Acausal Theories of Quantum Matter Coupled with Classical Gravity

hep-th/0611131

March 20, 2007
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Introduction

- Gravity: To quantize or not to quantize?
  - There are arguments that suggest gravity must be a quantum theory, but they are not conclusive.
  - There no experimental data showing a quatum behavior.
  - Quantize the Gravitational Field?[Callender& Hugget(2001)]
- Quantum gravity as QFT is not renormalizable.
  - WE do not know how to renormalize it.
  - Study alternatives to QFT
  - Consider coupling classical gravity with quantum field
- Although the ultimate theory of gravity was a quantum one, it still has sense to study quantum matter in a classical background as its results could generalize.

Coupling with Classical Gravity

$$\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left(R + aR^2 + bR_{\mu\nu}R^{\mu\nu}\right) + S_m$$

Ex. $S_m = \int d^4x\sqrt{-g}\,F^a_{\mu\nu}F^{a\mu\nu}$

- If we left gravity as classical,
  - Pure gravitational terms: by power counting and diffeomorphism invariance, only 3 counterterms can arise: $R^2;\ R_{\mu\nu}R^{\mu\nu};\ R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$
    - In 4 dimension only two remains due Gauss-Bonnet identity.
  - Matter: Same terms.
- As usual we can add required terms with independent constats to renormalize
- But it means to have a Higher derivative theory (In the gravitational sector)

Higher Derivative

- Problems at quantum and classical levels.
- Quantum: Stelle (1976)

$$\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left(R + aR^2 + bR_{\mu\nu}R^{\mu\nu}\right) + S_m$$

  - In a appropiate gauge, the gravitational propagator goes as $\approx \frac{1}{p^4}$
  - Improve the power counting.
  - Even in presence of matter, it is renormalizable.
  - But is unsatisfactory as physical theory as it propagate ghosts. It is not Unitary.
- Classically:
  - Instabilities: Runaway solutions
- Higher Derivative theories share this problems; not good classically (instabilities) neither as quantum theories (unitarity).

Coupling Matter to Gravity

- We do not want a HD theory. $R_{\mu\nu}R^{\mu\nu}; R^2?$
- What can we do with the divergences
- Instead include them as counterterms, transform them away by a field redefinition.

$$\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left(R + AR^2 + BR_{\mu\nu}R^{\mu\nu}\right) \longrightarrow \int d^4x\sqrt{-g'}\frac{R'}{2\kappa^2}$$

$$g'_{\mu\nu} \equiv g'_{\mu\nu}(g_{\mu\nu}; A; B)$$

- Note:
  - Also the matter sector will be modified: new vertices appear. It's a different theory

$$S_m[g;\varphi] \longrightarrow S'_m[g';\varphi]$$

- The idea is to implement a MAP that connect the renormalization of two different theories.

Map

$$\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left\{R + a_B R^2 + b_B R_{\mu\nu}R^{\mu\nu}\right\} + S_m[g;\varphi_B;\lambda_B]$$

$$\longrightarrow \quad g' \equiv G(g; a_B; b_B)$$

$$\int d^4x\sqrt{-g'}\,\frac{R'}{2\kappa^2} + S'_m[g';\varphi_B;\lambda_B]$$

Renormalization

Renormalization

$$\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left\{R + aR^2 + bR_{\mu\nu}R^{\mu\nu}\right\} + S_m[g;\varphi;\lambda]$$

$$\longrightarrow \quad g'' \equiv G(g; a; b)$$

$$\int d^4x\sqrt{-g''}\,\frac{R''}{2\kappa^2} + S'_m[g'';\varphi;\lambda]$$

$$\Rightarrow \quad g' \equiv g_B$$

Obs: g es renormalizado aunque es clasico!

Map perturbativo

- The redefinition of the metric can be written explicitly in a perturbative way. (Anselmi, **hep-th/0605205)**
- It uses this general property

$$S[\phi] + \frac{\delta S}{\delta \phi_i} F_{ij} \frac{\delta S}{\delta \phi_j} = S[\phi']$$

$$\phi'_i \equiv \phi'_i(\phi) = \phi_i + \Delta_{ij} \frac{\delta S}{\delta \phi_j}$$

$$\Delta_{ij} = \Delta_{ij}(F)$$

- In our case,

$$\underbrace{\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[R + \underbrace{aR^2 + bR_{\mu\nu}R^{\mu\nu}}_{\frac{\delta S}{\delta\phi_i}F_{ij}\frac{\delta S}{\delta\phi_j}}\right]}_{S} = \int d^4x\sqrt{-g'}\frac{R'}{2\kappa^2}$$

$$g'_{\mu\nu} \equiv g'_{\mu\nu}(g_{\mu\nu})$$

$$\begin{aligned} g'_{\mu\nu} = {} & g_{\mu\nu} + aR_{\mu\nu} - \frac{a+2b}{2}g_{\mu\nu}R + \frac{3a^2}{4}\Box R_{\mu\nu} - \frac{3a(a+2b)}{4}\nabla_\mu\nabla_\nu R - abRR_{\mu\nu} + \frac{1}{2}a^2R_\mu^{\ \lambda}R_{\lambda\nu} \\ & - \frac{3}{2}a^2R_{\mu\alpha\nu\beta}R^{\alpha\beta} + \frac{1}{8}g_{\mu\nu}\left[3(a+2b)(a+6b)\Box R + 2a(3a+4b)R_{\alpha\beta}R^{\alpha\beta} - a^2R^2\right] \\ & + O(a^3, a^2b, ab^2) \end{aligned}$$

Note this is a series in derivatives of g

Causality Violations

- Where it comes from? Example:

$$\mathcal{L}(\varphi;J)=\frac{1}{2}(\partial_\mu\varphi)(\partial^\mu\varphi)+\frac{1}{2}\alpha^2(\Box\varphi)^2+\varphi J$$

$$=\mathcal{L}'(\varphi';J)=(\partial_\mu\varphi')(\partial^\mu\varphi')+\varphi'\underbrace{\left(\frac{1}{\sqrt{1-\alpha^2\Box}}J\right)}_{\int d^4x'\,C(x-x')J(x')}$$

$$\varphi'=\sqrt{1-\alpha^2\Box}\,\varphi$$

  - Is the green function $C(x-x')$ that makes to integrate the source over points x' outside the past of x
  - That means, for example, that we need to know the source in the future to determinate the interaction in the present
  - Outside a radius of order $|\alpha|$ the green function is vanishing or rapidly oscillating.
  - The resummation of the series in derivatives is therefore the responsible of the causality violations at high energy.

Causality Violations

- The theories obtained from a map of this kind present causality problems of order of its parameters (a,b)
- The main feature that allows us demostrate the renormalizability of a AC theory from the HD is that the map depends only on g, which is classical

- In summary, we have
  - A map that relate the renormalization of two inequivalent theories.
    - One is higher derivate, has instabilities.
    - The other has causality violations at high energy.
  - The scale of the violation is given by the parameters (a,b).
  - The AC theory has new vertices coupling matter and gravity.

- After the map we are left with a theory that is acausal and has the form

$$S_{AC} = \frac{1}{2\kappa^2}\int d^4x \sqrt{-g}\,R + S_m(\varphi; g; \psi) + \Delta S_m(\varphi; g; \psi')$$

$$\Delta S_m = \int d^4x \sqrt{-g}\left(-\frac{a}{2} T^{\mu\nu} R_{\mu\nu} + \frac{1}{4}(a+2b) R T\right) + O(a^2; b^2; ab)$$

  - These theories have a term (the head of the deformation) that is proportional to the stress-tensor and the Ricci tensor.
  - Now we will use the map to prove the renormalization of more general theories.

First generalization

$$S_{AC} = \frac{1}{2\kappa^2}\int d^4x \sqrt{-g}\,R + S_m(\varphi; g; \lambda) + \Delta S_m(\varphi; g; \lambda')$$

$\Delta S_m(\varphi; g; \lambda')$

- Matter operators of dimensionalities less than or equal to 4
- Coupled to arbitrary functions of th metric (with the symmetries)

$$\Delta_1 S_m = \int d^4x \sqrt{-g}\, f_{\mu\nu}(g_{\rho\sigma})\, T^{\mu\nu}_m$$

$$\Delta_2 S_m = O(R^2_{\mu\nu}; \varphi)$$

$$f_{\mu\nu}(g_{\rho\sigma}) = \lambda'_1 R_{\mu\nu} + \lambda'_2 R g_{\mu\nu} + O(\nabla^2 R_{\rho\sigma})$$

Using the map is possible demostrate the renormalizability of this theory.

First generalization

Differential operator, can depend on R and the Dimensionalfull constants

$$S_{HD} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-g}R_{\mu\nu}T^{\mu\nu\rho\sigma}(\lambda')R_{\rho\sigma}$$

$$+ S_m(\varphi; g; \lambda) + \Delta S_m(\varphi; g; \lambda; \lambda')$$

Same restictions as before

- Analyzing generically all possible Feynman diagrams, inductively in loop expansion, one find that
  - Pure gravitational counterterms are squarly proportional to Ricci
  - The same form of matter term already present.
- This theory is renormalizable. The divergences can be eliminated order by order by redefinition of couplings and fields.
- Considering that in the map, the variation of g is proportional to Ricci, one demonstrates that this theory and the previous are connected by a map.

Second generalization

- All kinetic terms are proportional to Ricci^2 (Always possible, using Bianchi identities and integration by parts) $R_{\alpha\beta\gamma\delta}\nabla_\lambda\cdots\nabla^\epsilon R_{\rho\sigma\mu\nu}$

Only vertices

$$S_{HD} = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} + \underbrace{V(g) + R_{\mu\nu}T^{\mu\nu\rho\sigma}R_{\rho\sigma}}_{\text{Only vertices}}\right] + S_m(\varphi; g; \lambda) + \int d^4x\sqrt{-g}\sum_I O_I(\varphi; g)K^I(g)$$

Matter operator with dimensionality less than or equal to 4

Tensorial functions of the metric

- Renormalizable. No matter operator of dimension greater than for can be generated. Gravitational sector is the most general.
- Using the map, an acausal can be derived.
- Some consistent reduction of couplings can be applied.

Example 1:Acausal Einstein-Yang-Mills

$$\frac{L_{EYM-HD}}{\sqrt{-g}} = \frac{1}{2\kappa^2}\left\{R + aR^2 + bR_{\mu\nu}R^{\mu\nu}\right\} - \frac{1}{4\alpha}F^a_{\mu\nu}F^{a\mu\nu}$$

Applying the map:

$$\frac{L_{EYM-AC}}{\sqrt{-g}} = \frac{1}{2\kappa^2}R - \frac{1}{4\alpha}\left\{F^a_{\mu\nu}F^{a\mu\nu}H(g) + T_{\mu\nu}K^{\mu\nu}(g) + \Upsilon_{\mu\nu\rho\sigma}L^{\mu\nu\rho\sigma}(g)\right\}$$

With the matter operators

$$T_{\mu\nu} = -F^a_{\mu\alpha}F^{a\alpha}_{\nu} + \frac{1}{4}g_{\mu\nu}F^2;$$

$$\Upsilon_{\mu\nu\rho\sigma} = F^a_{\mu\nu}F^a_{\rho\sigma} + \frac{1}{2}(g_{\mu\rho}T_{\nu\sigma} - g_{\mu\sigma}T_{\nu\rho} - g_{\nu\rho}T_{\mu\sigma} + g_{\nu\sigma}T_{\mu\rho}) - \frac{1}{12}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})F^2$$

And these functions are determined by the map

$$H(g) = 1 + \frac{1}{6}a^2R_{\alpha\beta}R^{\alpha\beta} - \frac{1}{24}a^2R^2; \qquad L^{\mu\nu\rho\sigma} = a^2R^{\mu\rho}R^{\nu\sigma};$$

$$K^{\mu\nu}(g) = 2aR^{\mu\nu} + \frac{3}{2}a^2\Box R^{\mu\nu} + a^2RR^{\mu\nu} - 3a^2R^{\mu\alpha\nu\beta}R_{\alpha\beta} - \frac{3a(a+2b)}{2}\nabla^{\mu}\nabla^{\nu}R$$

Si las dejamos arbitrarias tenemos un ejemplo de esta generalizacion

Consistent reduction of couplings

- One way of restrict terms: Take a power-counting renormalizable theory in curved space and let the couplings depend on R.

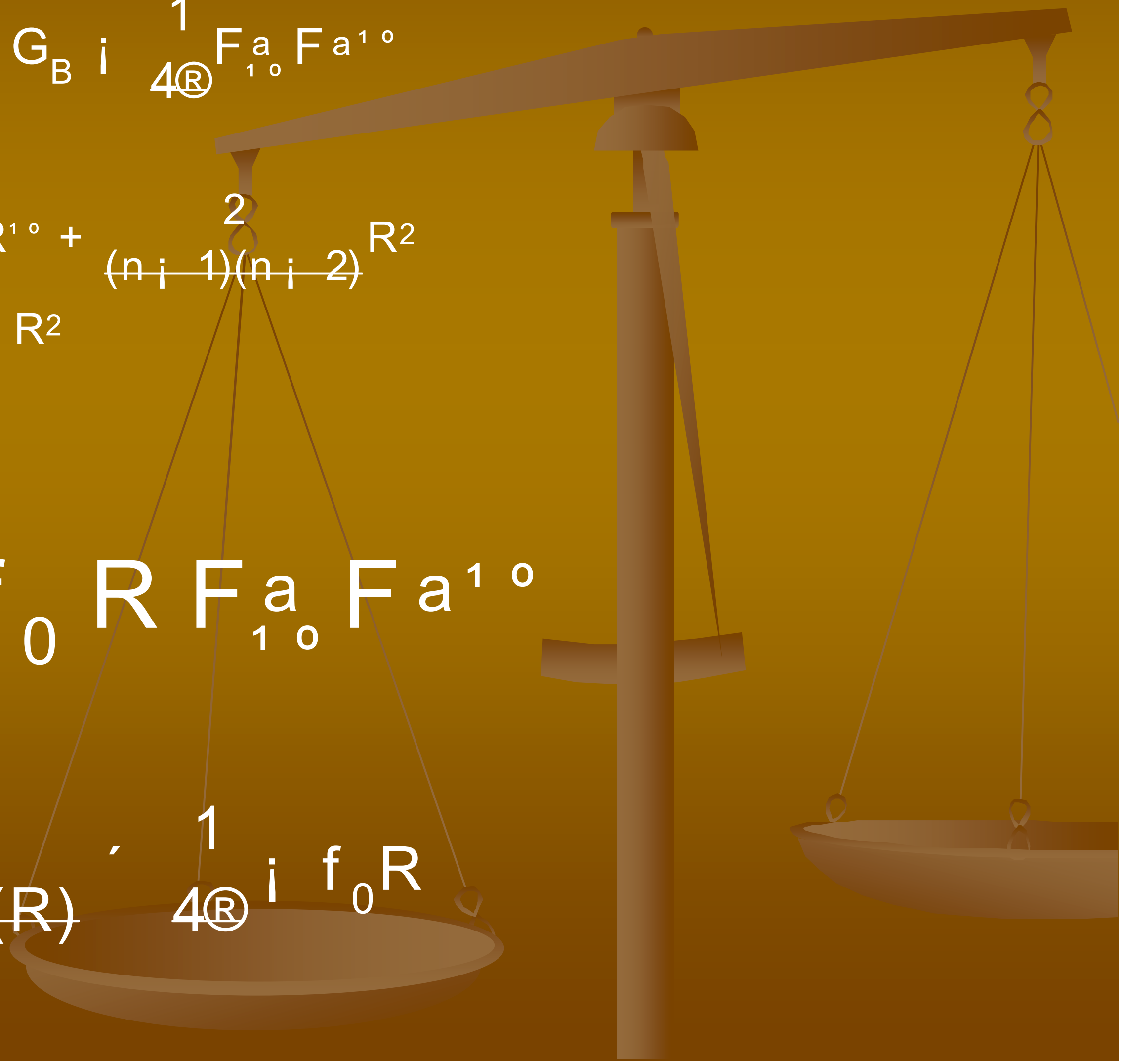
$$\frac{L_{EYM\text{-}HD}}{\sqrt{-g}} = \frac{R}{2\kappa^2} + \lambda W^2 + \gamma G_B - \frac{1}{4\alpha} F^a_{\mu\nu} F^{a\mu\nu}$$

$$W^2 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - \frac{4}{n-2} R_{\mu\nu}R^{\mu\nu} + \frac{2}{(n-1)(n-2)} R^2$$

$$G_B = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

Introduce an interaction: $f_0 R F^a_{\mu\nu} F^{a\mu\nu}$

Equivalent to $\frac{1}{4\alpha(R)} \equiv \frac{1}{4\alpha} - f_0 R$



$$\frac{L_{EYM\text{-}HD}}{\sqrt{-g}} = \frac{R}{2\kappa^2} + (\xi + \gamma R)W^2 + (\zeta + \rho R)G_B + \frac{\eta + \sigma R}{(n-1)^2}R^2 + \frac{\tau}{n-1}R * R - \left(\frac{1}{4\alpha} - f_0 R\right)F^a_{\mu\nu}F^{a\mu\nu}$$

- **Using the renormalization of F^2, in YM we obtain the terms of pure gravity that must be added.**
- **The renormalization of this parameters is connected. All order expresions can be obtained.**

$$\beta_\gamma = \frac{d\gamma}{d\ln\mu} = \varepsilon\gamma - f_0\alpha^2\frac{d\beta_\xi}{d\alpha}; \qquad \beta_\rho = \varepsilon\rho - f_0\alpha^2\frac{d\beta_\zeta}{d\alpha};$$

$$\beta_\sigma = \varepsilon\sigma - f_0\alpha^2\frac{d\beta_\eta}{d\alpha}; \qquad \beta_\tau = \varepsilon\tau - 4f_0\alpha\frac{d}{d\alpha}\left(\alpha^2\beta_\eta\right):$$

$$\frac{L_{EYM\text{-}HD}}{\sqrt{-g}} = \frac{R}{2\kappa^2} + (\xi + \gamma R) W^2 + (\zeta + \rho R) G_B + \frac{\eta + \sigma R}{(n-1)^2} R^2 + \frac{\tau}{n-1} R \Box R - \left(\frac{1}{4\alpha} - f_0 R \right) F^a_{\mu\nu} F^{a\mu\nu}$$

- **Using the Batalin-Vilkovisky formalism, we see that the divergences, order by order, have the form of**

$$f(R) F^a_{\mu\nu} F^{a\mu\nu}$$

- **Or pure gravity squarly proportional to Ricci tensor,**
- **Or sigma-exact, eliminted by a canonical transformation of the fields, of the form**

$$A_\mu \to L_A(R) A_\mu$$

$$\int d^n x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{4\alpha} F^a_{\mu\nu}(L_A A) F^{a\mu\nu}(L_A A)\,(1 + \alpha R f(R)) + (\xi + \gamma R) W^2 \right.$$

$$\left. + (\zeta + \rho R)\, G_B + \frac{\eta + \sigma R}{(n-1)^2} R^2 + \frac{\tau}{n-1} R * R + R_{\mu\nu} T_4^{\mu\nu\rho\sigma} R_{\rho\sigma} \right]$$

- **Now this theory is closed under renormalization, but HD.**
- **Is a particular case of couplings depending on the curvature scalar.**
- **Again, using the map we can generate from this an acausal theory**
- **The head (dimensionality 6) of the deformtion will be:**

$$\Delta S^{(HEAD)} = \int d^4x \sqrt{-g} \left[-\frac{f}{4} F^a_{\mu\nu} F^{a\mu\nu} R - \frac{a}{2} R_{\mu\nu} T^{\mu\nu} \right]$$

Conclusions

- In a wide class of theories, divergences are removed without introducing HD terms in the gravitational sector.
- No direct extension of the map to quantum gravity.
- In QG the resummation of infinite series of fields and derivatives (generated by renormalization) could similarly lead to causality violations at high energy.