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Scalar gravitational waves from Scalar-Tensor Gravity: production and response of interferometers

Miniworkshop

"Supersymmetry, Supergravity, Superstrings"

March 19-21 2007

Aula 131 - Dipartimento di Fisica - Largo B.Pontecorvo,3 - Pisa

Capozziello S and Corda C - Int. J. Mod. Phys. D **15** 1119 -1150 (2006)

Introduction

1) Mechanism of production of SGW from Scalar-Tensor Gravity

2) Massless case: invariance of the signal in three different gauges

3) Massless case: the frequency-dependent angular pattern

4) The small massive case

Generalized previous results analyzed in the low-frequencies approximation

Mechanism of production of SGW from Scalar-Tensor Gravity

Most general action for STG in literature

$$S = \int d^4x \sqrt{-g}\left[f(\phi)R + \frac{1}{2}g^{\mu\nu}\phi_{;\mu}\phi_{;\nu} - V(\phi) + \mathcal{L}_m\right].$$

Wands D. Class. Quant. Grav. **11** 269 (1994)
Capozziello S., R. De Ritis, C. Rubano, P. Scudellaro, Int. Jou. Mod. Phys. D, **4**, 767 (1995).

Considering the transformation

$$\varphi = f(\phi) \quad \omega(\varphi) = \frac{f(\phi)}{2'f(\phi)} \quad W(\varphi) = V(\phi(\varphi))$$

previous action reads

$$S = \int d^4x\sqrt{-g}[\varphi R - \frac{\omega(\varphi)}{\varphi}g^{\mu\nu}\varphi_{;\mu}\varphi_{;\nu} - W(\varphi) + \mathcal{L}_m]$$

BD-like theory

Capozziello S., R. De Ritis, C. Rubano, P. Scudellaro, Int. Jou. Mod. Phys. D, **5**, 85 (1996).

Field equations

$$G_{\mu\nu} = -\frac{4\pi\tilde{G}}{\varphi}T^{(m)}_{\mu\nu} + \frac{\omega(\varphi)}{\varphi^2}(\varphi_{;\mu}\varphi_{;\nu} - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\varphi_{;\alpha}\varphi_{;\beta}) +$$

$$+\frac{1}{\varphi}(\varphi_{;\mu\nu} - g_{\mu\nu}\Box\varphi) + \frac{1}{2\varphi}g_{\mu\nu}W(\varphi)$$

Klein-Gordon

$$\Box\varphi = \frac{1}{2\omega(\varphi)+3}(-4\pi\tilde{G}T^{(m)} + 2W(\varphi) + \varphi W'(\varphi) + \frac{d\omega(\varphi)}{d\varphi}g^{\mu\nu}\varphi_{;\mu}\varphi_{;\nu}$$

Linearized theory in vacuum

Minkowski background + $\varphi = \varphi_0$ **minimum for W**

$$W \simeq \frac{1}{2}\alpha\delta\varphi^2 \Rightarrow W' \simeq \alpha\delta\varphi$$

We assume

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\varphi = \varphi_0 + \delta\varphi.$$

obteining

$$\widetilde{R}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\widetilde{R} = \partial_\mu\partial_\nu\xi - \eta_{\mu\nu}\Box\xi$$

$$\Box\xi = -m^2\xi,$$

with

$$\xi \equiv \frac{\delta\varphi}{\varphi_0}, \qquad m^2 \equiv \frac{\alpha\varphi_0}{2\omega + 3}$$

The massless case

Effective BD $\omega = const$ and $W = 0$

Most simple case: $m = 0$:

Gauge transforms (Lorenz condition)

$$\Box \bar{h}_{\mu\nu} = 0$$

$$\Box \xi = 0 .$$

Solutions are plan waves

$$\bar{h}_{\mu\nu} = A_{\mu\nu}(\overrightarrow{k})\exp(ik^{\alpha}x_{\alpha}) + c.c. \qquad \xi = a(\overrightarrow{k})\exp(ik^{\alpha}x_{\alpha}) + c.c.$$

TT gauge extended to scalar waves

$$h_{\mu\nu}(t-z) = A^{+}(t-z)e_{\mu\nu}^{(+)} + A^{\times}(t-z)e_{\mu\nu}^{(\times)} + \Phi(t-z)e_{\mu\nu}^{(s)}.$$

Purely scalar wave: line element

$$ds^2 = -dt^2 + dz^2 + [1 + \Phi(t-z)][dx^2 + dy^2]$$

The response of an interferometer

Literature: low-frequencies approximation

Maggiore M and Nicolis A - Phys. Rev. D **62** 024004 (2000)

Method of "bouncing photon" : the variation of space-time due to the scalar field is computed in all the travel of the photon

Rakhmanov M - Phys. Rev. D **71** 084003 (2005)

Computation of the variation of proper time in presence of the SGW

$$\delta T(t) = \frac{1}{2}\int_{l}^{L+l} [\Phi(t - 2T + x - l) + \Phi(t - x + l)]dx.$$

In the Fourier domain

$$\frac{\delta \tilde{T}(\omega)}{T} = \Upsilon(\omega)\tilde{\Phi}(\omega)$$

$$\Upsilon(\omega) = \frac{\exp(2i\omega T) - 1}{2i\omega T}$$

The “Shibata, Nakao and Nakamura” gauge for SGW

Purely scalar wave: line element

$$ds^2 = (1 + \Phi)(-dt^2 + dz^2 + dx^2 + dy^2)$$

Shibata M, Nakao K and Nakamura T - Phys. Rev. D **50**, 7304 (1994)

Reanalyzed

Maggiore M and Nicolis A - Phys. Rev. D **62** 024004 (2000)

Used a time transform $d\tau^2 = g_{00} dt^2$

Same results of the TT gauge

$$\delta T(t) = \frac{1}{2} \int_{l}^{L+l} [\Phi(t - 2T + x - l) + \Phi(t - x + l)] dx.$$

In the Fourier domain

$$\frac{\delta \tilde{T}(\omega)}{T} = \Upsilon(\omega) \tilde{\Phi}(\omega)$$

$$\Upsilon(\omega) = \frac{\exp(2i\omega T) - 1}{2i\omega T}$$

The local Lorentz gauge for SGW: three different effects

The motion of test masses

$$\frac{\delta_1 T(t)}{T} = \Phi(t-T) - \frac{l}{2L}[\Phi(t) - 2\Phi(t-T) - \Phi(t-2T)]$$

The travel of photons in curved spacetime

$$\delta_2 T(t) = \int_{l+L}^{l} [\ddot{\Phi}(t-2T+x-l) + \ddot{\Phi}(t-x+l)]x^2 dx.$$

The shifting of time

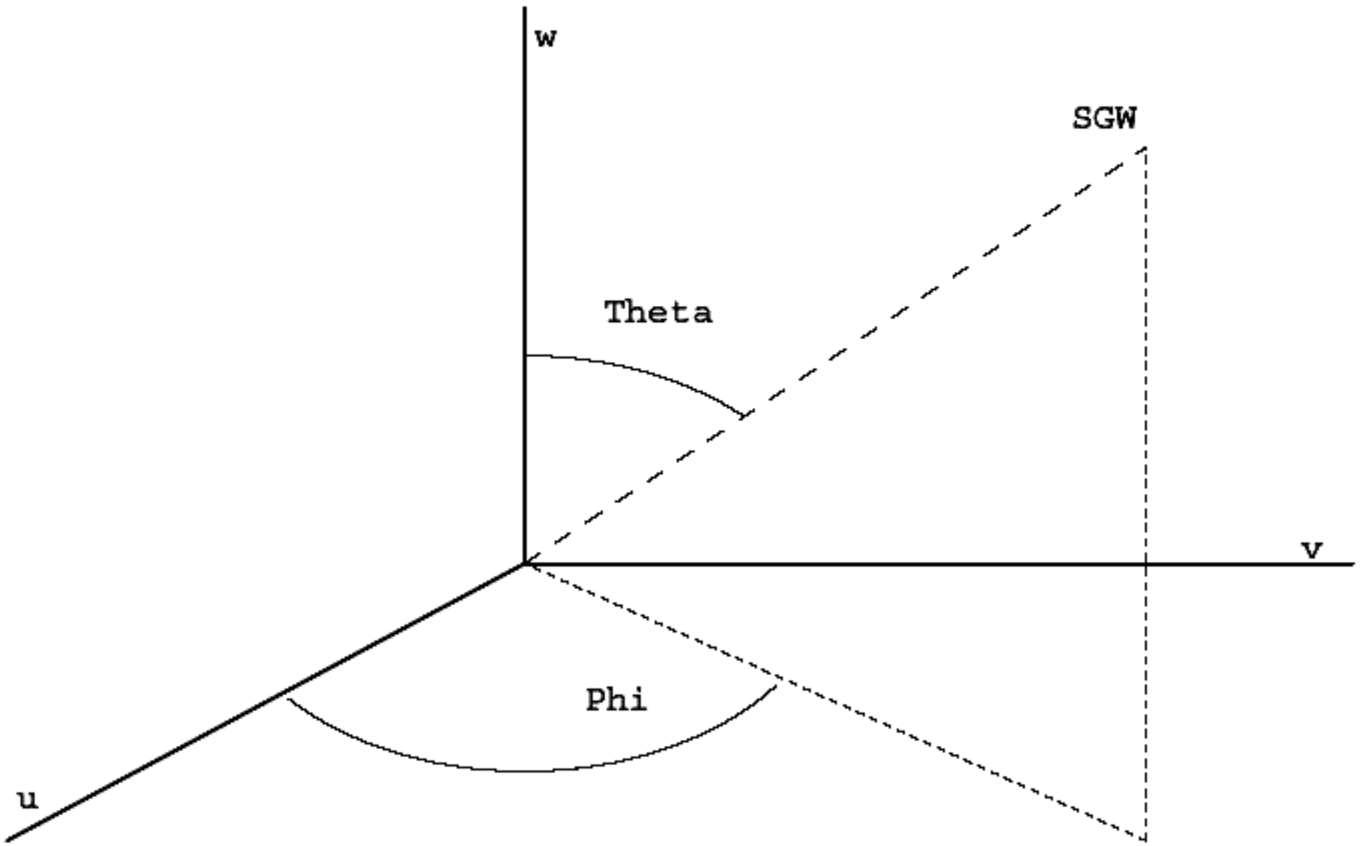
$$\delta_3 T(t) \approx \int_{t-T_{r.t.}}^{t} V(t', l)dt' = -l^2[\dot{\Phi}(t) - \dot{\Phi}(t-2T)]$$

In the Fourier domain

$$\frac{\delta\tilde{T}(\omega)}{T} = \Upsilon(\omega)\tilde{\Phi}(\omega) \qquad \Upsilon(\omega) = \frac{\exp(2i\omega T) - 1}{2i\omega T}$$

Gauge invariance recovered

Angular pattern for SGW



Line element in u direction

$$g^{ik} = \frac{\partial x^i}{\partial x'^l} \frac{\partial x^k}{\partial x'^m} g'^{lm}$$

$$ds^2 = -dt^2 + [1 + (1 - \sin^2\theta\cos^2\phi)\Phi(t - u\sin\theta\cos\phi)]du^2$$

variation of proper time in presence of the SGW in u direction

$$\delta T(t) = \frac{1-\sin^2\theta\cos^2\phi}{2} \int_0^L [\Phi(t - 2T + u(1 - \sin\theta\cos\phi))+$$

$$+\Phi(t - u(1 + \sin\theta\cos\phi))]du.$$

Response function in u direction

$$\Upsilon_u(\omega) = \frac{1}{2i\omega L}[-1 + \exp(2i\omega L) +$$

$$+ \sin\theta\cos\phi((1 + \exp(2i\omega L) - 2\exp i\omega L(1 + \sin\theta\cos\phi))].$$

Same analysis: response function in v direction

$$\Upsilon_v(\omega) = \frac{1}{2i\omega L}[-1 + \exp(2i\omega L) +$$

$$+ \sin\theta\sin\phi((1 + \exp(2i\omega L) - 2\exp i\omega L(1 + \sin\theta\sin\phi))].$$

Total frequency-dependent response function

$$\tilde{H}(\omega) = \frac{\sin\theta}{2i\omega L}\{\cos\phi[1+\exp(2i\omega L) - 2\exp i\omega L(1+\sin\theta\cos\phi)]+$$

$$-\sin\phi[1+\exp(2i\omega L) - 2\exp i\omega L(1+\sin\theta\sin\phi)]\}$$

Agrees with

Shibata M, Nakao K, Harada T, Kawamura S and Nakamura T - Phys. Rev. D **63** 082001 (2001)

Low frequencies $\tilde{H}(\omega \to 0) = -\sin^2\theta\cos 2\phi.$

Maggiore M and Nicolis A - Phys. Rev. D **62** 024004 (2000)

Bonasia N and Gasperini M - gr-qc/0504079 (2005)

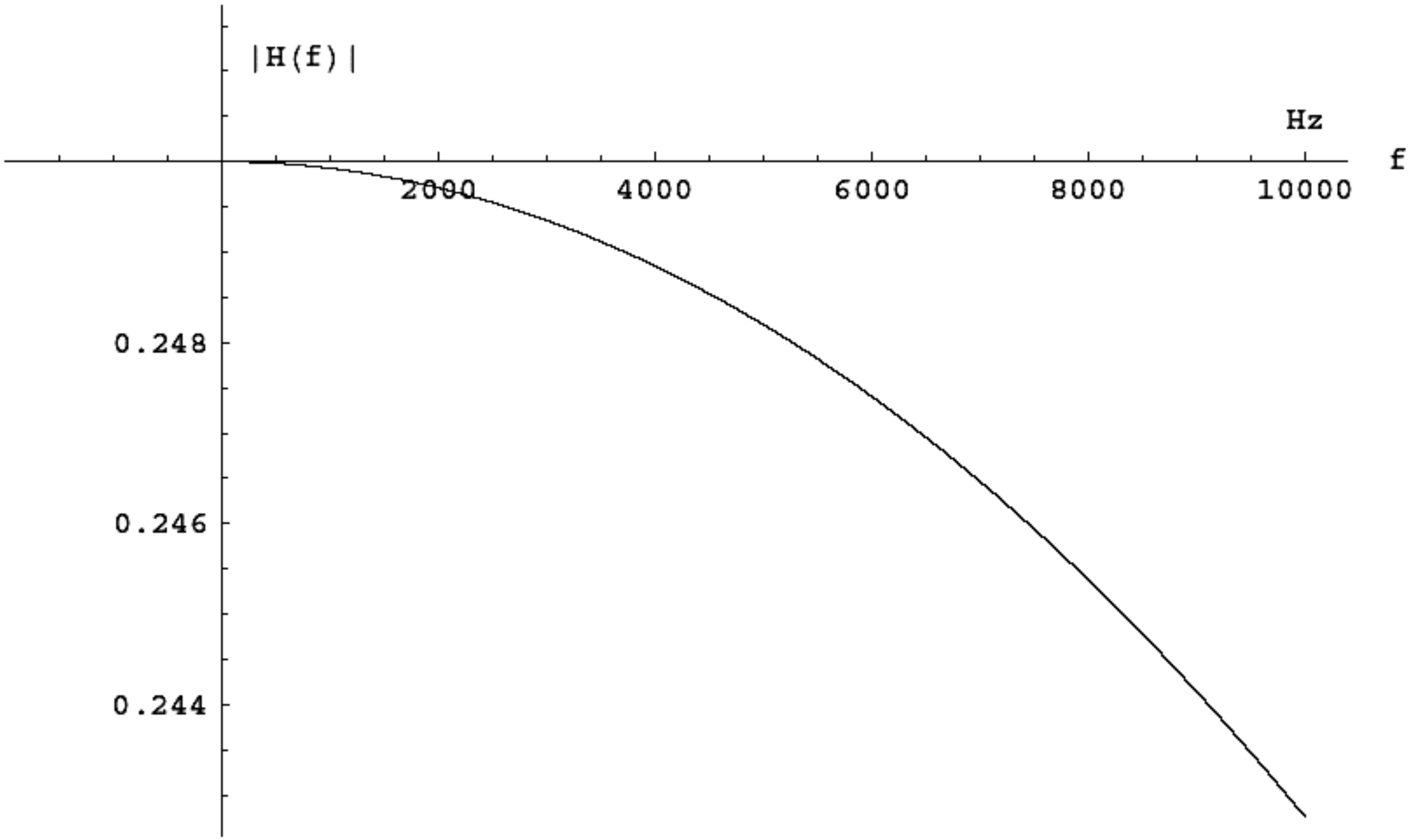


Figure 11: the absolute value of the total response function of the VIRGO interferometer to massless SGWs for $\theta = \frac{\pi}{4}$ and $\phi = \frac{\pi}{3}$.

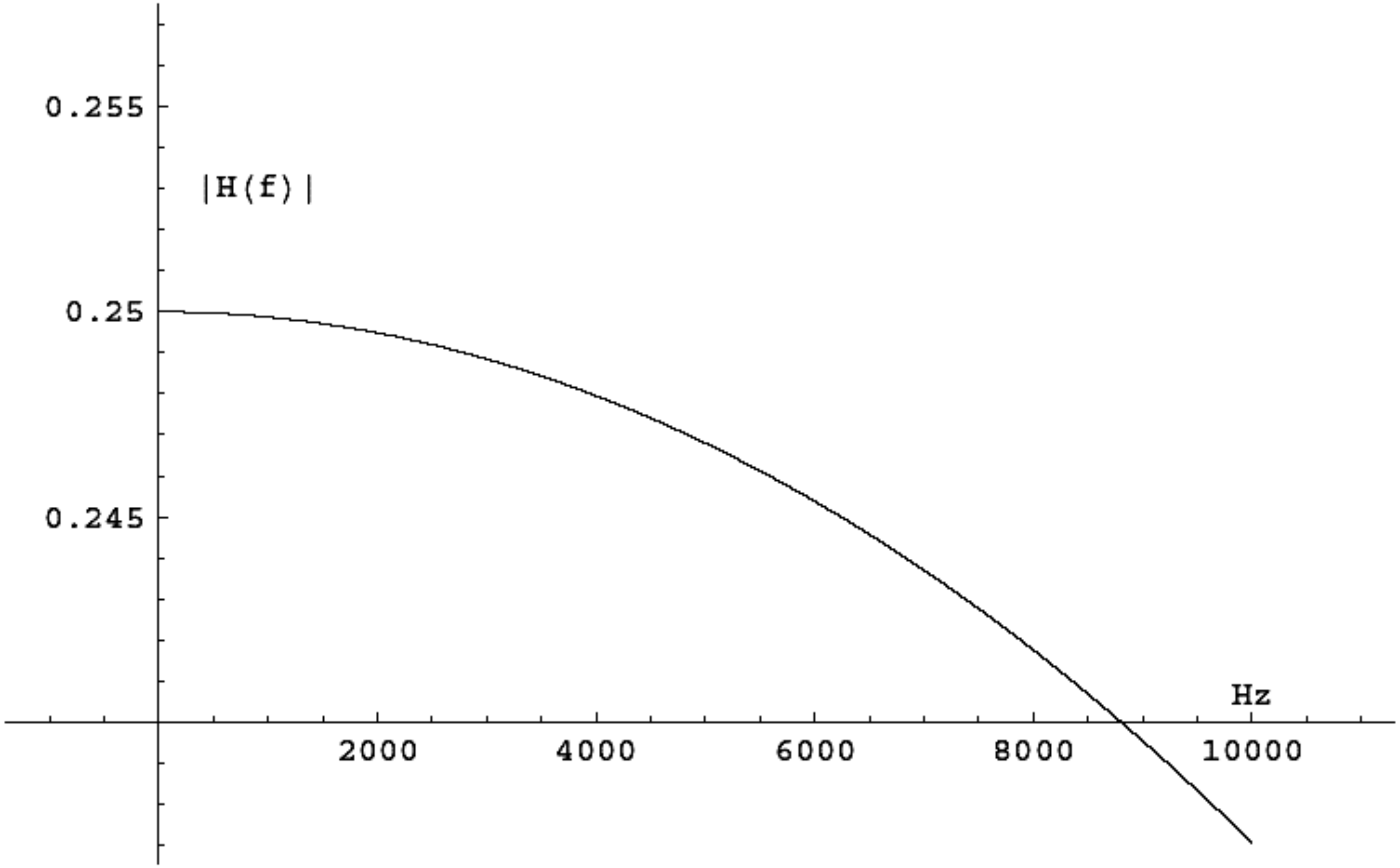


Figure 12: the absolute value of the total response function of the LIGO interferometer to massless SGWs for $\theta = \frac{\pi}{4}$ and $\phi = \frac{\pi}{3}$.

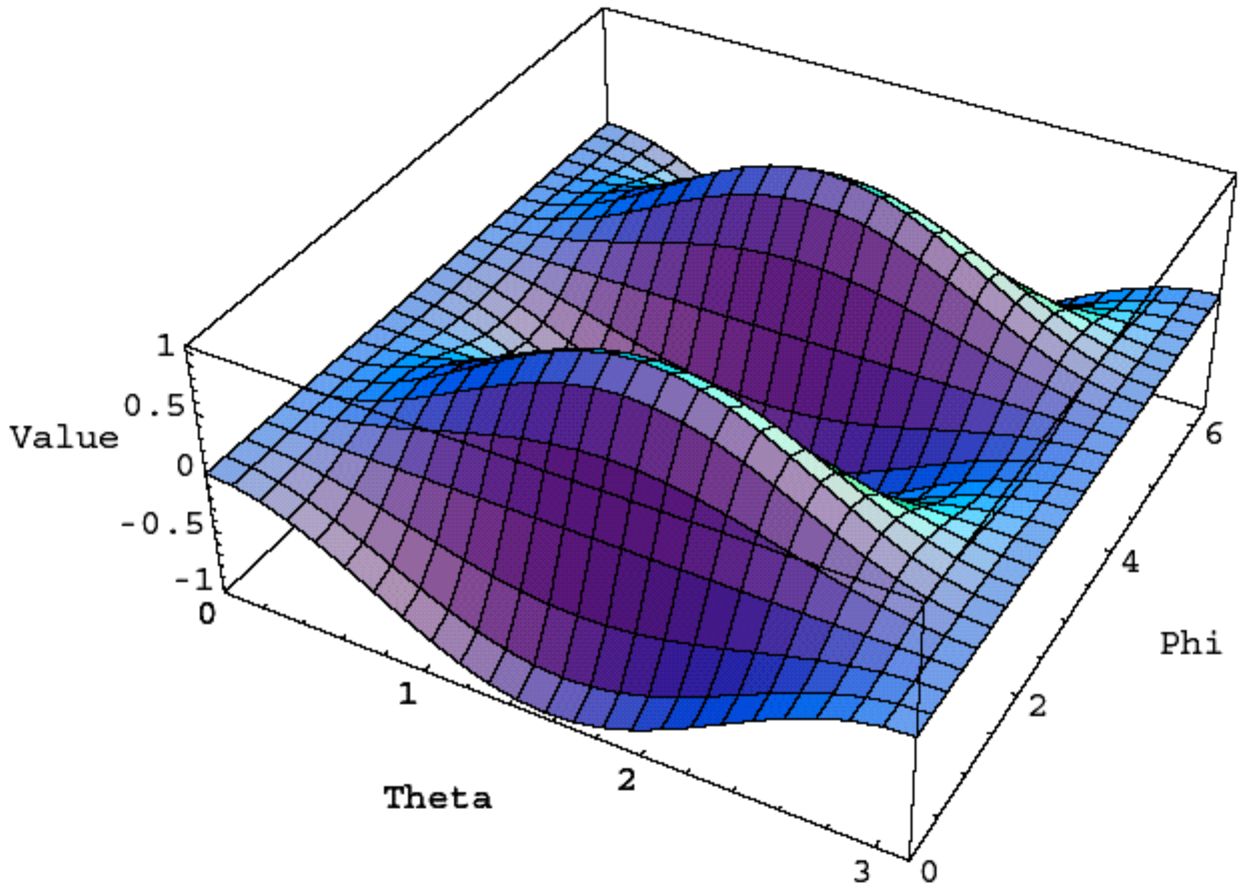


Figure 13: the angular dependance of the response of the VIRGO and LIGO interferometers for a SGW with a frequency of $f = 100Hz$

The small massive case

Treating scalars like classical waves $m \ll 1/L$

Frequencies have to fall in $10\,\mathrm{Hz} \le f \le 10\,\mathrm{kHz}.$

Interval for the mass $0\,\mathrm{eV} \le m \le 10^{-11}\,\mathrm{eV}$

Known for string-dilaton gravity

M. Gasperini, gr-qc/0301032.

Maggiore M and Nicolis A - Phys. Rev. D **62** 024004 (2000)

Parameterization of the field with the phase-velocity $\Phi(t - z/v_P)$

Presence of the mass: third component of Riemann

$$R^3_{030} = \frac{1}{2} m^2 \Phi \left(t - \frac{z}{v_P} \right)$$

Previous analysis in the local Lorentz gauge generalized with the aid of Fourier theorems, two effects

The motion of test masses

$$\frac{\delta_1 \tilde{T}(\omega)}{T} = -m^2 \Upsilon_1^*(\omega) \tilde{\psi}(\omega) \qquad \Upsilon_1^*(\omega) = \exp\left[i\omega \left(1 + \frac{1}{v_P} \right) L \right]$$

The travel of photons in curved spacetime

$$\delta_2\tilde{T}(\omega) = \frac{v_P(1-\frac{1}{v_P^2})\omega^2}{4\omega^3}\Bigg(\frac{\exp[2i\omega L](iv_P^2-(v_P-1)v_P\omega+iL(v_P-1)^2\omega^2}{(v_P-1)^3}$$
$$+\frac{2\exp[i\omega(1+\frac{1}{v_P})L](-2iv_P^2(3v_P^2+1)+2(1+L)v_P(v_P^4-1)\omega + iL^2(v_P+1)^2\omega^2)}{(v_P^2-1)^3}$$
$$-\frac{2iv_P^2+2v_P(v_P+1)\omega+2iL(v_P+1)^2\omega^2}{(v_P+1)^3}\Bigg)\tilde{\Phi}(\omega). \quad ($$

Total frequency-dependent longitudinal response function

$$\Upsilon_l(\omega) = \left(1-\frac{1}{v_P^2}\right)\exp\left[i\omega\left(1+\frac{1}{v_P}\right)L\right]$$
$$+\frac{v_P(1-\frac{1}{v_P^2})}{4L\omega}\Bigg(\frac{\exp[2i\omega L](iv_P^2-(v_P-1)v_P\omega+iL(v_P-1)^2\omega^2}{(v_P-1)^3}$$
$$+\frac{2\exp[i\omega(1+\frac{1}{v_P})L](-2iv_P^2(3v_P^2+1)+2(1+L)v_P(v_P^4-1)\omega + iL^2(v_P+1)^2\omega^2)}{(v_P^2-1)^3}$$
$$-\frac{2iv_P^2+2v_P(v_P+1)\omega+2iL(v_P+1)^2\omega^2}{(v_P+1)^3}\Bigg),$$

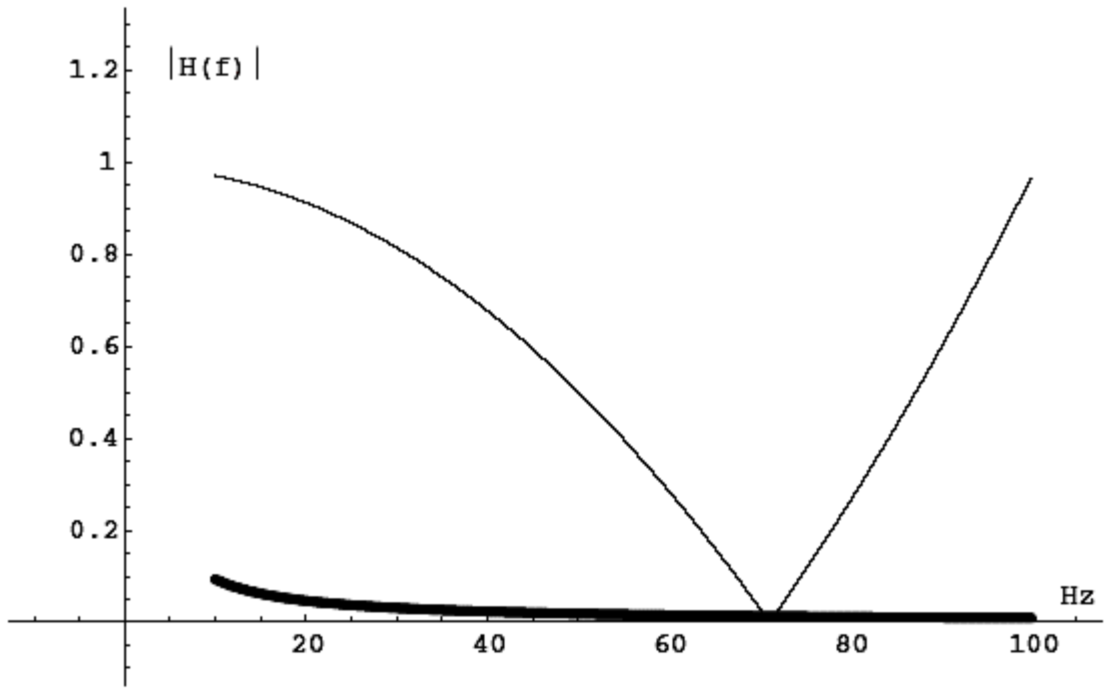


Fig. 4. The absolute value, at low frequencies, of the longitudinal response function of the VIRGO interferometer to two SGWs with speeds of $0.1c$ (non-relativistic case) and $0.9999c$ (ultra-relativistic case, thick line).

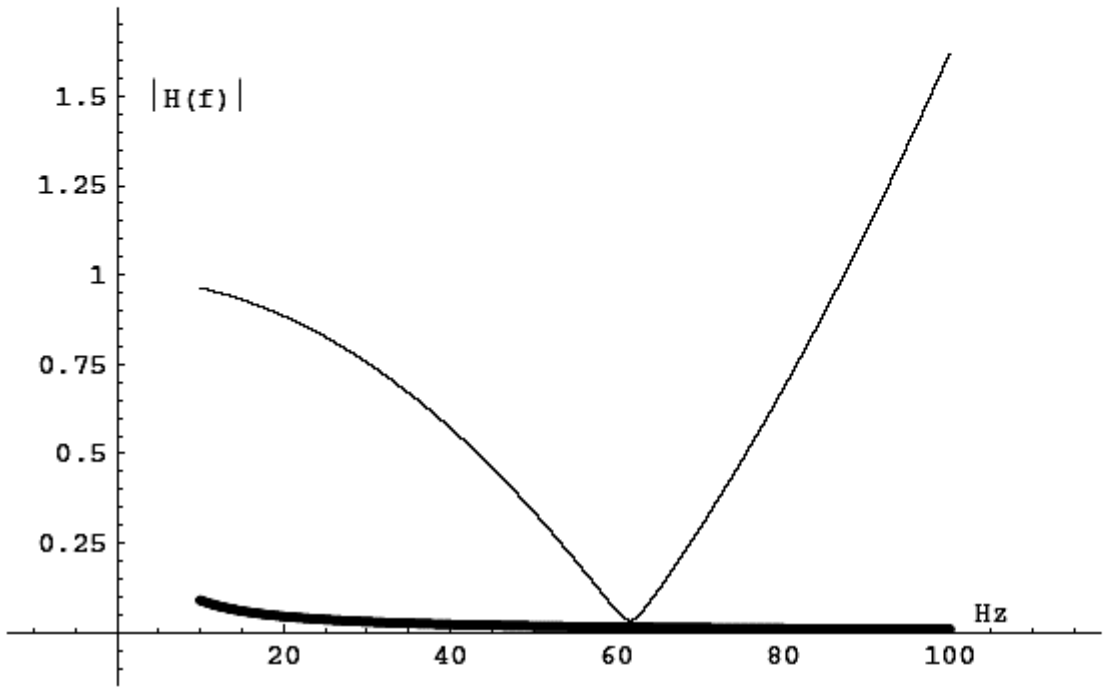


Fig. 5. The absolute value, at low frequencies, of the longitudinal response function of the LIGO interferometer to two SGWs with speeds of $0.1c$ (non-relativistic case) and $0.9999c$ (ultra-relativistic case, thick line).

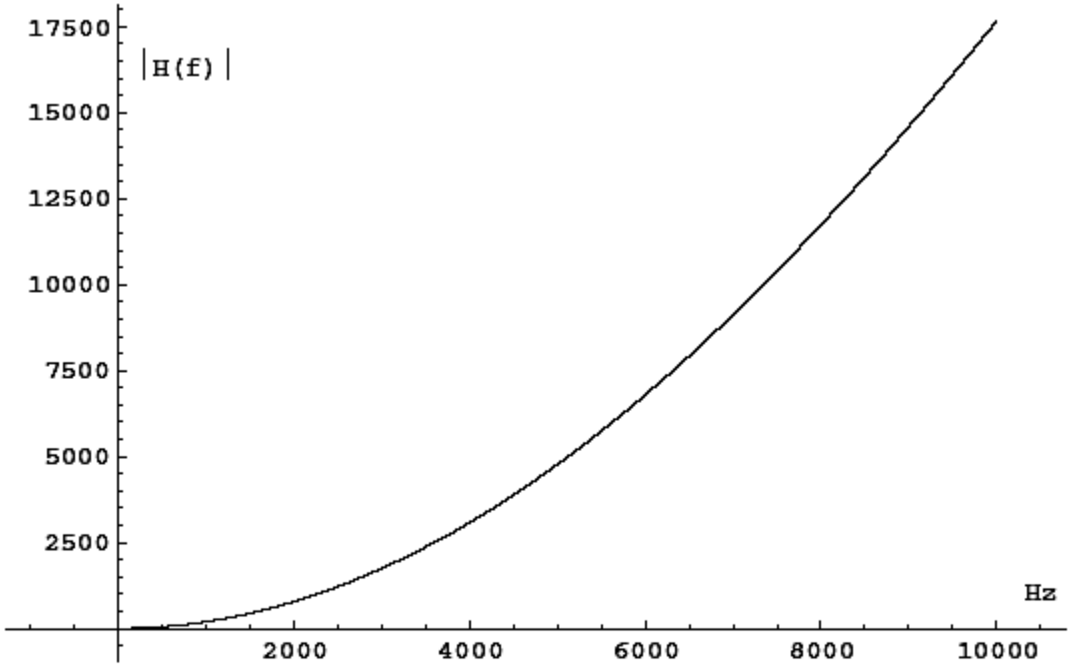


Fig. 6. The absolute value, at high frequencies, of the longitudinal response function of the VIRGO interferometer to an SGW with a speed of $0.1c$ (non-relativistic case).

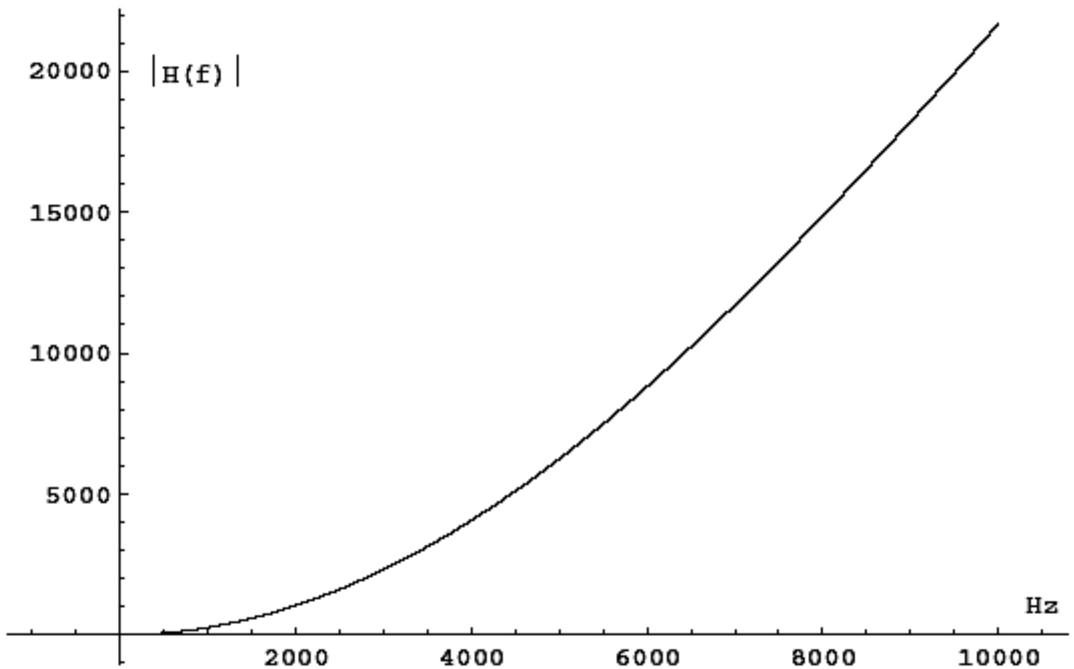


Fig. 7. The absolute value, at high frequencies, of the longitudinal response function of the LIGO interferometer to an SGW with a speed of $0.1c$ (non-relativistic case).

Conclusions

Realistic possibility to detect SGW in different gauges

Realistic possibility to detect a longitudinal component

The investigation of scalar components of GW could be a tool to discriminate among several theories of gravity

Next papers

Corda C. The "Shibata, Nakao and Nakamura gauge for SGW" submitted to GRG gr-qc 0610157

Corda C. "The importance of the magnetic components of gravitational waves in the response function of interferometers gr-qc 0702080 accepted for IJMPD

Capozziello S.,Corda C. and de Laurentis MF "On the correct frame for theories: Jordan frame versus Einstein frame"