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Non-perturbative and flux superpotentials on the Z_3 orientifold

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Bibliography and credits

- Work done in collaboration with:

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Work in progress with Massimo Bianchi, Giovanni Villadoro, Fabio Zwirner

Plan of the talk

- Introduction/Motivation

- Instantons in field theory

- Instantons in string theory

- The (blown-up) Z_3 orientifold

- Non-perturbative superpotentials from ED_3 and ED_1 instantons

- Superpotentials from fluxes

- Outlook

Introduction/Motivations

There are several obstacles to string theory making contact with experiment.

They are related to the explicit construction of vacua, that agree to a good degree with what we observe experimentally at low energies (aka Standard Model).

- Stringy vacua need at least N=1 susy for global stability. Supersymmetry breaking can be engineered, but the back-reaction must be neglected and perturbation theory therefore breaks down.

- The ones we know how to construct contain massless moduli coupling with gravitational strength (or worse).

- Getting a reasonable spectrum of masses is extremely hard (because at least solving the evaluation problem is hard)

To this one can add several conceptual issues, like:

- Is the vacuum fitting the SM unique?

- If more than one fit the SM model, is there any (in principle vs practical) selection mechanism?

etc.

What I will like to focus here on:

- Stabilizing the Moduli

- Getting correct masses.

and in particular a common potential ingredient to both: instantons.

Moduli stabilization

• String theory has no potential in maximal dimensions. In standard compactifications, it has no potential either for many scalars.

• As the metrics of scalars and other forms are non-trivial including fluxes in compactifications generates potentials for the massless scalars.

It was pointed out that in orientifold vacua:

♠A combination of closed string fluxes stabilizes complex structure moduli,

♠Kähler moduli can be stabilized by non-perturbative contributions to the superpotential, ending in an AdS vacuum

♠The addition of anti-D branes may lift the vacua to meta-stable dS Vacua.

Kachru+Kallosh+Linde+Trivedi

But as usual there is lot hidden as the details are worked out

Moduli stabilization: open problems

• In orientifolds, anomalous U(1) symmetries are abundant, they gauge the bulk axionic symmetries, and make all known non-perturbative superpotentials gauge-non-invariant: Open and closed string fields mix non-trivially here.

Binetruy+Dudas, Dudas+Vempati

• The uplifting procedure when done in supergravity via D-terms cannot work. Anti-D-branes do not fit in the supergravity description.

Zwirner+Villadoro

• This is correlated with the fact that for RR fluxes, only supergravity is currently the tool at hand.

The problem at hand

As a warmup for the harder problem (controlable moduli stabilisation) we will pick an class of IIB vacua and calculate the non-perturbative and flux superpotentials. This example is the Z_3 orbifold and its blow ups.

We focus on orientifolds because:

♠ It seems that one can implement a complete moduli stabilization procedure

Kachru+Kallosh+Linde+Trivedi

♣ Because it is possible to implement rather successfully a bottom-up algorithm of constructing the standard model.

Antoniadis+Kiritsis+Tomaras

Aldazabal+Ibanez+Quevedo+Uranga

Dijkstra+Huiszoon+Schellekens

♠ We chose the Z_3 orientifold as it is the simplest vacuum that contains all the complications in their simplest form (no complex structure moduli, +anomalous U(1), +chiral matter, +knowledge of closed Kähler potential and its blow up, existence of non-trivial instanton-induced superpotentials)

- We assume N=1 SUSY as a starting point.

- The technology for calculating the flux superpotentials is already established

- For the non-perturbative superpotentials it is just emerging.

Billo+Frau+Pesando+Fucito+Lerda+Liccardo, Billo+Frau+Fucito+Lerda
Blumenhagen+Cvetic+Weingand, Haack+Krefl+Lust+Van Proyen+Zagermann
Ibanez+Uranga, B. Florea+S. Kachru+Mac Greevy+Saulina
Akerblom+Blumenhagen+Lust+Plauschinn+Schmidt-Sommerfeld

Masses: another use of (stringy) instantons

• Orientifold vacua have typically a rich collection of (anomalous) U(1) symmetries.

• Their number constraints severely a number of important (phenomenologically) effective couplings. They include the μ term ($\mu H_1 H_2$), and masses for quarks and leptons: $Q^i \bar{q}_j H$, $L^i \begin{pmatrix} \epsilon_R \\ \nu_R \end{pmatrix}_j$ (including Majorana neutrino masses).

• It is probably important that some terms are zero to leading order, in order to achieve the observed hierarchies.

• Subleading contribution may come from higher order terms (eg. $(Q^i\,\bar{q}_j H)(H^{2n})$, with a suppression factor $\frac{v^{2n}}{M_s^{2n}}$)) or from instantons.

• For example masses in SU(5) orientifold vacua ($\sim \mathbf{10} \times \mathbf{10} \times \mathbf{5}$) are perturbatively forbidden and it seems that only instantons can generate them (if at all)

Anastasopoulos+Dijkstra+Kiritsis+Schellekens

• Similar remarks apply to Majorana masses for neutrinos, via the see-saw mechanism.

Instantons in field theory

• In N=1 theories some non-perturbative effects are due to instantons. The rest are due to strong IR dynamics.

• They can be used to calculate the gaugino condensate in pure SYM theories by noting that $\langle\lambda\lambda(x_1)\cdots\lambda\lambda(x_N)\rangle \simeq \Lambda^{3N}$ is dominated by a one instanton background.

• Example: SQCD with M=N-1 quarks. Instantons generate the ADS superpotential

$$W_{ADS}=\frac{\Lambda^{2N+1}}{det(Q\tilde{Q})} \quad , \quad W_m=m_{ij}Q^i\tilde{Q}^j$$

• From the Konishi anomaly:

$$\frac{1}{4}\bar{D}^2\Phi_I^{\dagger}\, e^{gV}\Phi^J=\frac{\partial W}{\partial\Phi_I}\Phi^J+\delta_I{}^J\frac{g^2}{32\pi^2}tr_R W^2 \quad\to\quad \sum_{ij}m_{ij}\langle Q^i\tilde{Q}^j\rangle=M\frac{g^2}{32\pi^2}\langle\lambda\lambda\rangle$$

- From one-instanton saturation

$$\frac{g^2}{32\pi^2}\langle\lambda\lambda(x_0)Q^{i_1}\tilde{Q}^{j_1}(x_1)...Q^{i_{N-1}}\tilde{Q}^{j_{N-1}}(x_{N-1})\rangle = \Lambda^{2N+1}\ \epsilon^{I_1\cdots I_{N-1}}\ \epsilon^{J_1\cdots J_{N-1}}$$

$$\frac{g^2}{32\pi^2}\langle\lambda\lambda\rangle \equiv \Lambda_L^3 = \frac{\Lambda^{2N+1}}{\det(Q\tilde{Q})} \quad , \quad \langle Q^i\tilde{Q}^j\rangle = (m^{-1})_{ij}\Lambda_L^3$$

- Using decoupling arguments the analysis generalizes to $M \leq N$.

$$\left(\frac{g^2}{32\pi^2}\right)^{N-M}\langle\lambda\lambda(y_1)\cdots\lambda\lambda(y_{N-M})Q^{i_1}\tilde{Q}^{j_1}(x_1)\cdots Q^{i_1}\tilde{Q}^{j_1}(x_M)\rangle_{k=1} = \Lambda^{3N-M}$$

with $3N-M=\beta_1 \to 3\ell(Adj)-\sum_I \ell(R_I)$ in general.

• The rule of thump is: two zero modes θ are appropriate to generate a correction to the superpotential $\to \int d^2\theta\; W$.

• In general there are $2\ell(Adj)$ gaugino zero modes and $2\sum_I \ell(R_I)$ matter zero modes.

• Matter and gaugino zero-modes can be lifted by Yukawa interactions $g\; \phi_I^\dagger \psi^I \lambda$.

• Correction to the superpotential therefore arise when

$$\ell(Adj) - \sum_I \ell(R_I) = 1 \quad , \quad W_{np} = \frac{\Lambda^{\beta_1}}{\mathcal{H}(\Phi)} \quad , \quad \Delta_{\mathcal{H}} = 2\sum_I \ell(R_I)$$

♠ In our problem (Z_3 orientifold) we will need this for $G = SU(4) \simeq SO(6)$ and three chiral multiplets in the antisymmetric $(\mathbf{6})$ representation.

String Theory Instantons

- World-sheet instantons $\to$ perturbative in the string coupling
- Non-perturbative: associated to Euclidean wrapped branes

Becker^2+Strominger

Initial understanding originates from non-perturbative dualities, when they can be mapped to world-sheet instantons:

- ED_{-1} corrections to the hypermultiplets (on the conifold)

Ooguri+Vafa

- ED_{-1} (for R^4) mapped to (p,q) string instantons+M-theory perturbative corrections

Green+Gutperle, Green+Vanhove, Kiritsis+Pioline

- $ED_{p=0,1,2,3}$ for R^4 from M-covariance and T-duality

Kiritsis+Pioline

- ENS_5 for heterotic R^2

Harvey+Moore

- $ED_1 + ED_5$ in type I ($R^4 + F^4$ couplings)

Bachas+Fabre+Kiritsis+Obers+Vanhove, Hammou+Morales

- $D_3 + ED_{-1}$ in orientifolds.

Billo+Frau+Pesando+Fucito+Lerda+Liccardo, Billo+Frau+Fucito+Lerda

String Theory Instantons in IIB orientifolds

• Potential instanton branes: $ED_{-1}, ED_1, ED_3, ED_5, ENS_5$.

• Survive the Ω projection: ED_1, ED_5 and must wrap, complex 2-cycles or all of CY.

• From the point of view of D_9 branes:

ED_5 have 4 ND directions $\to$ standard gauge instantons.

ED_1 have 8 ND directions $\to$ stringy (octonionic?) instantons.

• For D_5 branes:

ED_5 have 8 ND directions $\to$ stringy (octonionic?) instantons.

ED_1 have 4 ND directions $\to$ standard gauge instantons.

The general case involves a magnetized D_9 brane and a magnetized ED_5 brane

Stringy gauge theory instantons

- N D_p branes and k ED_{p-4} instanton branes, precisely reproduce the (gauge) k-instanton action, the ADHM data, the instanton profile, and the associated zero modes and amplitudes.

- The ADHM data are associated with ED-ED strings or D-ED strings.

The ED-ED zero mode bosonic vertex operators are of the form

$$V_a \sim a_\mu e^{-\varphi}\psi^\mu \, T_{K\times K} \quad , \quad V_\chi \sim \chi_i e^{-\varphi}\psi^i \, T_{K\times K}$$

The D-ED zero mode bosonic vertex operators are of the form

$$V_w \sim \sqrt{\frac{g_s}{v_{p-3}}}\, w_\alpha \, e^{-\varphi} \, \prod_\mu \sigma_\mu \, S^\alpha \, T_{K\times K}$$

- The instanton action coincides with the holomorphic gauge coupling

$$S_{\rm inst} = f(S,T,U,Z) \quad , \quad f_{D_9,{\rm orb}} = S + B_I Z^I + \Delta_{1-{\rm loop}}(T,U)$$

For the Z_3 orientifold, $\Delta_{1-{\rm loop}}(T,U)$ is constant.

New (ED_1) instantons

- Prototype: N D_9-k ED_1 pair.

Bachas+Fabre+Kiritsis+Obers+Vanhove

- The structure of zero modes closely resembles the D_9-D_1 system : reproduces the heterotic zero modes in the type I theory (32 chiral left 2d fermions from $D_9 - D_1$ strings and 8 bosons+8 right fermions from $D_1 - D_1$ strings)

- From the $ED_1 - ED_1$ strings we get the two massless Θ-zero modes

$$V_\Theta = \Theta_\alpha S^\alpha \Sigma_{+3/2}\, e^{-\varphi/2} \quad , \quad V_a = a_\mu e^{-\varphi}\ \psi^\mu$$

$S^a \to$ 4d spinor, $\Sigma_{+3/2} \to$ internal spinor, $V_a \to$ spacetime translation 0-modes. Extra massless bosonic modes may appear if the two-cycle is not rigid.

- From the $D_9 - ED_1$ strings we get a number of massless fermionic modes

$$V_\lambda = \sqrt{g_s}\ \lambda_R\ e^{-\varphi/2}\ S^- \prod_\mu \sigma_\mu \prod_I \sigma_I$$

$S^- \to$ 2d R-handed spinor, $\sigma_{\mu,I} \to$ ND twist fields. The number of λ-modes depends on N, k and the "intersections" Integrating out the zero modes we obtain W-corrections, that are not similar to gauge-instanton ones. $(S_{\text{instanton}} \neq f)$

The diagrams

♠The relevant diagrams are disks $(\frac{1}{g_s} \times (\sqrt{g_s})^2)$ or $\chi = 0$ surfaces (annulus/Möbius) with insertions of V_Θ and V_λ, with or without insertions of the physical massless fields V_{Φ_i}.

- Summation over disks without V_{Φ_i} generate the instanton action (including "λ-interactions")

- Summation over disks with one V_{Φ_i} generate the classical profile of the instanton.

- Summation over disks with more V_{Φ_i} implement higher order corrections.

- The summation over one-loop diagrams provides the one-loop determinants around the instanton.

- Around a supersymmetric instanton there are two Θ zero modes, and $2n$ λ zero modes. An F-term is obtained as: n disks with 2n λ insertions, (n-2) with V_ϕ and 2 with V_ψ, or n-1 with V_ϕ and one with V_F ($V_\Phi = V_\phi + \Theta V_\psi + \Theta^2 V_F$)

- Integrating Θ's and λ's yields superpotential terms of the form:

$$W = e^{-T_{ED_p}\ V_{ED_p}(Z)}\ (\Phi_i)^n$$

Compatibility of bulk isometries and instantons

• Isometries of (bulk) chiral fields Z, are gauged by anomalous U(1) symmetries: $\zeta = ImZ \to \zeta + \epsilon$. (Also by bulk fluxes)

• Z can only appear in "dressed" (gauge invariant) combinations in the superpotential.

• Instantons cannot spoil the isometry associated to Z (as it is protected by gauge invariance).

• This means that in practice instanton branes that could do this, are disallowed:

♠ Either because the wrapped ED brane is anomalous

Freed+Witten

♠ Or because its wrapping is destabilized because of flux

Kashani-Poor+Tomasiello

Such constraints can be obtained from the Bianchi identities:

$$DG = \Pi[branes] \wedge e^F \quad , \quad D \equiv d + H + \mathcal{T} + \mathcal{Q} + \mathcal{R}$$

Expanding

$$dG_1 + \mathcal{T}G_1 = Tr[F_2] \wedge \Pi_0(D_9) + \Pi_2(D_7)$$

$$dG_3 + \mathcal{T}G_3 + H_3 \wedge G_1 = \frac{Tr[F_2^2]}{2} \wedge \Pi_0(D_9) + Tr[F_2] \wedge \Pi_2(D_7) + \Pi_4(D_5)$$

In the absence of Scherk-Schwarz torsion

$$dG_1 = Tr[F_2] \quad , \quad \int_{ED_1} Tr[F_2] = \int_{ED_1} dG_1 = 0$$

• We cannot wrap an ED_1 on cycles $\mathcal{C}$ such that $\int_{\mathcal{C}} Tr[F_2] \neq 0$ because this will break the U(1) gauge symmetry.

• This remains true even if G_1 and F_2 are odd under Ω.

• For the Z_3 orientifold $\mathcal{C}$ is a democratic linear combination of the exceptional (twisted) cycles.

• Similar remarks apply to when closed string fluxes are turned on *Kashani-Poor+Tomasiello*

The Z_3 orbifold

It can be obtained by modding out T^6 by Z_3 as

$$z^I \to \alpha\ z^I \quad , \quad \alpha = e^{\frac{2\pi i}{3}}$$

This constraints the metric and B fields to be of the form

$$ds^2 = G_{I\bar{I}}\ dz^I d\bar{z}^{\bar{I}} \quad , \quad B_2 = B_{I\bar{I}}\ dz^I \wedge d\bar{z}^{\bar{I}}$$

$M = G + B$ is an arbitrary 3×3 complex matrix ($h^{\text{untwisted}}_{1,1} = 9$ untwisted Kähler moduli).

• There are no (untwisted) complex structure moduli as the Z_3 action completely fixes the complex structure.

$$\mathcal{M}^{\text{untwisted}}_{1,1} = \frac{SU(3,3)}{SU(3) \times SU(3) \times U(1)}$$

• It is a special Kähler manifold with prepotential

$$\mathcal{F}_{\text{untwisted}} = det(X) = \frac{1}{3!}\epsilon^{I_1 I_2 I_3}\epsilon^{J_1 J_2 J_3}\ X_{I_1 J_1} X_{I_2 J_2} X_{I_3 J_3} \quad , \quad K = -\log[\det[Re[X]]]$$

• Z_3 has 27 fixed points associated to 27 exceptional (rigid) divisors $E_i \to h^{\text{twisted}}_{1,1} = 27$, $h^{\text{twisted}}_{2,1} = 0$ The blown-up CY has $h_{1,1} = 36$, $h_{1,2} = 0$.

The Z_3 orientifold

Tadpole cancellation implies that (discrete magnetic flux through the collapsed cycles)

$$Tr[\mathbf{1}] = 32 \quad , \quad Tr[\gamma_3] = -4$$

Angelantonj+Bianchi+Sagnotti+Pradisi+Stanev

Together with $\gamma_3^3 = 1$, $\gamma_3 \to$ unitary, it implies

$$\gamma_3 = (\mathbf{1}_{N\times N}, \alpha\mathbf{1}_{M\times M}, \bar{\alpha}\mathbf{1}_{\bar{M}\times\bar{M}})$$

with

$$N + M + \bar{M} = 32 \quad , \quad M = \bar{M} \quad , \quad N + \alpha M + \bar{\alpha}\bar{M} = -4$$

leading to

$$N = 8, M = \bar{M} = 12 \quad \to \quad G = SO(8) \times U(12)$$

when all branes are at the origin.

• Spectrum=3 copies of $(\mathbf{8},\mathbf{12})_{+1} \oplus (\mathbf{1},\mathbf{66^*})_{-2}$ of $SO(8)\times U(12)$

• The U(1) of U(12) is anomalous: $t_3 \equiv Tr[QT^aT^a] \neq 0$.

• The U(1) mixes with the democratic combination of twisted chiral multiplets $Z=\sum_i Z_i$: $Z+\bar{Z} \to Z+\bar{Z}+M\ V$ (with $M=\frac{1}{2}3^{\frac{5}{4}}\pi^{-\frac{3}{4}}$)

Antoniadis+Kiritsis+Rizos

$$V \to V + i(\epsilon - \bar{\epsilon}) \quad , \quad Z \to Z + iM\epsilon$$

♠ Anomaly cancellation : $f_a = S + C_a Z + \cdots \quad , \quad M\ C_a = t_{3,a}$

• There are two T-dual versions related by six T-dualities. One has O_9 planes while the other O_3 planes.

The perturbative superpotential and phase structure

There is a disk-generated superpotential:

$$W = \frac{1}{2!3!} Y(T,S,Z)\ \epsilon_{IJK}\delta^{ij}\ C_i^{Ir} C_j^{Js}\ A^K_{[rs]}$$

where: $C \leftrightarrow (\mathbf{8},\mathbf{12})_{+1}$, $A \leftrightarrow (\mathbf{1},\mathbf{66^*})_{-2}$,

$i,j = SO(8)$ index, $r,s = SU(12)$ index, $I,J,K = 1,2,3 \rightarrow$ family index

• Y depends on dilaton only at tree level.

• Higher polynomials in (CCA) are allowed to appear (U(1) neutral), but not of Pf(A) (Q=12) in perturbation theory.

• Turning on general Wilson lines breaks G to $SO(8-2n)\times U(12-2n)\times U(n)^3$

• For $n=4$: $G=U(4)_{fp}\times U(4)^3$ with $U(4)^3$ a bulk $\mathcal{N}=4$ conformal gauge theory with 4 branes and their 6 copies under Z_3 and Ω

• 3 generation of chiral mater in $(\mathbf{6}_{-2};\mathbf{1}_0,\mathbf{1}_0,\mathbf{1}_0)$ plus

$$(\mathbf{1}_0;\mathbf{4}_{+1},\mathbf{4^*}_{-1},\mathbf{1}_0)\quad,\quad(\mathbf{1}_0;\mathbf{1}_0,\mathbf{4}_{+1},\mathbf{4^*}_{-1})\quad,\quad(\mathbf{1}_0;\mathbf{4^*}_{-1},\mathbf{1}_0,\mathbf{4}_{+1})$$

• Turning-on VEVs in bi-fundamentals $U(4)^3\to U(4)_{\text{diagonal}}$ a standard $\mathcal{N}=4$ conformal gauge theory, which in the Coulomb branch breaks further to $U(1)^4$.

• The non-trivial dynamic of the superpotential is therefore associated to the $U(4)_{fp}$

Wrapped ED_5 instantons

• When $G = U(4)_{fp} \times U(4)_{\text{diagonal}}$ with $3 \times \mathbf{6}_{-2}$, instanton calculus is reliable: we can turn on vevs of the A's so that the surviving group is SO(3), with no light charged matter $\to$ pure $\mathcal{N}=1$ sQCD $\to$ gaugino condensation with $W = \Lambda_L^3$. Matching between low and high energy we get

$$W = \Lambda_L^3 = \frac{\Lambda^9}{det_{IJ}(\delta^{ab} A_a^I A_b^J)} \quad , \quad A_a^I = \frac{1}{2}\Gamma_a^{rs} A_{[rs]}^I \quad , \quad a = 1,2\cdots,6 \quad , \quad I = 1,2,3$$

This is one of the "classic" ADS-like case with $\ell_A - \sum_C \ell_C = 1$, as $\ell_A = 4$, $\ell_C = 1$.

• In string theory:

$$W(S,T,Z) = \frac{\exp[f(S,T,Z)]}{\mathcal{H}(A)}$$

where

$$f(S,T,Z) = f_{\text{tree}}(S,Z) + f_{1-\text{loop}}(T,Z) \quad , \quad f_{\text{tree}}(S,Z) = S + CZ \quad , \quad f_{1-\text{loop}}(T,Z=0) = f_1$$

• This is invariant under the anomalous U(1) transformations, as $t_{144} = Tr[QT^aT^a] = -12$ and

$$Z \to Z - \frac{12i}{C}\epsilon \quad , \quad A \to e^{-2i\epsilon} A$$

• This parallels the derivations of the ADS superpotential in a local (oriented) brane configuration with bifundamentals. *Akerblom+Blumenhagen+Lust+Plauschinn+Schmidt-Sommerfeld*

Wrapped ED_1 instantons

• Similar to the calculation of $ED_1 - D_9$ in the T^6-compactified type-I string

Bachas+Fabre+Kiritsis+Obers+Vanhove

• We must wrap ED_1 on non-trivial two cycles $\mathcal{C}$, and count λ zero modes between ED_1 and D_9.

• There are two Θ zero modes, but the $\bar{\Theta}$ are projected out.

• The λ-modes depend on the cycle $\mathcal{C}$ and the restriction of the D_9 gauge bundle on $\mathcal{C}$. They transform as $\mathbf{4}_{+1}$. At the disk level we get:

$$L = m_I(\mathcal{C})\ A^I_{[rs]}\ \lambda^r_{\mathcal{C}}\ \lambda^s_{\mathcal{C}}$$

• We must interpret A, λ as sections of holomorphic line bundles.

• We also have

$$m(\mathcal{C}) \sim e^{-S_{\text{instanton}}}\ \frac{Pfaff(\bar{\partial}_{V(-1)})}{\det[\bar{\partial}_{\mathcal{O}(-1)}]^2\,\det'[\bar{\partial}_{\mathcal{O}}]^2}$$

$$V|_{CP^1} = \sum_{i=1}^{16} [\mathcal{O}(k_i) \oplus \mathcal{O}(-k_i)] \quad , \quad \mathcal{O}(-1) \otimes V|_{CP^1} = \sum_{i=1}^{16} [\mathcal{O}(k_i - 1) \oplus \mathcal{O}(-k_i - 1)]$$

Witten

$$dim Ker(\bar{\partial}_{V(-1)}) = \sum_i k_i$$

.

• There is the further constraint $c_2(T) = c_2(V)$ which amounts to $dG_3 = 0$, as there are no D_5 branes, which selects the orientifold gauge group.

• The minimal case corresponds to rigid two-cycles with $\sum_i k_i = 4$. Integrating the Θs and λ's we obtain supersymmetric mass terms

$$W_m = \sum_{\mathcal{C}} m_I(\mathcal{C}) \; m_J(\mathcal{C}) \; \epsilon^{rspq} \; A^I_{[rs]} \; A^J_{[pq]}$$

• U(1) invariance indicates that the instanton action must behave as $e^{-\frac{C}{3}Z}$. This is a fractional instanton.

• Indeed in the T-dual picture, we have D_7 branes wrapping one collapsed (twisted) 4-cycle, and ED_3 instantons wrapping the same cycle. Because of the $B = \frac{1}{3}$ trapped flux and the $\int C_2 \wedge B$ coupling, the instanton is fractional.

• The general expected behavior is

$$\sum_{n_a, n} g(n_a, n) \, \mathrm{exp} \left[- \sum_{n^a} Z'_a - \frac{n}{3} Z \right]$$

where n is correlated with the power of A^{2n} multiplets.

Closed string fluxes

• Apart from Z_2 bulk fluxes and open string magnetic fields the fluxes that are compatible with the orientifold are:
RR 3-form flux G_3, Scherk-Schwarz torsion $\mathcal{T}$, the non-geometric flux $\mathcal{R}$.

• $\mathcal{T}$ maps p-forms to $p+1$-forms while $\mathcal{R}$ maps p-forms to $(p-3)$-forms

$$\mathcal{T} \circ A_p = A_{p+1} \quad , \quad \mathcal{R} \bullet A_p = A_{p-3}$$

• The flux superpotential reads

$$W_{\text{flux}} = \int [G_3 - i\mathcal{T} \circ J_C + \mathcal{R} \bullet (*S)] \wedge \Omega_3$$

What next?

• It seems that we have more or less control over non-perturbative superpotentials for the special case of the (blown-up) Z_3 orbifold. Several results generalize to more complex cases.

• Numerical coefficients in front of the various terms need to be calculated carefully.

• There are still some points that need to be clarified in the absence of bulk fluxes, like the one-loop corrections to gauge couplings for non-zero twist fields as well as some open string instanton corrections to the superpotential.

• One can attempt to turn on the bulk fluxes and attempt a complete analysis of moduli stabilisation. It has been argued by Lust, Reffert, Scheidegger, Schulgin, Stieberger, that in orbifolds like Z_3 although the moduli can be stabilized in AdS, no uplift is possible to dS. This rests on $\mathcal{T} = \mathcal{R} = 0$, and neglecting the non-perturbative open string superpotentials presented here.

• An re-analysis of the instanton effects in the presence of fluxes is necessary. This has simplifications (and complications)

• A good control of the Kähler potential is necessary. The one-loop dependence on the open string fields needs to be calculated.

• An improved analysis of the D-terms is also needed and is under way.

• The hope is that this will give a completely controlled example of successful moduli stabilisation with positive vacuum energy. That remains to be seen.

What's new?

- A clear classification of all relevant instanton effects.

- The precise form of the vertex operators of the instanton fluctuations (including zero modes).

- Inclusion of the effect of orientifold projections and A-S representations.

- Rigid cycles are explicitly identified (do not exist in toroidal examples studied so far)

- Precise identification of consistency conditions (Bianchi identities), that constrain ED-instanton wrappings.

- Complete incorporation of anomalous U(1) symmetries and checks, via zero mode counting and anomaly calculations.

The Bianchi identities

Introducing the bulk "covariant" exterior derivative, we have NS identities:

$$D = (d + \mathcal{T}) + \mathcal{R} \quad , \quad D \cdot D = 0$$

For $\quad d\mathcal{T} = d\mathcal{R} = 0 \qquad \mathcal{T} \circ \mathcal{T} = 0 \quad$ and $\quad \mathcal{T} \circ \mathcal{R} = 0$

These can be solved as:

$$\mathcal{T} \circ \omega_i = -\alpha_i \, \Omega + \overline{\alpha}_i \, \overline{\Omega} \quad , \quad \mathcal{T} \circ \Omega = -\overline{\alpha}_i \, \tilde{\omega}_i \quad , \quad \mathcal{T} \circ \overline{\Omega} = -\alpha_i \, \tilde{\omega}_i \quad , \quad \mathcal{T} \circ \tilde{\omega}_i = 0$$

Action of non-geometrical fluxes:

$$\mathcal{R} \bullet \ \Omega = \kappa \quad , \quad \mathcal{R} \bullet \ \overline{\Omega} = \overline{\kappa} \quad , \quad \mathcal{R} \bullet \ \tilde{\omega}_i = 0 \quad , \quad \mathcal{R} \bullet \ V = \overline{\kappa} \, \Omega - \kappa \, \overline{\Omega}$$

- The RR identities

$$D \ G = Tr[e^{\mathcal{F}}] \ A_{Dp} \ [Dp] + A_{Op} \ [Op] \, ,$$

where $\mathcal{F} = F + B$ and

$$A_{Dp} = 1 - [p_1(R_T) - p_1(R_N)] + \dots \quad , \quad A_{Op} = 1 + \frac{1}{2}[p_1(R_T) - p_1(R_N)] + \dots$$

Expanding in our case:

$$\mathcal{R} \bullet G_3 = [D9] + [O9] \quad , \quad c_1(F_9)\ [D9] = 0 \quad ,$$

$$\mathcal{T} \circ G_3 = c_2(F_9) - p_1(R_9)\ [D9] + \frac{1}{2} p_1(R_9)\ [O9] + [D5] + [O5]$$

$$c_3(F_9)\ [D9] + c_1(F_5)\ [D5] = 0$$

- General form of the localized Bianchi identities, for each brane stack:

$$D\ \left(e^{F_p}\ [D_p]\right) = 0$$

that translate into:

$$\left(\mathcal{T} \circ F_9 + \mathcal{R} \bullet (F_9)^3\right)\ [D9] = 0 \quad , \quad \mathcal{R} \bullet\ (F_5\ [D5]) = 0$$

Detailed plan of the presentation

- Title page 1 minutes
- Bibliography 2 minutes
- Plan 3 minutes
- Introduction/Motivations 6 minutes
- Moduli stabilisation 7 minutes
- Moduli stabilisation:open problems 9 minutes
- The problem at hand 10 minutes
- Masses: another use of (stringy) instantons 12 minutes
- Instantons in Field Theory 15 minutes
- String Theory Instantons 17 minutes
- String Theory Instantons in IIB orientifolds 19 minutes
- Stringy gauge theory Instantons 22 minutes
- New (ED_1) instantons 24 minutes

- The diagrams 26 minutes
- Compatibility of bulk isometries and instantons 30 minutes
- The Z_3 orbifold 32 minutes
- The Z_3 orientifold 35 minutes
- The perturbative superpotential and phase structure 38 minutes
- Wrapped ED_5 instantons 41 minutes
- Wrapped ED_1 instantons 45 minutes
- Closed string fluxes 46 minutes
- What next? 49 minutes

- What's new? 51 minutes
- The Bianchi identities 55 minutes