On the moduli space of semilocal vortices and lumps

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Work in progress with M. Eto, J. Evslin, K. Konishi, G. Marmorini, M. Nitta, K. Ohashi, N. Yokoi

Outline

1. Overview

2. Non-Abelian Semilocal Vortices
   - The Moduli Space
   - A Duality for Semilocal Vortices

3. The Lump Limit
   - Vortices and Lumps

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Semilocal Vortices and Lumps

- Vortices are codimension 2 objects stabilized by $\pi_1(G_{gauge})$;
- Lump solutions are codimension 2 objects stabilized by $\pi_2(\mathcal{M}_{target})$.

Much is known about abelian semilocal vortices:

- the term "semilocal" means that both global and local symmetries are relevant;
- semilocal vortices emerge when "large" global symmetries are present;
- they have size moduli like lump solutions;
- they interpolate between ANO ("local") vortices and lumps.

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The same and much more is going to happen in the non-abelian case!

Motivations

Semilocal strings and lumps have fundamental role in broad area of physics. Just two examples that we investigated:

- Cosmology: Cosmic Strings
  - GUT models have typically large global symmetries;
    See PRL 98:091602,2007, M. Eto et al. (hep-th/0609214), about the issue of reconnection.
- Strongly Coupled Gauge Theories:
  - Large flavor symmetries are needed to preserve non abelian gauge symmetry;
    See hep-th/0611313, M. Eto et al.
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The Model

Non abelian $U(N_{\rm C})$ gauge theory with $N_{\rm F}$ "fundamental" flavour

$$\mathcal{L} = \mathrm{Tr}\left[-\frac{1}{2g^2}F_{\mu\nu}\,F^{\mu\nu} - \mathcal{D}_\mu\,H\,\mathcal{D}^\mu H^\dagger - \frac{g^2}{4}\left(\xi\,\mathbf{1}_{N_{\rm C}} - H\,H^\dagger\right)^2\right]$$

where H is the $N_{\rm C}\times N_{\rm F}$ matrix of squark fields;

- Bosonic sector of $\mathcal{N}=2$ SUSY theory;
- The FI term ξ puts the theory on a Higgs branch: $\mathcal{V}_{Higgs} = Gr_{N_{\rm C},N_{\rm F}}$;
- Non abelian BPS vortices supported by $\pi_1[U(N_{\rm C})] = \mathbb{Z}$.

To have semilocal vortices we must take $N_{\rm F} \equiv N_{\rm C} + \tilde{N}_{\rm C} > N_{\rm C}$

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The Moduli Matrix: $H_0(z)$

The BPS equation for the vortices can be put in the following form:

$$\partial_z(\Omega^{-1}\bar{\partial}_z\Omega) = \frac{g^2}{4}\left(\xi \mathbf{1}_{N_\mathrm{C}} - \Omega^{-1}H_0 H_0^\dagger\right); \quad W_1 + i\, W_2 = -2\, i\, S^{-1}(z,\bar{z})\,\bar{\partial}_z S(z,\bar{z}),$$

where we defined:

$$H \equiv S^{-1}(z,\bar{z})\, H_0(z), \quad \Omega \equiv S(z,\bar{z})S^\dagger(z,\bar{z}), \quad z \equiv x_1 + i\, x_2$$

- $H_0(z)$ is an arbitrary $N_\mathrm{C} \times N_\mathrm{F}$ holomorphic matrix which contains all the moduli of the BPS equations as coefficients of its polynomial entries;
- The number of vortices, k, is defined by: $\det H_0 H_0^\dagger \sim |z|^{2k}$ for large z;
- The set of physically inequivalent H_0 define the moduli space of vortices:

$$\mathcal{M}_{N_\mathrm{C},N_\mathrm{F};k}$$

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$$\mathcal{M}_{N_{\rm C}, N_{\rm F}; k}$$

Moduli Space of Semilocal Vortices

The Kähler quotient construction

$\mathcal{M}_{N_{\rm C},N_{\rm F};k}$, is isomorphic to the quotient:

$$\mathcal{M}_{N_{\rm C},N_{\rm F};k} = \{(\mathbf{Z},\mathbf{\Psi},\tilde{\mathbf{\Psi}}) : GL(k,\mathbf{C}) \text{ free on } (\mathbf{Z},\mathbf{\Psi})\}/GL(k,\mathbf{C}).$$

- $\mathbf{Z}_{k\times k}$, $\mathbf{\Psi}_{N_{\rm C}\times k}$ and $\tilde{\mathbf{\Psi}}_{k\times \tilde{N}_{\rm C}}$ are constant matrices;
- The action of $\mathcal{V} \in GL(k,\mathbf{C})$ is: $(\mathcal{V}\mathbf{Z}\mathcal{V}^{-1}, \mathbf{\Psi}\mathcal{V}^{-1}, \mathcal{V}\tilde{\mathbf{\Psi}})$;

- These matrices collect all the parameters contained in the moduli matrix H_0;
- they contain all zero modes of squarks and gauge fields

Parent Spaces and "Dual" Regularizations

Consider the "parent" set:

$$\hat{\mathcal{M}}^{parent}_{N_{\rm C},\tilde{N}_{\rm C};k} = \{\mathbf{Z},\mathbf{\Psi},\tilde{\mathbf{\Psi}}\}/GL(k,\mathbf{C})$$

- This quotient space is in general singular and non-Hausdorff;
- A non-Hausdorff space has distinct points with no distinct neighborhoods;

The set $\hat{\mathcal{M}}^{parent}_{N_{\rm C},\tilde{N}_{\rm C};k}$ is symmetric under a kind of $\mathcal{N}=2$ Seiberg duality:

$$N_{\rm F} \leftrightarrow N_{\rm F}, \quad N_{\rm C} \leftrightarrow \tilde{N}_{\rm C} = N_{\rm F} - N_{\rm C}$$

$$\Downarrow$$

Two dual regularizations

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Two dual regularizations

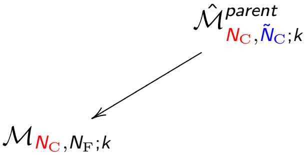
The "half" Duality-Diagram

$$\hat{\mathcal{M}}^{parent}_{N_{\rm C},\tilde{N}_{\rm C};k}$$

- We must regularize the parent set keeping only a regular subspace...

$$\hat{\mathcal{M}}^{parent}_{N_{\rm C},\tilde{N}_{\rm C};k} = \{\mathbf{Z}, \boldsymbol{\Psi}, \tilde{\boldsymbol{\Psi}}\}/GL(k,\mathbf{C})$$

The "half" Duality-Diagram



- ...we can choose the moduli space of semilocal vortices when the gauge group is N_C...

$$\mathcal{M}_{N_C,N_F;k} = \{\mathbf{Z}, \mathbf{\Psi}, \tilde{\mathbf{\Psi}}\}/GL(k,\mathbf{C}) \quad \text{with } (\mathbf{Z}, \mathbf{\Psi}) \text{ free}$$

The "half" Duality-Diagram

$$\hat{\mathcal{M}}^{parent}_{N_{\rm C},\tilde{N}_{\rm C};k} \longrightarrow \mathcal{M}_{\tilde{N}_{\rm C},N_{\rm F};k}$$

- ...or take the moduli space of semilocal vortices when the gauge group is $\tilde{N}_{\rm C}$

$$\mathcal{M}_{\tilde{N}_{\rm C},N_{\rm F};k} = \{\mathbf{Z},\mathbf{\Psi},\tilde{\mathbf{\Psi}}\}/GL(k,\mathbf{C}) \quad \text{with } (\mathbf{Z},\tilde{\mathbf{\Psi}}) \text{ free}$$

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$$\mathcal{M}_{N_{\rm C},N_{\rm F};k} \leftarrow\cdots\cdots (N_{\rm C} \leftrightarrow \tilde{N}_{\rm C}) \cdots\cdots\rightarrow \mathcal{M}_{\tilde{N}_{\rm C},N_{\rm F};k}$$

Deep relation between the dual spaces:

- They are "birationally" equivalent;
- They are linked by geometric transitions.

The Simplest Example: $N_{\rm C} = \tilde{N}_{\rm C} = 1\,(N_{\rm F} = 2),\ k = 1$

A single semilocal vortex in an abelian self dual theory

The parent space contains a weighted projective space with mixed weights:

$$\hat{\mathcal{M}}^{parent}_{1,1;1} = \{Z, \Psi, \tilde{\Psi}\}/\mathbf{C}^* = \mathbf{C}(Z) \times W\mathbf{C}P^1_{(1,-1)}(\Psi, \tilde{\Psi})$$

$W\mathbf{C}P^1_{(1,-1)} = \{(\Psi, \tilde{\Psi}) \sim (\lambda\Psi, \lambda^{-1}\tilde{\Psi})\}$ is non-Hausdorff:

- It contains two distinct points: $(1,0) \neq (0,1)$...
  - ... with no distinct neighborhood: $(1,\epsilon) \sim (\epsilon, 1)$, with $\epsilon \ll 1$
- To regularize this space we throw away $(0,1)_{\Psi\,not\ free}$ or $(1,0)_{\tilde{\Psi}\,not\ free}$
- In both case the regularized spaces are: $W\mathbf{C}P^1_{(1,-1)}|_{regul.} = (1, \Psi\tilde{\Psi}) \sim (\Psi\tilde{\Psi}, 1) = \mathbf{C}(\Psi\tilde{\Psi})$

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This gives us the moduli space for a semilocal vortex:

$$\mathcal{M}_{1,2;1} = \mathcal{M}_{1,2;1} = \mathbf{C}^2 = \mathbf{C}(Z)|_{position} \times \mathbf{C}(\Psi\tilde{\Psi})|_{size}$$

The Lump Limit

In the limit $g \to \infty$ we get a non linear sigma model on the Higgs branch:

$$\mathcal{V}_{Higgs} = Gr_{N_{\rm C},N_{\rm F}}\,,$$

which supports lump solutions: $\pi_2(Gr_{N_{\rm C},N_{\rm F}}) = \mathbb{Z}$.

- Semilocal vortices, at g finite, are mapped into lumps in the limit $g \to \infty$;
  - Some vortex configurations are mapped into zero size lumps... the limit develops singularities (small lumps singularities).
- The sigma model inherits the natural duality property of Grassmanians:
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$$\mathcal{M}^{lump}_{N_{\mathrm{C}},\tilde{N}_{\mathrm{C}};k} = \mathcal{M}_{N_{\mathrm{C}},N_{\mathrm{F}};k}/\{\text{singular points}\} = \mathcal{M}_{\tilde{N}_{\mathrm{C}},N_{\mathrm{F}};k}/\{\text{singular points}\}$$

The Duality Diagram

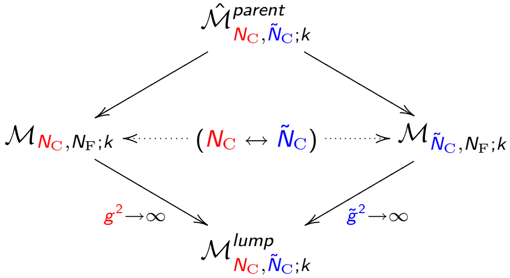
$$\hat{\mathcal{M}}^{parent}_{N_{\rm C},\tilde{N}_{\rm C};k}$$

$$\swarrow \qquad\qquad\qquad \searrow$$

$$\mathcal{M}_{N_{\rm C},N_{\rm F};k} \leftarrow\cdots\cdots (N_{\rm C} \leftrightarrow \tilde{N}_{\rm C}) \cdots\cdots\rightarrow \mathcal{M}_{\tilde{N}_{\rm C},N_{\rm F};k}$$

- From the moduli space of semilocal vortices $\mathcal{M}_{N_{\rm C},N_{\rm F};k}$ or $\mathcal{M}_{\tilde{N}_{\rm C},N_{\rm F};k}$

The Duality Diagram



- We can eliminate the sick points in this simple (and duality invariant) way:

$$\mathcal{M}^{lump}_{N_C,\tilde{N}_C;k} = \mathcal{M}_{N_C,N_F;k} \cap \mathcal{M}_{\tilde{N}_C,N_F;k}$$

Non Abelian Semilocal Vortex: $N_{\rm C} = 2,\ N_{\rm F} = 3$

Dual To An Abelian Theory: $\tilde{N}_{\rm C} = 1$

$$\mathbf{C} \times W\mathbf{C}P^2_{[1,1,-1]}$$

- The space $W\mathbf{C}P^2_{[1,1,-1]}(\Psi_1, \Psi_2, \tilde{\Psi})$ has two overlapping subsets:
  - $\mathbf{C}P^1 = W\mathbf{C}P^2(\Psi_1, \Psi_2, 0)$
  - *point* $= W\mathbf{C}P^2(0, 0, \tilde{\Psi}) \sim (0, 0, 1)$

$$(\Psi_1, \Psi_2, 0) \simeq (\Psi_1, \Psi_2, \epsilon\tilde{\Psi}) \sim (\epsilon\Psi_1, \epsilon\Psi_2, \tilde{\Psi}) \simeq (0, 0, \tilde{\Psi}) \quad \epsilon \ll 1$$

Non Abelian Semilocal Vortex: $N_{\mathrm{C}} = 2,\ N_{\mathrm{F}} = 3$

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$$\mathbf{C} \times W\mathbf{C}P^2_{[1,1,-1]}$$

$$\Big\downarrow\ /point$$

$$\mathbf{C} \times \tilde{\mathbf{C}}^2$$

- If we throw away the point $(0, 0, 1)$ we get, for the moduli space of lumps:

$$\mathcal{M}_{2,3;k} = \mathbf{C} \times \tilde{\mathbf{C}}^2 \equiv \mathbf{C}(Z)|_{position} \times \tilde{\mathbf{C}}^2(\Psi_2/\Psi_1, \tilde{\Psi}\Psi_1)|_{orientation,\, size},$$

where $\tilde{\mathbf{C}}^2$ is the blow up of $\mathbf{C}^2$

Non Abelian Semilocal Vortex: $N_{\rm C} = 2,\ N_{\rm F} = 3$

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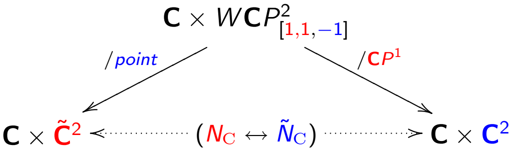
$$\mathbf{C} \times W\mathbf{C}P^2_{[1,1,-1]} \xrightarrow{/\mathbf{C}P^1} \mathbf{C} \times \mathbf{C}^2$$

- While if we throw away the $\mathbf{C}P^1$...

$$\mathcal{M}_{1,3;k} = \mathbf{C} \times \mathbf{C}^2 \equiv \mathbf{C}(Z)|_{position} \times \mathbf{C}^2(\tilde{\Psi}\Psi_1, \tilde{\Psi}\Psi_2)|_{2\ sizes}$$

Non Abelian Semilocal Vortex: $N_{\rm C} = 2$, $N_{\rm F} = 3$

Dual To An Abelian Theory: $\tilde{N}_{\rm C} = 1$

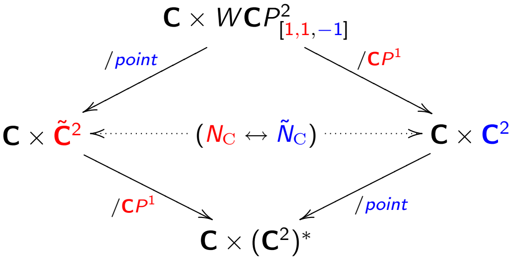


- Here it is easy to see the geometric relation between the moduli spaces of the dual theories:

The moduli space of the non abelian theory is the blow-up of that of the abelian dual theory.

Non Abelian Semilocal Vortex: $N_{\rm C} = 2,\ N_{\rm F} = 3$

Dual To An Abelian Theory: $\tilde{N}_{\rm C} = 1$



- To obtain the moduli space of lumps:
  - eliminate the $\mathbf{C}P^1$ from $\tilde{\mathbf{C}}^2 \rightarrow (\mathbf{C}^2)^* = \mathbf{C}^2/(0,0)$
  - eliminate the point from $\mathbf{C}^2 \rightarrow (\mathbf{C}^2)^* = \mathbf{C}^2/(0,0)$

$$\mathcal{M}^{lump}_{2,1;k} = \mathbf{C}(Z) \times (\mathbf{C}^2)^* (\tilde{\Psi}\Psi_1, \tilde{\Psi}\Psi_2)$$

Summary

- The moduli space of semilocal vortices of "dual", ($N_{\rm C} \leftrightarrow \tilde{N}_{\rm C}$), theories descend, after a process of regularization, from the same parent space;
- These dual spaces are linked by geometric transitions;
- In the lump limit they reduce to the same space of lumps;
- They are obtained from the moduli space of lumps by eliminating small lump singularities with insertions of "local" vortices.

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Outlook:

- There is still much to learn about dynamics: effective actions, non-normalizable modes...
- It would be very interesting to generalize to other gauge groups: $SO(N)$, $Usp(N)$...