Abelian Vortices and Junctions in the Large Magnetic Flux Limit

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Mini-workshop:
Supersymmetry, Supergravity and Superstrings
Work done in collaboration with
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Motivation

- Field theoretic analog of D-brane physics?

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- Confinement in a $SU(N)$, can it be understood by studying non-Abelian vortices?
- Is there some limit that simplifies the problem/structure of the solitons?

Characteristics of Solitons

- Vortices: map:
  $f : \mathcal{S}^1$ (spatial infinity) $\mapsto \mathcal{S}^1$(vacuum manifold)
  - Characterization: $\pi_1(\mathcal{S}^1) = \mathbb{Z} \;\Rightarrow$ winding number: n

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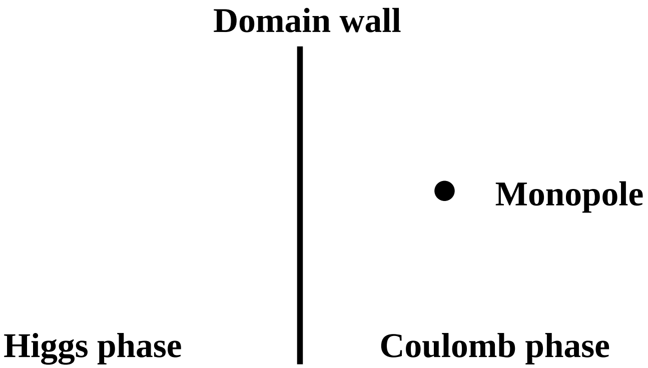
- . . . it turns out also to be the case for solitons.

Setup

Domain wall

Higgs phase | **Coulomb phase**

Setup



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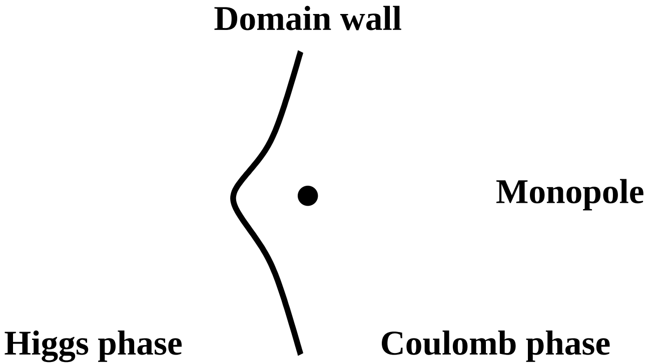

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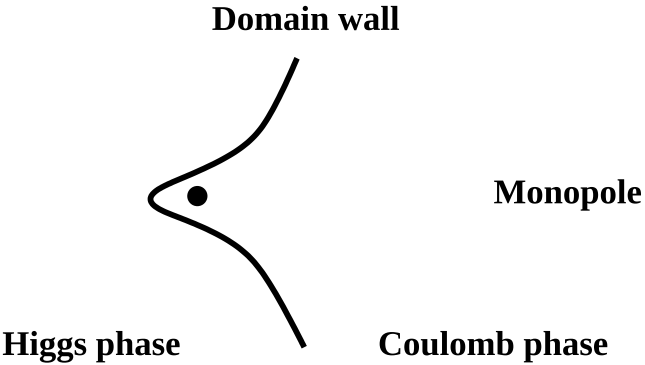

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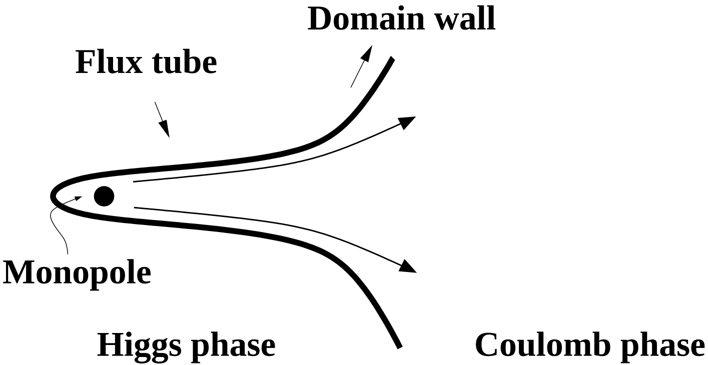
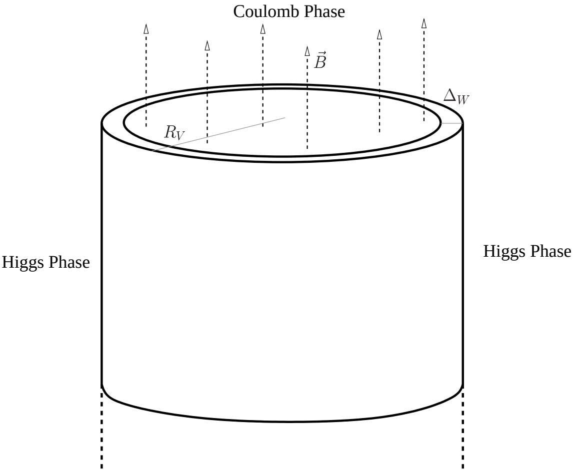


Wrapped-up Wall



Taken from [hep-th/0507273] by Stefano Bolognesi

The ANO Vortex

- The simplest example we can think of is a $U(1)$ gauge theory

$$\mathcal{L} = -\frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} - |(\partial_\mu - iA_\mu) q|^2 - V(|q|) \ , \qquad (1)$$

So this we will consider throughout this talk.

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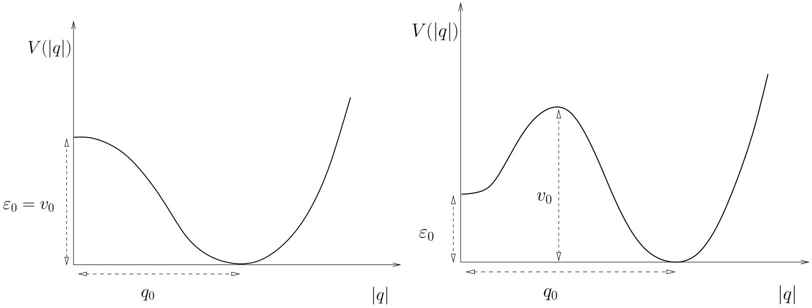
- But these wall vortices will also appear in non-Abelian gauge theories, specifically every time a domain wall interpolates between two vacua with different confinement indices.

Potential



We will see, that independent of the potential, only by given ε_0 and q_0, we can find the solution in the large-n limit.

Tension of the Vortex

$$T(R) = \frac{2\pi n^2}{e^2 R^2} + T_W 2\pi R + \varepsilon_0 \pi R^2 \ . \qquad (2)$$

Two regimes arise by increasing n:

- **SLAC Bag Regime**: $R_V \gg \Delta_W$ *and* the surface term dominates (only for some intermediate n)
  - $T_{\mathrm{SLAC}} \propto n^{2/3}$ and $R_{\mathrm{SLAC}} \propto n^{2/3}$

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- **MIT Bag Regime**: $R_V \gg \Delta_W$ *and* the volume term dominates (always wins)
  - $T_{\mathrm{MIT}} \propto n$ and $R_{\mathrm{MIT}} \propto \sqrt{n}$

Taking the Large-n Limit

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- For the large magnetic flux-limit (large-n), it will prove convenient to scale the radius, such that R_V remains constant.

Conjecture

Consider the Abelian Higgs model with a general potential that has a true vacuum at $|q| = q_0 \neq 0$ *and a Coulomb phase with energy density* $V(0) = \varepsilon_0 \neq 0$*. Call* $T_V(n)$ *the tension of the vortex with* n *units of magnetic flux. The conjecture is that*

$$\lim_{n\to\infty} T_V(n) = T_{\mathrm{MIT}}(n)\ . \tag{3}$$

Non-trivial check

- For the BPS potential,

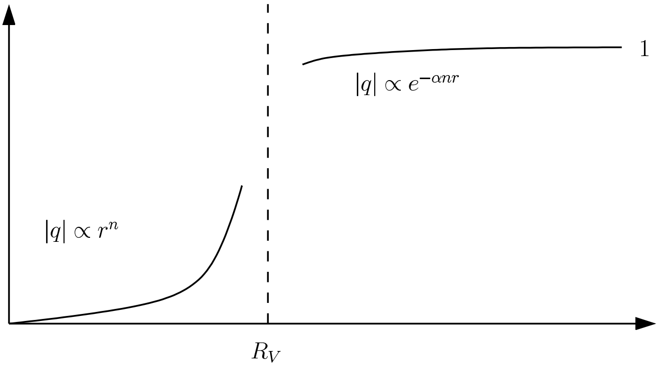
$$V(|q|) = \frac{e^2}{2}\left(|\phi|^2 - \xi\right)^2 \ , \tag{4}$$

we know already the tension:

$$T_{\rm BPS} = 2\pi n \xi \ , \tag{5}$$

which is exactly what we obtain, using our large-n formula. This could hardly be just a coincidence.

Indications of the limiting shape



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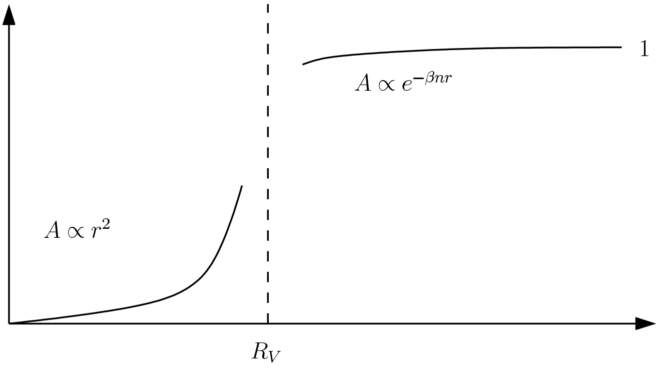
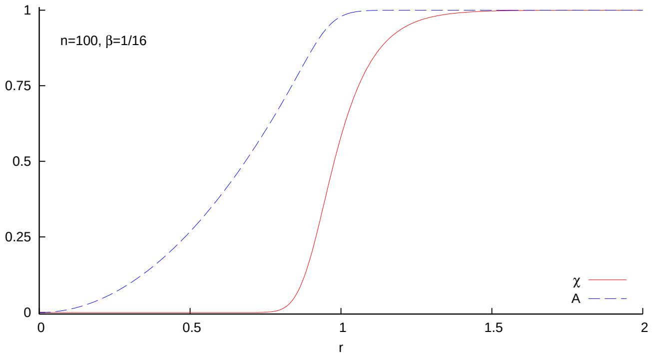
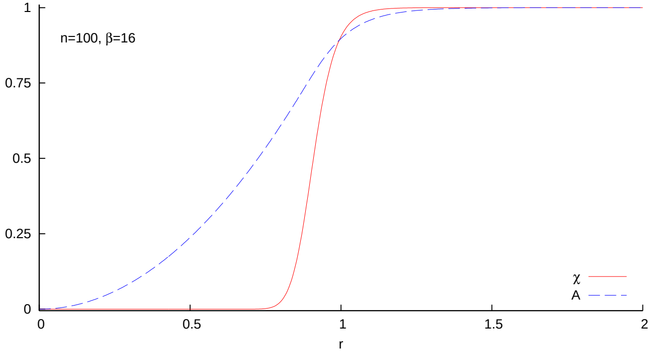


Numerical solution, Type I, $n = 100$



Numerical solution, Type II, $n = 100$



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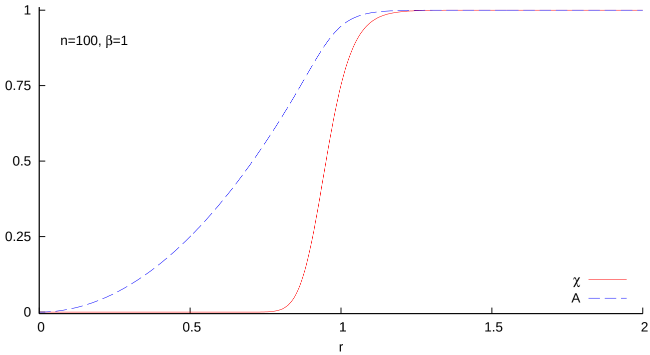
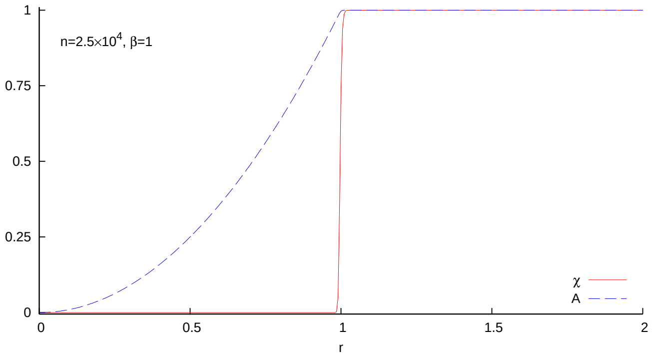
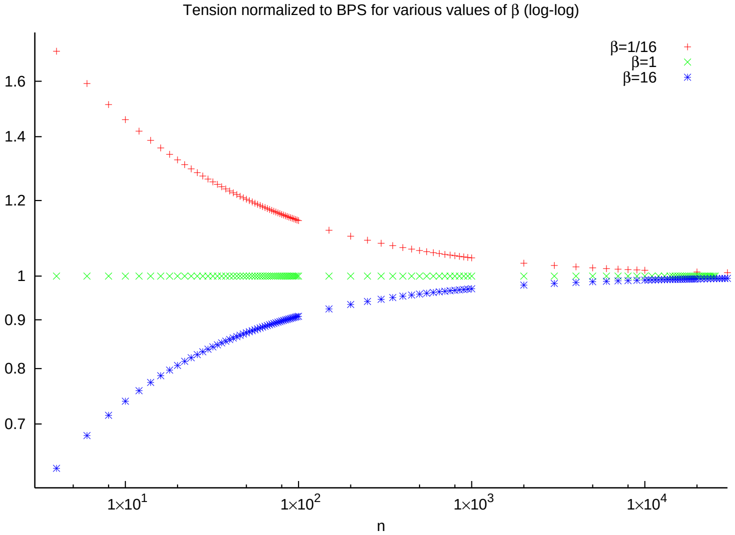


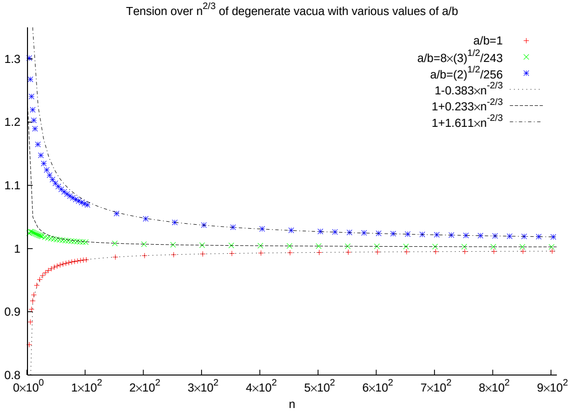
Numerical solution, BPS, $n = 2.5 \times 10^4$



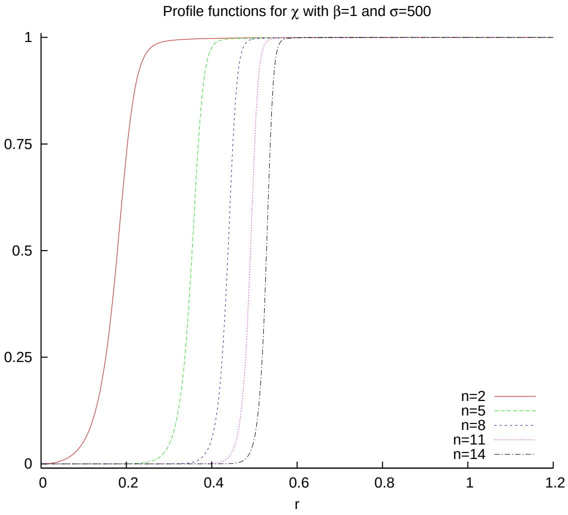
Tension for various n



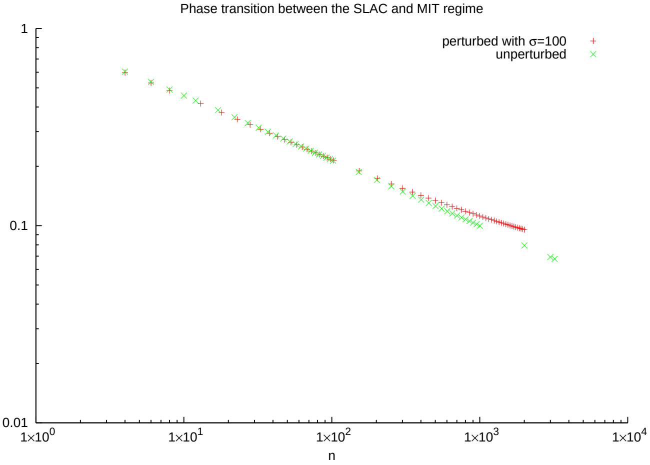
Investigation of the SLAC-bag regime



Investigation of the SLAG-bag regime



Investigation of the SLAC-bag regime/Search for a phase transition



So far only an indication.

Junctions in the Large-n Limit

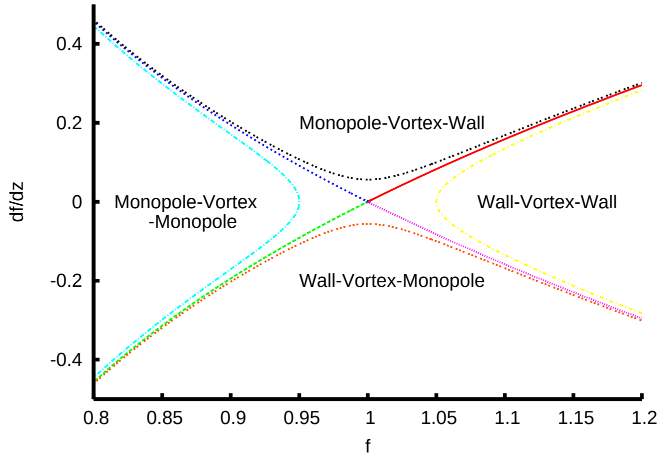
Now we relax the translation invariance along the direction of the vortex ($\hat{z}$-direction) and obtain

$$\Delta\varphi = 0 \; , \qquad \vec{\nabla}\varphi \parallel \mathbf{i} \; , \qquad \Phi_B = \frac{2\pi n}{e} \; . \tag{6}$$

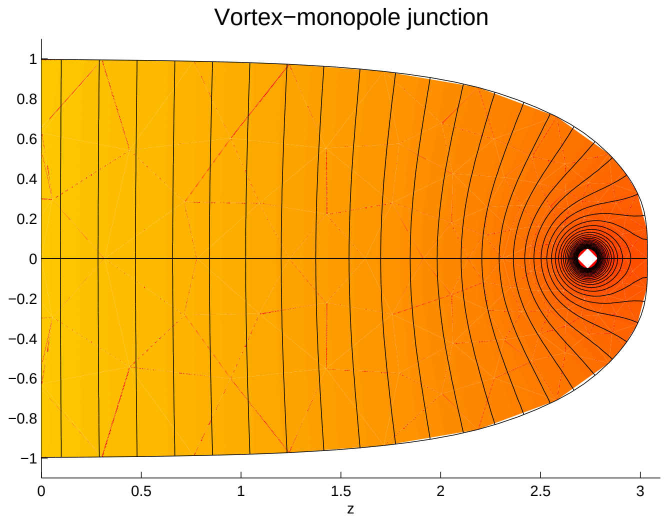
and

$$-\left.\frac{B^2}{2}\right|_{\mathrm{wall}} + T_{\mathrm{W}} \frac{1 + f'^2 - \; f'' f}{f(1+f'^2)^{3/2}} + \varepsilon_0 = 0 \; , \tag{7}$$

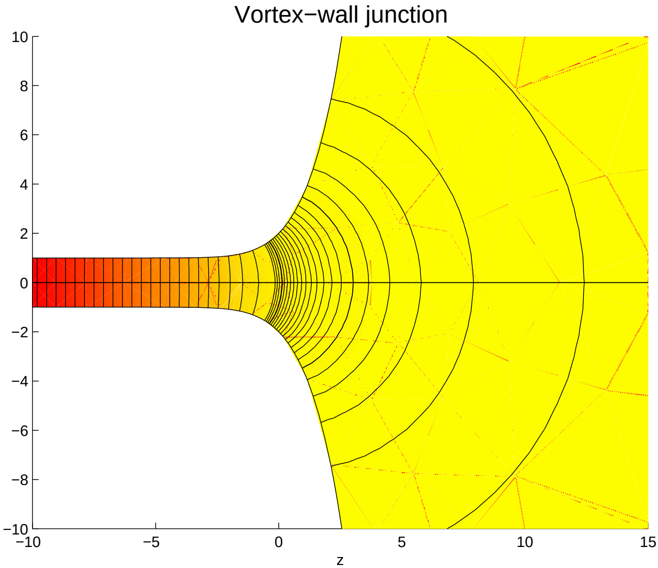
Phase Diagram of the Junctions



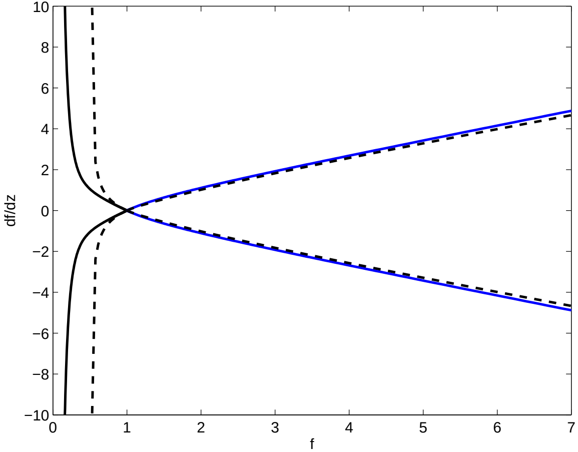
The Vortex-Monopole Junction



The Vortex-Wall Junction



The Full Phase Diagram of the Junctions



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- How about $1/n$-corrections?

Bibliography

- S. Bolognesi, "Domain walls and flux tubes," Nucl. Phys. B **730**, 127 (2005) [arXiv:hep-th/0507273].
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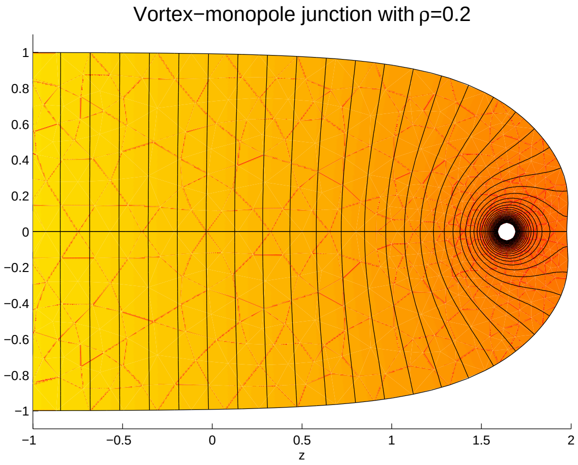
The Beast


Vortex−monopole junction with ρ=0.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−1
−0.5
0
0.5
1
1.5
2
z

The Phase Diagram for the Beast