Higher spin from a world line prespective

Emanuele Latini
Bologna University and LNF, INFN

Pisa, 20 March 2007

Summary

- Higher Spin and SO(N) Spinning particle
- Gauge fixing
- Gauge fixed partition function
  - Even N
  - Odd N
- Pashnev and Sorokin model
- Conclusions and outlook

- Higher Spin and SO(N) Spinning particle
- Gauge fixing
- Gauge fixed partition function
  - Even N
  - Odd N
- Pashnev and Sorokin model
- Conclusions and outlook

Summary

- Higher Spin and SO(N) Spinning particle
- Gauge fixing
- Gauge fixed partition function
  - Even N
  - Odd N
- Pashnev and Sorokin model
- Conclusions and outlook

- Higher Spin and SO(N) Spinning particle
- Gauge fixing
- Gauge fixed partition function
  - Even N
  - Odd N
- Pashnev and Sorokin model
- Conclusions and outlook

- Higher Spin and SO(N) Spinning particle
- Gauge fixing
- Gauge fixed partition function
  - Even N
  - Odd N
- Pashnev and Sorokin model
- Conclusions and outlook

- Higher Spin and SO(N) Spinning particle
- Gauge fixing
- Gauge fixed partition function
  - Even N
  - Odd N
- Pashnev and Sorokin model
- Conclusions and outlook

Summary

- Higher Spin and SO(N) Spinning particle
- Gauge fixing
- Gauge fixed partition function
  - Even N
  - Odd N
- Pashnev and Sorokin model
- Conclusions and outlook

- Higher Spin and SO(N) Spinning particle
- Gauge fixing
- Gauge fixed partition function
    - Even N
    - Odd N
- Pashnev and Sorokin model
- Conclusions and outlook

$SO(N)$ spinning particle model

$SO(N)$ SPINN. PARTICLE WITH SUGRA MULTIPLET ON THE WORLDLINE	$\Longleftrightarrow$	SPIN $N/2$ IN $D=4$, CONFORMAL PARTICLES IN "D" DIMENSION

$SO(N)$ spinning particle model

$SO(N)$ SPINN. PARTICLE WITH SUGRA MULTIPLET ON THE WORLDLINE	$\Longleftrightarrow$	SPIN $N/2$ IN $D=4$, CONFORMAL PARTICLES IN "D" DIMENSION

The action for this models reads

$$S[x,\psi_i^\mu,G] = \int_0^1 d\tau \left[\frac{1}{2}e^{-1}(\dot{x}^\mu - \chi_i\psi_i^\mu)^2 + \frac{1}{2}\psi_i^\mu(\delta_{ij}\partial_\tau - a_{ij})\psi_{j\mu}\right]$$

$SO(N)$ spinning particle model

$SO(N)$ SPINN. PARTICLE WITH SUGRA MULTIPLET ON THE WORLDLINE	$\Longleftrightarrow$	SPIN $N/2$ IN $D = 4$, CONFORMAL PARTICLES IN "D" DIMENSION

The action for this models reads

$$S[x, \psi_i^\mu, G] = \int_0^1 d\tau \left[\frac{1}{2} e^{-1} (\dot{x}^\mu - \chi_i \psi_i^\mu)^2 + \frac{1}{2} \psi_i^\mu (\delta_{ij} \partial_\tau - a_{ij}) \psi_{j\mu} \right]$$

The N world line supergravity multiplet contains

$SO(N)$ SPINN. PARTICLE WITH SUGRA MULTIPLET ON THE WORLDLINE	$\Longleftrightarrow$	SPIN $N/2$ IN $D=4$, CONFORMAL PARTICLES IN "D" DIMENSION

The action for this models reads

$$S[x,\psi_i^\mu,G]=\int_0^1 d\tau\left[\frac{1}{2}e^{-1}(\dot{x}^\mu-\chi_i\psi_i^\mu)^2+\frac{1}{2}\psi_i^\mu(\delta_{ij}\partial_\tau-a_{ij})\psi_{j\mu}\right]$$

The N world line supergravity multiplet contains

- the enbein e which gauges worldline translations

$SO(N)$ SPINN. PARTICLE WITH SUGRA MULTIPLET ON THE WORLDLINE	$\Longleftrightarrow$	SPIN $N/2$ IN $D=4$, CONFORMAL PARTICLES IN "D" DIMENSION

The action for this models reads

$$S[x,\psi_i^\mu,G] = \int_0^1 d\tau \left[\frac{1}{2}e^{-1}(\dot{x}^\mu - \chi_i\psi_i^\mu)^2 + \frac{1}{2}\psi_i^\mu(\delta_{ij}\partial_\tau - a_{ij})\psi_{j\mu}\right]$$

The N world line supergravity multiplet contains

- the enbein e which gauges worldline translations
- gravitini χ_i which gauges worldline susy

$SO(N)$ spinning particle model

$SO(N)$ SPINN. PARTICLE WITH SUGRA MULTIPLET ON THE WORLDLINE	$\Longleftrightarrow$	SPIN $N/2$ IN $D=4$, CONFORMAL PARTICLES IN "D" DIMENSION

The action for this models reads

$$S[x,\psi_i^\mu,G] = \int_0^1 d\tau \left[\frac{1}{2} e^{-1}(\dot{x}^\mu - \chi_i \psi_i^\mu)^2 + \frac{1}{2}\psi_i^\mu(\delta_{ij}\partial_\tau - a_{ij})\psi_{j\mu}\right]$$

The N world line supergravity multiplet contains

- the enbein e which gauges worldline translations
- gravitini χ_i which gauges worldline susy
- gauge field a_{ij} which gauges $SO(N)$ symmetry

$SO(N)$ spinning particle model

$SO(N)$ SPINN. PARTICLE WITH SUGRA MULTIPLET ON THE WORLDLINE	$\Longleftrightarrow$	SPIN $N/2$ IN $D=4$, CONFORMAL PARTICLES IN "D" DIMENSION

The action for this models reads

$$S[x,\psi_i^\mu,G] = \int_0^1 d\tau \left[\frac{1}{2}e^{-1}(\dot{x}^\mu - \chi_i\psi_i^\mu)^2 + \frac{1}{2}\psi_i^\mu(\delta_{ij}\partial_\tau - a_{ij})\psi_{j\mu}\right]$$

The N world line supergravity multiplet contains

- the enbein e which gauges worldline translations
- gravitini χ_i which gauges worldline susy
- gauge field a_{ij} which gauges $SO(N)$ symmetry

OBJECTIVE

$SO(N)$ spinning particle model

$SO(N)$ SPINN. PARTICLE WITH SUGRA MULTIPLET ON THE WORLDLINE	$\Longleftrightarrow$	SPIN $N/2$ IN $D = 4$, CONFORMAL PARTICLES IN "D" DIMENSION

The action for this models reads

$$S[x, \psi_i^\mu, G] = \int_0^1 d\tau \left[\frac{1}{2} e^{-1} (\dot{x}^\mu - \chi_i \psi_i^\mu)^2 + \frac{1}{2} \psi_i^\mu (\delta_{ij} \partial_\tau - a_{ij}) \psi_{j\mu} \right]$$

The N world line supergravity multiplet contains

- the enbein e which gauges worldline translations
- gravitini χ_i which gauges worldline susy
- gauge field a_{ij} which gauges $SO(N)$ symmetry

OBJECTIVE: we want to study the partition function on the circle

$$Z \sim \int_{T^1} \frac{\mathcal{D}X\mathcal{D}G}{\boxed{\mathrm{Vol}\,(\mathrm{Gauge})}} e^{-S[X,G]}$$

Gauge fixing

gauge symmetry $\Bigg\{$

$$\text{gauge symmetry} \begin{cases} \delta e = \dot{\xi} + 2\chi_i \epsilon_i \\ \end{cases}$$

gauge symmetry
$$\begin{cases} \delta e &= \dot{\xi} + 2\chi_i\epsilon_i \\ \delta\chi_i &= \dot{\epsilon}_i - a_{ij}\epsilon_j + \alpha_{ij}\chi_j \end{cases}$$

gauge symmetry
$$\begin{cases} \delta e &= \dot{\xi} + 2\chi_i \epsilon_i \\ \delta \chi_i &= \dot{\epsilon}_i - a_{ij}\epsilon_j + \alpha_{ij}\chi_j \\ \delta a_{ij} &= \dot{\alpha}_{ij} + \alpha_{im} a_{mj} + \alpha_{jm} a_{im} \end{cases}$$

gauge symmetry
$$\begin{cases} \delta e &= \dot{\xi} + 2\chi_i\epsilon_i \\ \delta\chi_i &= \dot{\epsilon}_i - a_{ij}\epsilon_j + \alpha_{ij}\chi_j \\ \delta a_{ij} &= \dot{\alpha}_{ij} + \alpha_{im}a_{mj} + \alpha_{jm}a_{im} \end{cases}$$

We take fermions and gravitino with antiperiodic boundary condition (ABC)

$\Downarrow$

gravitinos can be completely gauged away

gauge symmetry
$$\begin{cases} \delta e &= \dot{\xi} + 2\chi_i\epsilon_i \\ \delta\chi_i &= \dot{\epsilon}_i - a_{ij}\epsilon_j + \alpha_{ij}\chi_j \\ \delta a_{ij} &= \dot{\alpha}_{ij} + \alpha_{im}a_{mj} + \alpha_{jm}a_{im} \end{cases}$$

We take fermions and gravitino with antiperiodic boundary condition (ABC)

$\Downarrow$

gravitinos can be completely gauged away

Our gauge choice

Gauge fixing

$$\text{gauge symmetry} \begin{cases} \delta e &= \dot{\xi} + 2\chi_i \epsilon_i \\ \delta \chi_i &= \dot{\epsilon}_i - a_{ij}\epsilon_j + \alpha_{ij}\chi_j \\ \delta a_{ij} &= \dot{\alpha}_{ij} + \alpha_{im}a_{mj} + \alpha_{jm}a_{im} \end{cases}$$

We take fermions and gravitino with antiperiodic boundary condition (ABC)

$$\Downarrow$$

gravitinos can be completely gauged away

Our gauge choice

$$\begin{aligned} e &\longrightarrow \beta \in [0, \infty) \\ \chi_i &\longrightarrow 0 \\ a_{ij} &\longrightarrow \hat{a}_{ij}(\theta_k) \text{ with } \theta_k \in [0, 2\pi] \end{aligned}$$

gauge symmetry
$$\begin{cases} \delta e &= \dot{\xi} + 2\chi_i \epsilon_i \\ \delta \chi_i &= \dot{\epsilon}_i - a_{ij}\epsilon_j + \alpha_{ij}\chi_j \\ \delta a_{ij} &= \dot{\alpha}_{ij} + \alpha_{im}a_{mj} + \alpha_{jm}a_{im} \end{cases}$$

We take fermions and gravitino with antiperiodic boundary condition (ABC)

$$\Downarrow$$

gravitinos can be completely gauged away

Our gauge choice

$$\begin{aligned} e &\longrightarrow \beta \in [0, \infty) \\ \chi_i &\longrightarrow 0 \\ a_{ij} &\longrightarrow \hat{a}_{ij}(\theta_k) \text{ with } \theta_k \in [0, 2\pi] \end{aligned}$$

fixes the supergravity multiplet up to some moduli

gauge symmetry
$$\begin{cases} \delta e &= \dot{\xi} + 2\chi_i \epsilon_i \\ \delta \chi_i &= \dot{\epsilon}_i - a_{ij}\epsilon_j + \alpha_{ij}\chi_j \\ \delta a_{ij} &= \dot{\alpha}_{ij} + \alpha_{im} a_{mj} + \alpha_{jm} a_{im} \end{cases}$$

We take fermions and gravitino with antiperiodic boundary condition (ABC)

$$\Downarrow$$

gravitinos can be completely gauged away

Our gauge choice

$$\begin{aligned} e &\longrightarrow \beta \in [0, \infty) \\ \chi_i &\longrightarrow 0 \\ a_{ij} &\longrightarrow \hat{a}_{ij}(\theta_k) \text{ with } \theta_k \in [0, 2\pi] \end{aligned}$$

fixes the supergravity multiplet up to some moduli

- β is the usual proper time

$$\text{gauge symmetry} \left\{ \begin{array}{lcl} \delta e & = & \dot{\xi} + 2\chi_i\epsilon_i \\ \delta\chi_i & = & \dot{\epsilon}_i - a_{ij}\epsilon_j + \alpha_{ij}\chi_j \\ \delta a_{ij} & = & \dot{\alpha}_{ij} + \alpha_{im}a_{mj} + \alpha_{jm}a_{im} \end{array} \right.$$

We take fermions and gravitino with antiperiodic boundary condition (ABC)

$$\Downarrow$$

gravitinos can be completely gauged away

Our gauge choice

$$\begin{array}{rcl} e & \longrightarrow & \beta \in [0, \infty) \\ \chi_i & \longrightarrow & 0 \\ a_{ij} & \longrightarrow & \hat{a}_{ij}(\theta_k) \text{ with } \theta_k \in [0, 2\pi] \end{array}$$

fixes the supergravity multiplet up to some moduli

- β is the usual proper time
- $\hat{a}_{ij}(\theta_k)$ is some block diagonal matrix we discuss later

The gauge fixed partition function reads

$$
\begin{aligned}
Z \qquad &= -\frac{1}{2}\int_0^\infty \frac{d\beta}{\beta}\int \frac{d^D x}{(2\pi\beta)^{\frac{D}{2}}} \\
& K_N \prod_{k=1}^{r}\int_0^{2\pi}\frac{d\theta_k}{2\pi}\mathrm{Det}\left(\partial_\tau - \hat{a}_{vec}\right)^{\frac{D}{2}-1}_{ABC}\mathrm{Det}'\left(\partial_\tau - \hat{a}_{adj}\right)_{PBC}
\end{aligned}
$$

The gauge fixed partition function reads

$$Z = -\frac{1}{2}\int_0^\infty \frac{d\beta}{\beta}\int \frac{d^D x}{(2\pi\beta)^{\frac{D}{2}}} \boxed{K_N} \prod_{k=1}^{r}\int_0^{2\pi}\frac{d\theta_k}{2\pi}\mathrm{Det}\left(\partial_\tau - \hat{a}_{vec}\right)^{\frac{D}{2}-1}_{ABC}\mathrm{Det}'\left(\partial_\tau - \hat{a}_{adj}\right)_{PBC}$$

♮ this is a normalization factor we will discuss later

The gauge fixed partition function reads

$$Z = -\frac{1}{2}\int_0^\infty \frac{d\beta}{\beta}\int \frac{d^Dx}{(2\pi\beta)^{\frac{D}{2}}} \boxed{K_N} \prod_{k=1}^{r}\int_0^{2\pi}\frac{d\theta_k}{2\pi} \boxed{\mathrm{Det}\left(\partial_\tau - \hat{a}_{vec}\right)^{\frac{D}{2}-1}_{ABC}} \mathrm{Det}'\left(\partial_\tau - \hat{a}_{adj}\right)_{PBC}$$

♭ determinants of the susy ghosts and Majorana fermions which all have antiperiodic boundary conditions (ABC) and transform in the vector representation of SO(N)

The gauge fixed partition function reads

$$Z = -\frac{1}{2}\int_0^\infty \frac{d\beta}{\beta}\int \frac{d^Dx}{(2\pi\beta)^{\frac{D}{2}}}$$

$$K_N \prod_{k=1}^{r}\int_0^{2\pi}\frac{d\theta_k}{2\pi}\, \mathrm{Det}\,(\partial_\tau - \hat{a}_{vec})^{\frac{D}{2}-1}_{ABC}\; \mathrm{Det}'\,(\partial_\tau - \hat{a}_{adj})_{PBC}$$

◇ This determinant is due to the ghosts for the SO(N) gauge symmetry; they transform in the adjoint representation and have periodic boundary conditions (PBC). It's indicated by Det' because it contains zero modes we have to exclude from the determinant

The gauge fixed partition function reads

$$Z = -\frac{1}{2}\int_0^\infty \frac{d\beta}{\beta}\int \frac{d^Dx}{(2\pi\beta)^{\frac{D}{2}}}$$

$$\boxed{K_N \prod_{k=1}^r \int_0^{2\pi} \frac{d\theta_k}{2\pi} \mathrm{Det}\left(\partial_\tau - \hat{a}_{vec}\right)_{ABC}^{\frac{D}{2}-1} \mathrm{Det}'\left(\partial_\tau - \hat{a}_{adj}\right)_{PBC}}$$

THIS LINE COMPUTES THE NUMBER OF DEGREES OF FREEDOM (Dof) NORMALIZED TO ONE FOR A REAL SCALAR FIELD

Using constant gauge transformations $\Rightarrow$ a_{ij} is block diagonal

Using constant gauge transformations $\Rightarrow$ a_{ij} is block diagonal

$$\hat{a}_{ij} = \begin{pmatrix} 0 & \theta_1 & . & 0 & 0 \\ -\theta_1 & 0 & . & 0 & 0 \\ . & . & . & . & . \\ 0 & 0 & . & 0 & \theta_r \\ 0 & 0 & . & -\theta_r & 0 \end{pmatrix} \qquad r = \frac{N}{2} = SO(N) \text{ rank}$$

Using constant gauge transformations $\Longrightarrow$ a_{ij} is block diagonal

$$\hat{a}_{ij} = \begin{pmatrix} 0 & \theta_1 & . & 0 & 0 \\ -\theta_1 & 0 & . & 0 & 0 \\ . & . & . & . & . \\ 0 & 0 & . & 0 & \theta_r \\ 0 & 0 & . & -\theta_r & 0 \end{pmatrix} \qquad r = \frac{N}{2} = SO(N) \text{ rank}$$

Using large gauge transformation $\Longrightarrow$ $\theta_r \sim \theta_r + 2\pi n \quad \theta_k$=angles

Using constant gauge transformations $\Rightarrow$ a_{ij} is block diagonal

$$\hat{a}_{ij} = \begin{pmatrix} 0 & \theta_1 & . & 0 & 0 \\ -\theta_1 & 0 & . & 0 & 0 \\ . & . & . & . & . \\ 0 & 0 & . & 0 & \theta_r \\ 0 & 0 & . & -\theta_r & 0 \end{pmatrix} \qquad r = \frac{N}{2} = SO(N) \text{ rank}$$

Using large gauge transformation $\Rightarrow$ $\theta_r \sim \theta_r + 2\pi n \quad \theta_k$=angles

- $$\text{Det}' \left(\partial_\tau - \hat{a}_{adj}\right)_{PBC} = \prod_{k<l} (2\sin\tfrac{\theta_k+\theta_l}{2})^2 (2\sin\tfrac{\theta_k-\theta_l}{2})^2$$

Using constant gauge transformations $\Rightarrow$ a_{ij} is block diagonal

$$\hat{a}_{ij} = \begin{pmatrix} 0 & \theta_1 & . & 0 & 0 \\ -\theta_1 & 0 & . & 0 & 0 \\ . & . & . & . & . \\ 0 & 0 & . & 0 & \theta_r \\ 0 & 0 & . & -\theta_r & 0 \end{pmatrix} \qquad r = \frac{N}{2} = SO(N) \text{ rank}$$

Using large gauge transformation $\Rightarrow$ $\theta_r \sim \theta_r + 2\pi n \quad \theta_k$=angles

- ◇ $\text{Det}'\left(\partial_\tau - \hat{a}_{adj}\right)_{PBC} = \prod_{k<l}(2\sin\frac{\theta_k+\theta_l}{2})^2(2\sin\frac{\theta_k-\theta_l}{2})^2$
- ♭ $\text{Det}\left(\partial_\tau - \hat{a}_{vec}\right)_{ABC}^{\frac{D}{2}-1} = \prod_{k=1}^{r}(2\cos\frac{\theta_k}{2})^{D-2}$

Using constant gauge transformations $\Rightarrow$ a_{ij} is block diagonal

$$\hat{a}_{ij} = \begin{pmatrix} 0 & \theta_1 & . & 0 & 0 \\ -\theta_1 & 0 & . & 0 & 0 \\ . & . & . & . & . \\ 0 & 0 & . & 0 & \theta_r \\ 0 & 0 & . & -\theta_r & 0 \end{pmatrix} \qquad r = \frac{N}{2} = SO(N) \text{ rank}$$

Using large gauge transformation $\Rightarrow$ $\theta_r \sim \theta_r + 2\pi n \quad \theta_k$=angles

- ⋄ $\mathrm{Det}' \left(\partial_\tau - \hat{a}_{adj}\right)_{PBC} = \prod_{k<l} (2 \sin \frac{\theta_k + \theta_l}{2})^2 (2 \sin \frac{\theta_k - \theta_l}{2})^2$
- ♭ $\mathrm{Det} \left(\partial_\tau - \hat{a}_{vec}\right)_{ABC}^{\frac{D}{2}-1} = \prod_{k=1}^{r} (2 \cos \frac{\theta_k}{2})^{D-2}$
- ♮ $K_N = \frac{2}{2^r} \frac{1}{r!}$

Using constant gauge transformations $\Longrightarrow$ a_{ij} is block diagonal

$$\hat{a}_{ij} = \begin{pmatrix} 0 & \theta_1 & . & 0 & 0 \\ -\theta_1 & 0 & . & 0 & 0 \\ . & . & . & . & . \\ 0 & 0 & . & 0 & \theta_r \\ 0 & 0 & . & -\theta_r & 0 \end{pmatrix} \qquad r = \frac{N}{2} = SO(N) \text{ rank}$$

Using large gauge transformation $\Longrightarrow$ $\theta_r \sim \theta_r + 2\pi n$ $\quad \theta_k$=angles

- ◇ $$\mathrm{Det}'\left(\partial_\tau - \hat{a}_{adj}\right)_{PBC} = \prod_{k<l}(2\sin\frac{\theta_k+\theta_l}{2})^2(2\sin\frac{\theta_k-\theta_l}{2})^2$$
- ♭ $$\mathrm{Det}\left(\partial_\tau - \hat{a}_{vec}\right)^{\frac{D}{2}-1}_{ABC} = \prod_{k=1}^{r}(2\cos\frac{\theta_k}{2})^{D-2}$$
- ♮ $$K_N = \frac{2}{2^r}\frac{1}{r!}$$

Using constant SO(n) transformation one can only change signs to pairs of angles simultaneously

$$\begin{pmatrix} +\Theta_i & . & 0 \\ . & . & . \\ 0 & . & +\Theta_j \end{pmatrix}$$

Using constant gauge transformations $\Longrightarrow$ a_{ij} is block diagonal

$$\hat{a}_{ij} = \begin{pmatrix} 0 & \theta_1 & . & 0 & 0 \\ -\theta_1 & 0 & . & 0 & 0 \\ . & . & . & . & . \\ 0 & 0 & . & 0 & \theta_r \\ 0 & 0 & . & -\theta_r & 0 \end{pmatrix} \qquad r = \frac{N}{2} = SO(N) \text{ rank}$$

Using large gauge transformation $\Longrightarrow$ $\theta_r \sim \theta_r + 2\pi n \quad \theta_k$=angles

- ◇ $\mathrm{Det}'\left(\partial_\tau - \hat{a}_{adj}\right)_{PBC} = \prod_{k<l}(2\sin\frac{\theta_k+\theta_l}{2})^2(2\sin\frac{\theta_k-\theta_l}{2})^2$
- ♭ $\mathrm{Det}\left(\partial_\tau - \hat{a}_{vec}\right)^{\frac{D}{2}-1}_{ABC} = \prod_{k=1}^{r}(2\cos\frac{\theta_k}{2})^{D-2}$
- ♮ $K_N = \frac{2}{2^r}\frac{1}{r!}$

Using constant SO(n) transformation one can only change signs to pairs of angles simultaneously

$$\begin{pmatrix} -\Theta_i & . & 0 \\ . & . & . \\ 0 & . & -\Theta_j \end{pmatrix}$$

Using constant gauge transformations $\Rightarrow$ a_{ij} is block diagonal

$$\hat{a}_{ij} = \begin{pmatrix} 0 & \theta_1 & . & 0 & 0 \\ -\theta_1 & 0 & . & 0 & 0 \\ . & . & . & . & . \\ 0 & 0 & . & 0 & \theta_r \\ 0 & 0 & . & -\theta_r & 0 \end{pmatrix} \qquad r = \frac{N}{2} = SO(N) \text{ rank}$$

Using large gauge transformation $\Rightarrow$ $\theta_r \sim \theta_r + 2\pi n \quad \theta_k$=angles

- $\diamond$ $\quad \mathrm{Det}'\left(\partial_\tau - \hat{a}_{adj}\right)_{PBC} = \prod_{k<l}(2\sin\frac{\theta_k+\theta_l}{2})^2(2\sin\frac{\theta_k-\theta_l}{2})^2$
- $\flat$ $\quad \mathrm{Det}\left(\partial_\tau - \hat{a}_{vec}\right)^{\frac{D}{2}-1}_{ABC} = \prod_{k=1}^{r}(2\cos\frac{\theta_k}{2})^{D-2}$
- $\natural$ $\quad K_N = \frac{2}{2^r}\frac{1}{r!}$

this factor is due to the fact that with a $SO(N)$ transformation one can permute the angles θ_i

$$Dof(D,N=2r) = \frac{2}{2^r r!}\prod_{k=1}^{r}\int_0^{2\pi}\frac{d\theta_k}{2\pi}(2cos\frac{\theta}{2})^{D-2} \prod_{k<l}[(2cos\frac{\theta_k}{2})^2-(2cos\frac{\theta_l}{2})^2]$$

- ◇ $\mathrm{Det}'\,(\partial_\tau - \hat{a}_{adj})_{PBC} = \prod_{k<l}(2\sin\frac{\theta_k+\theta_l}{2})^2(2\sin\frac{\theta_k-\theta_l}{2})^2$
- ♭ $\mathrm{Det}\,(\partial_\tau - \hat{a}_{vec})^{\frac{D}{2}-1}_{ABC} = \prod_{k=1}^{r}(2\cos\frac{\theta_k}{2})^{D-2}$
- ♮ $K_N = \frac{2}{2^r}\frac{1}{r!}$

in a way somewhat similar to the even case one finds:

Odd N

in a way somewhat similar to the even case one finds:

$$\hat{a}_{ij} = \begin{pmatrix} 0 & \theta_1 & . & 0 & 0 & 0 \\ -\theta_1 & 0 & . & 0 & 0 & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & \theta_r & 0 \\ 0 & 0 & 0 & -\theta_r & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad r = \frac{N-1}{2} = SO(N) \text{ rank}$$

in a way somewhat similar to the even case one finds:

$$\hat{a}_{ij} = \begin{pmatrix} 0 & \theta_1 & . & 0 & 0 & 0 \\ -\theta_1 & 0 & . & 0 & 0 & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & \theta_r & 0 \\ 0 & 0 & 0 & -\theta_r & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad r = \frac{N-1}{2} = SO(N) \text{ rank}$$

- $\mathrm{Det}'\left(\partial_\tau - \hat{a}_{adj}\right)_{PBC} = 64 \prod_{k=1}^{r} \sin^2 \frac{\theta_k}{2} \prod_{k<l} \sin^2 \frac{\theta_k + \theta_l}{2} \sin^2 \frac{\theta_k - \theta_l}{2}$

in a way somewhat similar to the even case one finds:

$$\hat{a}_{ij} = \begin{pmatrix} 0 & \theta_1 & . & 0 & 0 & 0 \\ -\theta_1 & 0 & . & 0 & 0 & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & \theta_r & 0 \\ 0 & 0 & 0 & -\theta_r & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad r = \frac{N-1}{2} = SO(N) \text{ rank}$$

- $\mathrm{Det}'\left(\partial_\tau - \hat{a}_{adj}\right)_{PBC} = 64\prod_{k=1}^{r}\sin^2\frac{\theta_k}{2}\prod_{k<l}\sin^2\frac{\theta_k+\theta_l}{2}\sin^2\frac{\theta_k-\theta_l}{2}$
- $\mathrm{Det}\left(\partial_\tau - \hat{a}_{vec}\right)_{ABC}^{\frac{D}{2}-1} = 2^{\frac{D}{2}-1}\prod_{k=1}^{r}(2\cos\frac{\theta_k}{2})^{D-2}$

in a way somewhat similar to the even case one finds:

$$\hat{a}_{ij} = \begin{pmatrix} 0 & \theta_1 & . & 0 & 0 & 0 \\ -\theta_1 & 0 & . & 0 & 0 & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & \theta_r & 0 \\ 0 & 0 & 0 & -\theta_r & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad r = \frac{N-1}{2} = SO(N) \text{ rank}$$

- $\text{Det}'\left(\partial_\tau - \hat{a}_{adj}\right)_{PBC} = 64 \prod_{k=1}^{r} \sin^2 \frac{\theta_k}{2} \prod_{k<l} \sin^2 \frac{\theta_k+\theta_l}{2} \sin^2 \frac{\theta_k-\theta_l}{2}$
- $\text{Det}\left(\partial_\tau - \hat{a}_{vec}\right)^{\frac{D}{2}-1}_{ABC} = 2^{\frac{D}{2}-1} \prod_{k=1}^{r} (2\cos\frac{\theta_k}{2})^{D-2}$
- $K_N = \frac{1}{2^r}\frac{1}{r!}$

in a way somewhat similar to the even case one finds:

$$\hat{a}_{ij} = \begin{pmatrix} 0 & \theta_1 & . & 0 & 0 & 0 \\ -\theta_1 & 0 & . & 0 & 0 & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & \theta_r & 0 \\ 0 & 0 & 0 & -\theta_r & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad r = \frac{N-1}{2} = SO(N) \text{ rank}$$

◇ $\mathrm{Det}'\,(\partial_\tau - \hat{a}_{adj})_{PBC} = 64 \prod_{k=1}^{r} \sin^2 \frac{\theta_k}{2} \prod_{k<l} \sin^2 \frac{\theta_k + \theta_l}{2} \sin^2 \frac{\theta_k - \theta_l}{2}$

♭ $\mathrm{Det}\,(\partial_\tau - \hat{a}_{vec})_{ABC}^{\frac{D}{2}-1} = 2^{\frac{D}{2}-1} \prod_{k=1}^{r} (2\cos\frac{\theta_k}{2})^{D-2}$

♮ $K_N = \frac{1}{2^r}\frac{1}{r!}$

the factor 2 that appeared in the even case is not included here since one can always reflect the last coordinate to obtain an $SO(N)$ transformation that changes θ_k into $-\theta_k$

$$\begin{pmatrix} +\Theta_i & . & 0 \\ . & . & . \\ 0 & . & +\Theta_j \end{pmatrix}$$

in a way somewhat similar to the even case one finds:

$$\hat{a}_{ij} = \begin{pmatrix} 0 & \theta_1 & . & 0 & 0 & 0 \\ -\theta_1 & 0 & . & 0 & 0 & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & \theta_r & 0 \\ 0 & 0 & 0 & -\theta_r & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad r = \frac{N-1}{2} = SO(N) \text{ rank}$$

- ◇ $\mathrm{Det}'\left(\partial_\tau - \hat{a}_{adj}\right)_{PBC} = 64 \prod_{k=1}^{r} \sin^2 \frac{\theta_k}{2} \prod_{k<l} \sin^2 \frac{\theta_k+\theta_l}{2} \sin^2 \frac{\theta_k-\theta_l}{2}$
- ♭ $\mathrm{Det}\left(\partial_\tau - \hat{a}_{vec}\right)_{ABC}^{\frac{D}{2}-1} = 2^{\frac{D}{2}-1} \prod_{k=1}^{r} (2\cos\frac{\theta_k}{2})^{D-2}$
- ♮ $K_N = \frac{1}{2^r}\frac{1}{r!}$

the factor 2 that appeared in the even case is not included here since one can always reflect the last coordinate to obtain an $SO(N)$ transformation that changes θ_k into $-\theta_k$

$$\begin{pmatrix} -\Theta_i & . & 0 \\ . & . & . \\ 0 & . & +\Theta_j \end{pmatrix}$$

in a way somewhat similar to the even case one finds:

$$\hat{a}_{ij} = \begin{pmatrix} 0 & \theta_1 & . & 0 & 0 & 0 \\ -\theta_1 & 0 & . & 0 & 0 & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & \theta_r & 0 \\ 0 & 0 & 0 & -\theta_r & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad r = \frac{N-1}{2} = SO(N) \text{ rank}$$

- ◇ $\mathrm{Det}'\left(\partial_\tau - \hat{a}_{adj}\right)_{PBC} = 64\prod_{k=1}^{r}\sin^2\frac{\theta_k}{2}\prod_{k<l}\sin^2\frac{\theta_k+\theta_l}{2}\sin^2\frac{\theta_k-\theta_l}{2}$
- ♭ $\mathrm{Det}\left(\partial_\tau - \hat{a}_{vec}\right)^{\frac{D}{2}-1}_{ABC} = 2^{\frac{D}{2}-1}\prod_{k=1}^{r}(2\cos\frac{\theta_k}{2})^{D-2}$
- ♮ $K_N = \frac{1}{2^r}\frac{1}{r!}$

the factor 2 that appeared in the even case is not included here since one can always reflect the last coordinate to obtain an $SO(N)$ transformation that changes θ_k into $-\theta_k$

$$\begin{pmatrix} +\Theta_i & . & 0 \\ . & . & . \\ 0 & . & -\Theta_j \end{pmatrix}$$

in a way somewhat similar to the even case one finds:

$$\hat{a}_{ij} = \begin{pmatrix} 0 & \theta_1 & . & 0 & 0 & 0 \\ -\theta_1 & 0 & . & 0 & 0 & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & \theta_r & 0 \\ 0 & 0 & 0 & -\theta_r & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad r = \frac{N-1}{2} = SO(N) \text{ rank}$$

- ◇ $\text{Det}'\left(\partial_\tau - \hat{a}_{adj}\right)_{PBC} = 64 \prod_{k=1}^{r} \sin^2 \frac{\theta_k}{2} \prod_{k<l} \sin^2 \frac{\theta_k + \theta_l}{2} \sin^2 \frac{\theta_k - \theta_l}{2}$

- ♭ $\text{Det}\left(\partial_\tau - \hat{a}_{vec}\right)_{ABC}^{\frac{D}{2}-1} = 2^{\frac{D}{2}-1} \prod_{k=1}^{r} (2\cos\frac{\theta_k}{2})^{D-2}$

- ♮ $K_N = \frac{1}{2^r} \frac{1}{r!}$

due to the permutation

$$Dof(D,N=2r+1) \quad = \quad \frac{2^{\frac{D}{2}-1}}{2^r r!}\prod_{k=1}^{r}\int_0^{2\pi}\frac{d\theta_k}{2\pi}(2cos\frac{\theta}{2})^{D-2}(2sin\frac{\theta}{2})^2 \prod_{k<l}[(2sin\frac{\theta_k+\theta_l}{2})^2(2sin\frac{\theta_k-\theta_l}{2})^2]$$

- $\diamond$ $\mathrm{Det}'\left(\partial_\tau - \hat{a}_{adj}\right)_{PBC} = 64\prod_{k=1}^{r}\sin^2\frac{\theta_k}{2}\prod_{k<l}\sin^2\frac{\theta_k+\theta_l}{2}\sin^2\frac{\theta_k-\theta_l}{2}$
- $\flat$ $\mathrm{Det}\left(\partial_\tau - \hat{a}_{vec}\right)^{\frac{D}{2}-1}_{ABC} = 2^{\frac{D}{2}-1}\prod_{k=1}^{r}(2\cos\frac{\theta_k}{2})^{D-2}$
- $\natural$ $K_N = \frac{1}{2^r}\frac{1}{r!}$

We have used "ortoghonal polynomial method" to compute the integral:

We have used "ortoghonal polynomial method" to compute the integral:

$$Dof(2d,2r) = 2^{r-1}\frac{(2d-2)!}{[(d-1)!]^2}\prod_{k=1}^{r-1}\frac{k\ (2k-1)!\ (2k+2d-3)!}{(2k+d-2)!\ (2k+d-1)!}$$

$$Dof(2d,2r+1) = \frac{2^{d-2+r}(2d-2)!}{d[(d-1)!]^2}\prod_{k=1}^{r-1}\frac{(k+d-1)\ (2k+1)!\ (2k+2d-3)!}{(2k+d-1)!\ (2k+d)!}$$

We have used "ortoghonal polynomial method" to compute the integral:

$$Dof(2d,2r) = 2^{r-1}\frac{(2d-2)!}{[(d-1)!]^2}\prod_{k=1}^{r-1}\frac{k\;(2k-1)!\;(2k+2d-3)!}{(2k+d-2)!\;(2k+d-1)!}$$

$$Dof(2d,2r+1) = \frac{2^{d-2+r}(2d-2)!}{d[(d-1)!]^2}\prod_{k=1}^{r-1}\frac{(k+d-1)\;(2k+1)!\;(2k+2d-3)!}{(2k+d-1)!\;(2k+d)!}$$

OBSERVATION:

We have used "ortoghonal polynomial method" to compute the integral:

$$Dof(2d,2r) = 2^{r-1}\frac{(2d-2)!}{[(d-1)!]^2}\prod_{k=1}^{r-1}\frac{k\,(2k-1)!\,(2k+2d-3)!}{(2k+d-2)!\,(2k+d-1)!}$$

$$Dof(2d,2r+1) = \frac{2^{d-2+r}(2d-2)!}{d[(d-1)!]^2}\prod_{k=1}^{r-1}\frac{(k+d-1)\,(2k+1)!\,(2k+2d-3)!}{(2k+d-1)!\,(2k+d)!}$$

OBSERVATION:

$$Dof(2d+1,N)=0 \qquad \forall N>1$$

We have used "ortoghonal polynomial method" to compute the integral:

$$Dof(2d, 2r) = 2^{r-1} \frac{(2d-2)!}{[(d-1)!]^2} \prod_{k=1}^{r-1} \frac{k\ (2k-1)!\ (2k+2d-3)!}{(2k+d-2)!\ (2k+d-1)!}$$

$$Dof(2d, 2r+1) = \frac{2^{d-2+r}(2d-2)!}{d[(d-1)!]^2} \prod_{k=1}^{r-1} \frac{(k+d-1)\ (2k+1)!\ (2k+2d-3)!}{(2k+d-1)!\ (2k+d)!}$$

SPECIAL CASES:

We have used "ortoghonal polynomial method" to compute the integral:

$$Dof(2d,2r) = 2^{r-1}\frac{(2d-2)!}{[(d-1)!]^2}\prod_{k=1}^{r-1}\frac{k\,(2k-1)!\,(2k+2d-3)!}{(2k+d-2)!\,(2k+d-1)!}$$

$$Dof(2d,2r+1) = \frac{2^{d-2+r}(2d-2)!}{d[(d-1)!]^2}\prod_{k=1}^{r-1}\frac{(k+d-1)\,(2k+1)!\,(2k+2d-3)!}{(2k+d-1)!\,(2k+d)!}$$

SPECIAL CASES:

$$Dof(2,N) = 1, \quad \forall N$$
$$Dof(4,N) = 2, \quad \forall N$$
$$Dof(2d,2) = \frac{(2d-2)!}{[(d-1)!]^2}$$
$$Dof(2d,3) = \frac{2^{d-1}}{d}\frac{(2d-2)!}{[(d-1)!]^2}$$

Pashnev and Sorokin model

For $N = 4$ the gauge group is $SO(4) = SU(2) \times SU(2)$. In the literature, by Pashnev and Sorokin, also the model with a factor $SU(2)$ gauged and the other $SU(2)$ left as a global symmetry has been considered

For $N = 4$ the gauge group is $SO(4) = SU(2) \times SU(2)$. In the literature, by Pashnev and Sorokin, also the model with a factor $SU(2)$ gauged and the other $SU(2)$ left as a global symmetry has been considered.

Quantization of this model seems to produce an inconsistency:

For $N = 4$ the gauge group is $SO(4) = SU(2) \times SU(2)$. In the literature, by Pashnev and Sorokin, also the model with a factor $SU(2)$ gauged and the other $SU(2)$ left as a global symmetry has been considered.

Quantization of this model seems to produce an inconsistency:

Dirac operatorial quantization

seems to describe

- three scalar
- a spin 2

$Dof(D = 4, PS) = 5$

Gupta Bleuler quantization

seems to describe

- two scalar
- a spin 2

$Dof(D = 4, PS) = 4$

In this case the number of degrees of freedom is given by

In this case the number of degrees of freedom is given by

$$Dof(D,PS) \quad = \quad \frac{1}{2}\int_0^{2\pi} \frac{d\theta}{2\pi}\,(2\cos\frac{\theta}{2})^{2(D-2)}(2\sin\theta)^2 =$$

In this case the number of degrees of freedom is given by

$$Dof(D, PS) \quad = \quad \frac{1}{2}\int_0^{2\pi} \frac{d\theta}{2\pi}\,(2\cos\frac{\theta}{2})^{2(D-2)}(2\sin\theta)^2 = \frac{2^{D-1}(2D-3)!!}{D!}$$

In this case the number of degrees of freedom is given by

$$Dof(D,PS) \quad = \quad \frac{1}{2}\int_0^{2\pi} \frac{d\theta}{2\pi}\,(2\cos\frac{\theta}{2})^{2(D-2)}(2\sin\theta)^2 = \frac{2^{D-1}(2D-3)!!}{D!}$$

D	Dof
2	1
3	2
4	5
5	14
.	.
.	.

In this case the number of degrees of freedom is given by

$$Dof(D, PS) = \frac{1}{2}\int_0^{2\pi} \frac{d\theta}{2\pi}\,(2\cos\frac{\theta}{2})^{2(D-2)}(2\sin\theta)^2 = \frac{2^{D-1}(2D-3)!!}{D!}$$

D	Dof
2	1
3	2
4	5
5	14
.	.
.	.

OBSERVATION:

In this case the number of degrees of freedom is given by

$$Dof(D,PS) = \frac{1}{2}\int_0^{2\pi}\frac{d\theta}{2\pi}\,(2\cos\frac{\theta}{2})^{2(D-2)}(2\sin\theta)^2 = \frac{2^{D-1}(2D-3)!!}{D!}$$

D	Dof
2	1
3	2
4	5
5	14
.	.
.	.

OBSERVATION:

- Our analysis gives 5 degrees of freedom in $D=4$, corresponding presumably to a graviton and three scalars.

In this case the number of degrees of freedom is given by

$$Dof(D,PS) = \frac{1}{2}\int_0^{2\pi}\frac{d\theta}{2\pi}(2\cos\frac{\theta}{2})^{2(D-2)}(2\sin\theta)^2 = \frac{2^{D-1}(2D-3)!!}{D!}$$

D	Dof
2	1
3	2
4	5
5	14
.	.
.	.

OBSERVATION:

- Our analysis gives 5 degrees of freedom in $D=4$, corresponding presumably to a graviton and three scalars.
- The latter is non vanishing for *any* space-time dimension D, implying that - in this case - odd-dimensional models are non empty.

- We have studied the one-loop quantization of spinning particles with a gauged SO(N) extended supergravity on the worldline, propagating on flat target space

Conclusion

- We have studied the one-loop quantization of spinning particles with a gauged SO(N) extended supergravity on the worldline, propagating on flat target space
- We have obtained the measure on the moduli space of the SO(N)

- We have studied the one-loop quantization of spinning particles with a gauged SO(N) extended supergravity on the worldline, propagating on flat target space
- We have obtained the measure on the moduli space of the SO(N)
- and we have used it to compute the propagating physical degrees of freedom

Conclusion

- We have studied the one-loop quantization of spinning particles with a gauged SO(N) extended supergravity on the worldline, propagating on flat target space
- We have obtained the measure on the moduli space of the SO(N)
- and we have used it to compute the propagating physical degrees of freedom
- We have also studied the $N = 4$ case with an $SU(2)$ symmetry left as a global one and shown that this propagate 5 degrees of freedom in $D = 4$ corresponding probably to 3 scalars and a spin 2 field

- Would be interesting to study the one-loop partition function of this model coupled to Ads background

- Would be interesting to study the one-loop partition function of this model coupled to Ads background
- and study from the worldline point of view how one could introduce more general couplings.

- Would be interesting to study the one-loop partition function of this model coupled to Ads background
- and study from the worldline point of view how one could introduce more general couplings.
- One could also enlarge the analysis to osp(2p,Q) spinning particle and try to understand if they are related to partially massless HS in AdS

THANK YOU FOR YOUR ATTENTION