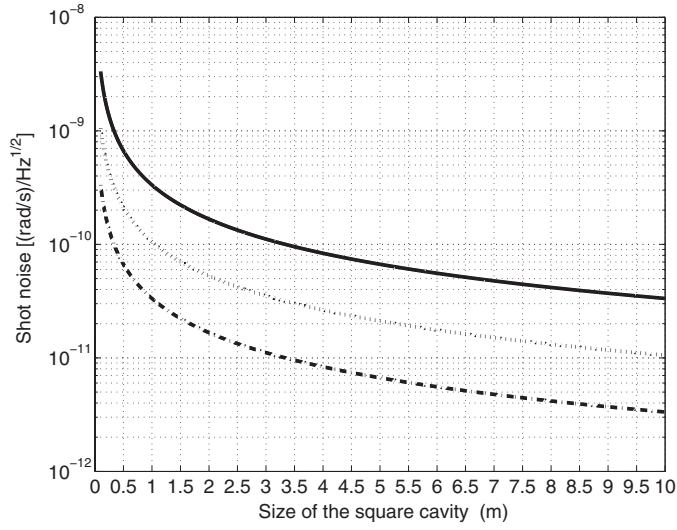
In large gyros the shot noise of light gives the fundamental limit to sensitivity:

$$\Omega_{sn} = \frac{c}{2\pi K L} \sqrt{h\nu\mu \frac{T}{2Pt}}$$

where  $c$  is the speed of light,  $L$  is the side of the square ring,  $h$  is the Plank Constant,  $\nu$  is the frequency of the light,  $\mu$  is the total cavity losses,  $T$  is the transmission of each mirror,  $P$  is the total radiation power exiting from the cavity,  $t$  is the observation time and  $K$  is the scale factor of the instrument, which depends on the geometry and the wavelength, whose value in a square cavity at a wavelength of 632.8 nm is  $1.58 \times 10^6 \times L$ . In figure 1 the shot noise limit is shown for various values of the parameters  $\mu$ ,  $T$  and  $P$  and an observation time of 1 s.

For the continuous line (good mirrors, but not the best ones)  $\mu = 40$  ppm,  $T = 0.2$  ppm and  $P = 10^{-8}$  W, for the dotted line absorption has been reduced to 4 ppm and for the point-dotted line the transmitted power has also been increased to  $10^{-7}$  W (top quality mirrors). It should be noted that with equivalent mirror quality the laser cavity losses increase by decreasing its length; as a consequence for very small rings it becomes more difficult to obtain a reasonably high output power. For a ring side of the order of 1 m or lower, it would be unlikely to have an output power of the order of  $10^{-8}$  W, independently from the mirror quality. In short, a device with  $L$  below 1 m could reasonably reach a sensitivity of  $10^{-9}$  rad s $^{-1}$  Hz $^{-1/2}$ , while to reach  $10^{-11}$  rad s $^{-1}$  Hz $^{-1/2}$  devices larger than 2 m are required. Backscattering-induced frequency pulling and lock-in pose severe limitation to the ring laser performances. It is usually avoided by introducing a bias, which separates the wavelengths of the two counter-propagating laser modes. The Earth rotation can be used in order to give the necessary bias. The parameter which sets the magnitude of the mode pulling is the lock-in threshold frequency  $l$ , where  $l \approx c s \lambda / (\pi d P)$ , where  $c$  is the speed of light,  $s$  is the fraction of the laser field amplitude which is scattered by each mirror,  $\lambda$  is the laser wavelength,  $d$  is the beam waist and  $P$  is the ring perimeter.

In figure 2 the dashed lines show the lock-in limit as a function of the side of a square ring, for a beam waist of 0.5 mm and different values of the scattering coefficient  $s$ . The continuous line shows the Sagnac frequency given by the Earth rotation for a device horizontally located at latitude  $43^\circ$  as a function of the ring size. In general the lock-in problem, figure 2, poses more severe limits at the size of a gyrolaser with the requirements given for AdVirgo, and a device with the side at least 1 m is necessary, at least until the mirror losses are not substantially decreased. A larger perimeter ring has several advantages: the larger cavity provides a more powerful laser, since the Earth bias depends on the relative alignment of the ring with respect to the Earth rotational axis, a larger ring allows higher

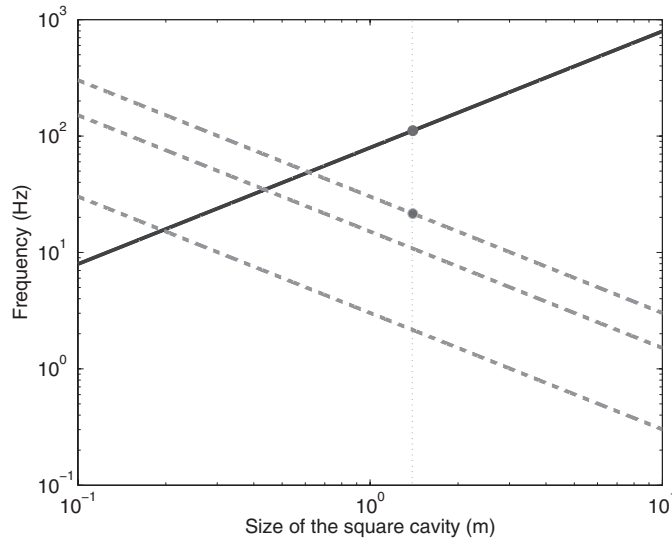


**Figure 1.** Shot noise limit as a function of the square cavity side. The three different traces correspond to sets of mirrors of different quality. Continuous line absorption  $\mu = 40$  ppm, transmission  $T = 0.2$  ppm and power  $P = 10^{-8}$  W, dotted line  $\mu = 4$  ppm, point-dotted line  $T = 10^{-7}$ .

flexibility, i.e. there is a larger choice of how and where to locate the instrument, a larger volume of gas provides a long-term stability of the gas itself. For this reason the 1 m side seems a reliable choice, while the 0.5 m side seems risky. The sensitivity requirement for our application is not as high as that of the top quality gyrolaser, but, in terms of the sensitivity and bandwidth response, it is not in the range of ordinary commercially available inertial sensors. As it has been said in the introduction that G-Pisa has been financed in order to learn the technique and study possible applications, it has not been so far part of the AdVirgo program; implementation of this instrument for AdVirgo is a purely an engineering problem: given the available space and the relative orientation with the Earth axis it is possible to provide a mechanical design. In order to fit inside the available space such a ring should be rectangular rather than squared, the real parameter being the perimeter, but the square device is the optimal one, since in this case the ratio *perimeter/area* is minimum, and the device is more symmetric in general.

### 3. Experimental set-up

G-Pisa is a square cavity 5.60 m in perimeter and 1.96 m<sup>2</sup> in area. The mechanical design is flexible and each side of the square can be scaled from 1.40 m down to 0.90 m with minor changes. The mechanical system is mounted onto an optical table and has a stainless steel modular structure: four boxes, located at the corner of the square containing the mirror holders inside, are connected by tubes, so as to form a ring vacuum chamber with a total volume of about  $5 \times 10^{-3}$  m<sup>3</sup>. The vacuum chamber is entirely filled with a mixture of He and a 50% isotopic mixture of <sup>20</sup>Ne and <sup>22</sup>Ne. The total pressure of the gas mixture is set to 560 Pa with a partial pressure of Neon of 20 Pa. In the center of one of the tubes there is a Pyrex insertion, a capillary with a 4 mm internal diameter, approximately 15 cm long, which is the discharge tube. The capillary internal diameter has been chosen in order to favor selectively *TEM00*

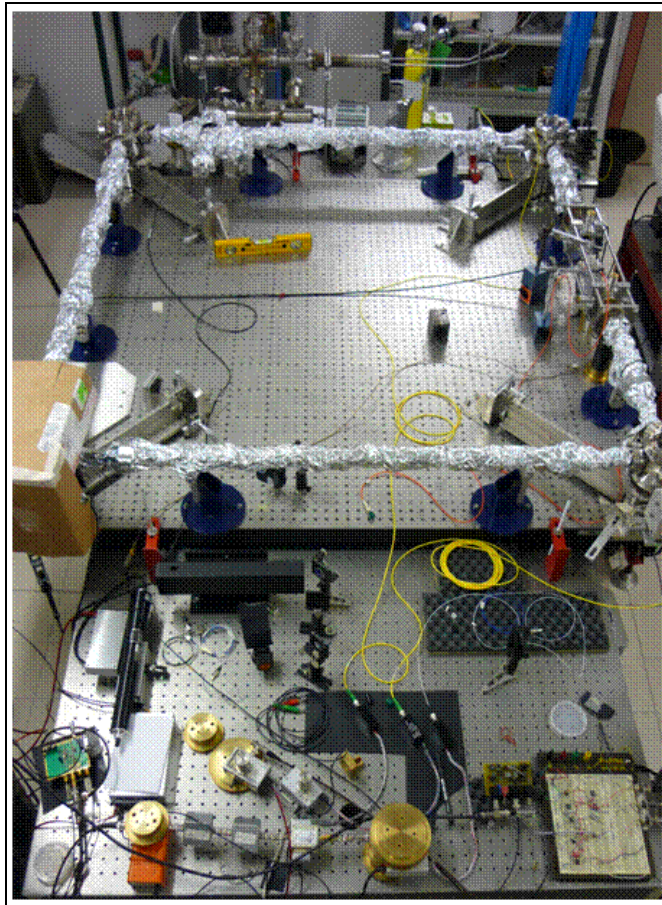


**Figure 2.** Continuous line shows the Sagnac frequency, the three dashed lines show the lock-in threshold frequency for three different sets of mirrors (from bottom to top:  $s = 10^{-4}$ ,  $5 \times 10^{-4}$  and  $10^{-3}$ ). Highlighted intersections represent the G-Pisa case,  $s = 10^{-3}$ .

cavity mode excitation. While the discharge in the laser medium of ‘G’ and analog systems is excited by a RF source, where two coils couple the RF oscillator to the gas, in our system we choose a capacitive coupling. Two halves of a copper cylinder are used as electrodes and are part of the resonant circuit of the RF source. The laser medium in this way is included in the active circuit and the fluctuations in the plasma density do not affect the coupling of the RF source to the discharge, but only the oscillator frequency, about 115 MHz. This kind of discharge provides a very good passive stability and makes it possible to regulate the laser output power very close to the laser threshold since the first runs. The discharge is 5 cm long, but different lengths will be tested in the next months to minimize the effect of plasma intensity fluctuations around the laser threshold. The discharge tube has four micro-metric screws used to align the Pyrex capillary with the mirrors and the optical cavity.

Four spherical mirrors with a 6 m radius of curvature were chosen for the resonator, and two micro-metric lever arms acting on the tilts of each mirror make the fine tuning of the cavity alignment possible. Mirrors reflectivity is optimized for the emission line around 632.8 nm. The free spectral range of the cavity is 53.6 MHz, the horizontal beam waist is 0.68 mm, the sagittal beam waist is 0.56 mm and the intra-cavity ring-down time is 20  $\mu$ s. In order to achieve long-term stability of the perimeter the laser gyro optical frequency will be locked to a reference laser. This will involve the measurement of the radio frequency beat note between the gyrolaser output and the reference laser. For such an application the capacitive coupling will introduce less noise than the inductive one. The two outputs (clockwise and anticlockwise sense of circulation) from one cavity mirror are combined by means of a 50% intensity beam splitter and detected by a photodiode. The photodiode output is amplified with a trans-impedance stage with a gain of  $10^9$   $v/A$  and 1 ms rise time. The two single-beam outputs are also monitored by means of two fiber-coupled photomultipliers. The laser modal structure has been detected injecting the output beams in a high finesse linear cavity. The signals are acquired and analyzed off-line. Figure 3 shows a picture of the apparatus.

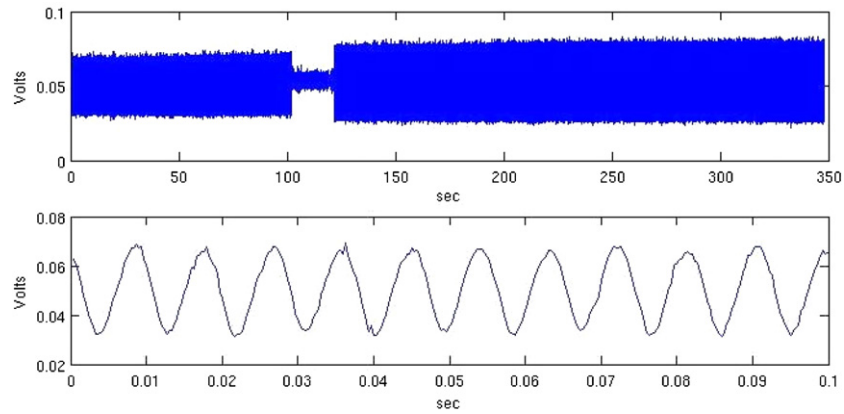




**Figure 3.** The G-Pisa apparatus is on top of a standard breadboard, optical fibers are used to handle signals and the laser used as reference to actively control the perimeter is visible.

#### 4. Behavior of the gyrolaser signal and sensitivity measurements

In the present set-up our instrument is basically free running; this means that because of thermal expansion it exhibits different regimes: normal single-mode operation, split mode and chaotic regime. For thermal expansion the perimeter changes and the laser sometimes has to change the number of wavelengths contained in the perimeter (mode jumps). Sometimes the two counter propagating modes are separated by a free spectral range, which is 53.6 MHz for the present set-up of G-Pisa. In this regime (split mode) the instrument could be useful as well, but requires a much different analysis and data acquisition since the beat note is as high as the free spectral range plus or minus the Sagnac frequency [16], and the clockwise or anticlockwise mode can be split. By increasing the discharge voltage it is possible to operate the laser in single mode or multi-mode. A stable operation has been observed up to four longitudinal modes. A detailed description of the behavior of the apparatus is given in [17, 18]. Mode jumps and splits can be easily avoided with thermal stabilization and with the perimeter active control, which we plan to implement in the near future. In the following the focus will be on the normal operation and relative sensitivity. G-Pisa is located in a

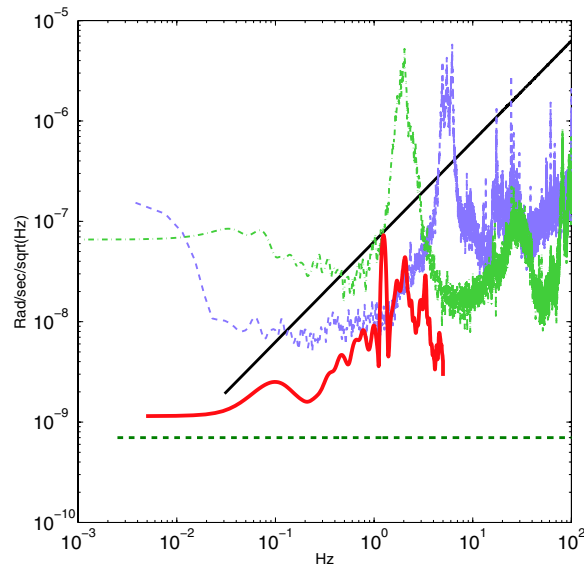


**Figure 4.** Standard operation Sagnac signal. After approximately 100 s a mode jump of one of the two laser modes takes place and for about 20 s the instrument is blind. In the bottom figure, 0.1 s time span is shown and the beat note is well visible.

very noisy environment without thermal control. Typically it works in normal operation for 10–20 min before a mode jump takes place, and from time to time we observe a split mode, which persists usually for some minutes, until both beams return to the same longitudinal mode and then the standard operation and Sagnac signal are recovered. Figure 4 shows a typical raw signal, approximately 6 min.

The rotational sensitivity is obtained by reconstructing the phase of the beat note, and differentiating it, in order to obtain the angular velocity. The mean value of this signal gives Earth rotational speed, and it has been checked that this mean value is compatible with the Earth rotation and the latitude in Pisa. To evaluate the sensitivity the mean value is usually subtracted. The power spectrum of the signal makes the estimate of sensitivity, which is the relevant parameter for the possible improvements of the gravitational waves interferometer suspension; since G-Pisa especially at low frequency is limited by environmental noise it has to be considered an upper limit, and it is natural to take the best measurement taken in a quieter environment. Typical spectra are reported in figure 5. The horizontal line is the expected shot noise limit, and the transverse line shows the requirement for the Virgo suspensions [1], expressed as angular velocity. Three different measurements are shown, taken at different times and under different conditions. The blue line is one of our first measurements, and the large structure around 7 Hz is due to the resonance of the legs of the optical table supporting the ring laser. After the first set of measurements the optical table legs have been fixed to the floor and the table as well has been seismically isolated with a set of pneumatic dampers. The green curve shows a typical spectrum, taken during the day, when there is a lot of activity around the laboratory right inside the Department of Physics of the University of Pisa, which is located in the town center. This spectrum shows a very large peak around 2 Hz which has been proved to be due to a horizontal rotation resonance of the table itself. The level of this peak depends of course on the environmental activity. We have carried at a very long, unattended, measurement all the night long, with a low acquisition rate; from this long run the pieces in which the system was optimally working have been extracted, analyzed, and among them the spectrum with the lowest profile below 1 Hz is shown in the figure in red. It corresponds to 2 o'clock in the morning and shows that the measured sensitivity of G-Pisa is even better than the AdVirgo requirements [1]. We note that the set of mirrors used for these preliminary tests were not ‘top quality’ mirrors (from ring-down measurement the mean loss of these mirrors





**Figure 5.** Comparison between the best low-frequency measurements. The dashed curves refer respectively to the case of the rigid and the passively air-damped optical table. The continuous curve shows the best measurement obtained at night, in a quiet environment. The horizontal dotted line is the expected G-Pisa shot noise, the transverse line shows the AdVirgo requirement for IP tilt control.

has been evaluated to be of the order of  $4 \times 10^{-5}$ ). A set of top quality mirrors will be installed in the near future; accordingly improvement of the performances of G-Pisa is expected. It is important to remind that the sensitivity measured with the 4 m side gyrolaser G, located in Wettzel (Germany), is more than a factor 1000 better than the measurements shown in figure 5. By means of a simultaneous measurement of rotational noise, performed with the G-Pisa gyrolaser, and translational noise obtained by a three axial linear accelerometer, it was possible to set an upper limit to the correlation between rotations and translations to a level of 0.5% [6]. The perimeter control has been successfully closed by moving one of the four mirrors using a piezo, showing that controlling the perimeter G-Pisa runs continuously in the standard operation, mode jumps and splits have not been observed so far. Acting on a single mirror, the relative phase between the backward scattered radiation from the different mirrors is affected in different ways. Because the coupling between the two counter propagating beams is given by the interferometric sum of the single backscattering sources, the lock-in frequency can be strongly affected by this correction [18].

## 5. Conclusions

The gyrolaser G-Pisa has been operational for one year. Interruptions of the normal operation are due to thermal expansion induced mode jumps and split mode regimes. The preliminary active perimeter control of G-Pisa has shown that the ring laser can run continuously, without interruptions.

The low-frequency measured power spectrum (below 1 Hz) is typically around  $10^{-8}$  ( $\text{rad s}^{-1}$ )  $\text{Hz}^{-1/2}$ , which looks like a real motion. In a very quiet environment sensitivity measurements down to  $10^{-9}$  ( $\text{rad s}^{-1}$ )  $\text{Hz}^{-1/2}$  have been obtained, which is much better than

the limit needed to control the tilts of the Virgo suspensions, planned for AdVirgo. A device with perimeter 4 m is adequate for this purpose, a design with a rectangular shape rather than square is feasible in order to fit inside the available space. A gyrolaser is a device more sensitive than the present requirements of AdVirgo. It cannot be excluded that in the very near future a relatively small and compact fiber baser Sagnac gyro (FOG) can fulfill the requirements.

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