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Nonabelian Duality and Confinement:
-- Monopoles Seventy-Five Years Later --

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Collaboration with

Carlino, Murayama, Yung, Auzzi, Bolognesi, Evslin, Marmorini, Vinci, Ferretti, Nitta, Yokoi, Ohashi, Eto, ...

Monopoles and Vortices

- Magnetic Monopols in QFT (Dirac 1931)
- Vortex in superconductor (Abrikosov, 1957)
- Vortices in rel. QFT (Nielsen-Olesen, 1973)
- Monopoles in QFT ('t Hooft, Polyakov, 1974)
- Confinement in QCD ('t Hooft, Mandelstam 1980)
- Nonabelian vortices (2003-), monopoles, walls
- Quantum mech. non-Abelian monopoles (1994 - 2006)

The strategy: a two-front attack

How

General ideas versus
Concrete models: softly broken N=2 supersymmetric SU, SO, USp models

- Fully quantum mechanical analysis. Possible in N=2 susy SU, SO, USp theories with N_f quarks;

  ☛ Light monopoles in $\underline{r}$ of various SU(r) magnetic gauge groups

  ☛ Class of nontrivial superconformal vacua (QCD?)

  Seiberg's duality ; Olive-Montonen duality in N=4 susy
  Cachazo-Douglas-Seiberg-Witten / Dijkgraaf-Vafa '03

- Semi-classical monopoles: homotopy-map, RG, exact symmetries; monopole transformations from vortex moduli

Nonabelian monopoles

Review

$$G \xrightarrow{\langle\phi\rangle\neq 0} H$$

H nonabelian

Goddard-Nuyts-Olive,E.Weinberg, Bais, Schroer, ...

$$F_{ij} = \epsilon_{ijk}\frac{r_k}{r^3}(\beta\cdot \mathrm{T}),\qquad 2\,\beta\cdot\alpha\in \mathbf{Z}$$

cfr. (Dirac)

$2\ \mathrm{m}\cdot \mathrm{e} \in \mathrm{Z}$

"Monopoles are multiplets of $\widetilde{\mathrm{H}}$ (GNOW)"

$\widetilde{\mathrm{H}}$= group generated by $\alpha^* \equiv \frac{\alpha}{\alpha\cdot\alpha}.$

$<\Phi> = v_I = h \cdot T$

H	$\widetilde{\mathrm{H}}$
U(N)	U(N)
SU(N)	$SU(N)/Z_N$
SO(2N)	SO(2N)
SO(2N+1)	USp(2N)

$$A_i(\mathrm{r}) = A_i^a(\mathrm{r}, \mathrm{h}\cdot\alpha)\,S_a;\quad \phi(\mathrm{r}) = \chi^a(\mathrm{r}, \mathrm{h}\cdot\alpha)\,S_a + [\,\mathrm{h} - (\mathrm{h}\cdot\alpha)\,\alpha^*]\cdot \mathrm{T},$$

$$S_1 = \frac{1}{\sqrt{2\alpha^2}}(E_\alpha + E_{-\alpha});\quad S_2 = -\frac{i}{\sqrt{2\alpha^2}}(E_\alpha - E_{-\alpha});\quad S_3 = \alpha^*\cdot \mathrm{T},$$

Simple Example:

$$SU(3) \xrightarrow{\langle\phi\rangle} \frac{SU(2)\times U(1)}{\mathbb{Z}_2}, \qquad \langle\phi\rangle = \begin{pmatrix} v & 0 & 0 \\ 0 & v & 0 \\ 0 & 0 & -2v \end{pmatrix}$$

$$S^1 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}; \quad S^2 = \frac{1}{2}\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}; \quad S^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\begin{aligned} \phi(\mathbf{r}) &= \begin{pmatrix} -\frac{1}{2}v & 0 & 0 \\ 0 & v & 0 \\ 0 & 0 & -\frac{1}{2}v \end{pmatrix} + 3\,v\hat{S}\cdot\hat{r}\phi(r), \\ \vec{A}(\mathbf{r}) &= \hat{S}\wedge\hat{r}A(r), \end{aligned}$$

$\phi(r)$ and $A(r)$ are BPS- 't Hooft's fnc with

$$\phi(\infty) = 1, \qquad \phi(0) = 0, \qquad A(\infty) = -1/r.$$

$\Rightarrow$ two *degenerate* $SU(3)$ solutions: Topology: $\Pi_1(\frac{SU(2)\times U(1)}{\mathbb{Z}_2}) = \mathbb{Z}$

Difficulties

① Topological obstructions (Abouelsaad et.al)

Φ = diag(v,v,-2v) ↵

e.g., In the system SU(3) → SU(2)×U(1),

∄ monopoles ~ (2, 1*)

"No colored dyons exist" (Abouelsaood, Coleman, ...)

② Non-normalizable gauge zero modes:

Monopoles not multiplets of H

(Weinberg, Coleman, Nelson, Dorey...)

cfr.
Jackiw-Rebbi
Flavor Q.N. of monopoles
via
fermion zeromodes

The real issue:
how do they transform under $\tilde{H}$?

N=1, 2, or ∞ ?

N.B. : H and $\tilde{H}$ relatively nonlocal

General considerations $G \to H$

General ideas

- Very concept of the dual group to be understood

- H nonabelian → Dynamics

  H can break itself dynamically
  (e.g., in pure N=2 YM, all monopoles are Abelian)

- "a $\tilde{H}$ multiplet" well defined if $\tilde{H}$ weakly coupled (or conformally inv)

- Duality → H strongly coupled (tough!)

  N=2 SU(N) with N_f quarks: ➞ "r" vacua with

  Monopoles in $\underline{r}$ of SU(r) !!! $r < N_f / 2$

  $SU(3) \to SU(2) \times U(1)$, $\exists$ monopoles ~ $(2^*, 1^*)$ do exist

- Phase: study $\tilde{H}$ in confinement phase: need H in Higgs phase

  $\Rightarrow$

Study:

$$G \xrightarrow{v_1} H \xrightarrow{v_2} \emptyset, \qquad v_1 \gg v_2,$$

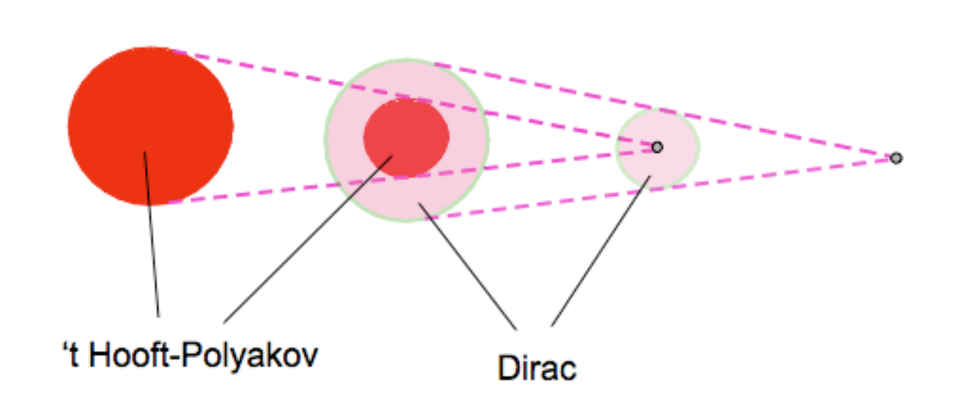
with

$$H_{C+F} \subset H_{color} \otimes G_F$$

- Flavor symmetry essential:
  - H_{C+F} necessary not to break the degeneracy;
  - H_{C+F} ⇔ H dual
  - H does not grow strong in IR
- v_2 as an infrared cut-off
- Low-energy system ($v_1 \approx \infty$) has vortices if $\pi_1(H) \neq \emptyset$
- High-energy system ($v_2 \approx 0$) has regular monopoles if $\pi_2(G/H) \neq \emptyset$
- Transformation properties under H_{C+F} from those of nonabelian vortices (Eto et.al., ASY, HT, '06)

Homotopy exact sequence

$$\cdots \longrightarrow \pi_2(G) \longrightarrow \pi_2(G/H) \longrightarrow \pi_1(H) \longrightarrow \pi_1(G) \longrightarrow \cdots$$



- Regular monopoles ⇔ Kernel of the map $\pi_1(H) \Rightarrow \pi_1(G)$ Coleman

e.g., G=SU(N), USp(2N): $\pi_1=\varnothing \Rightarrow$ No Dirac monopoles (Wu-Yang)

G=SO(N): $\pi_1=Z_2$, Z_2 monopoles; G=SU(N)/Z_N: Z_N monopoles;

Apply to: $G \xrightarrow{v_1} H \xrightarrow{v_2} \emptyset,$

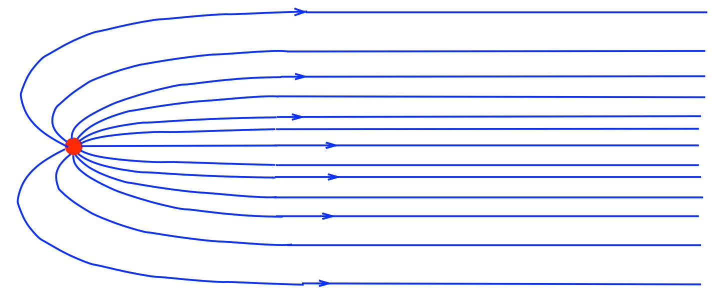
- $\pi_2(G) = \varnothing \Rightarrow$ No regular monopoles: confined by vortices

{
- If $\pi_1(G) = \varnothing \Rightarrow$ No vortices: they "end" at reg monopoles
- If $\pi_1(G) = Z_2 \Rightarrow$ No k=2 vortices: they "end" at reg monopoles!

't Hooft SO(3)/U(1)

k=1 vortices are there: they would confine Dirac monopoles

$SU(N+1) \Rightarrow U(N) \Rightarrow \emptyset$



- Unbroken exact $SU(N)_{C+F} \subset SU(N)_C \times SU(N)_F$

- Individual vortex breaks $SU(N)_{C+F} \,/\, SU(N-1) \times U(1)$

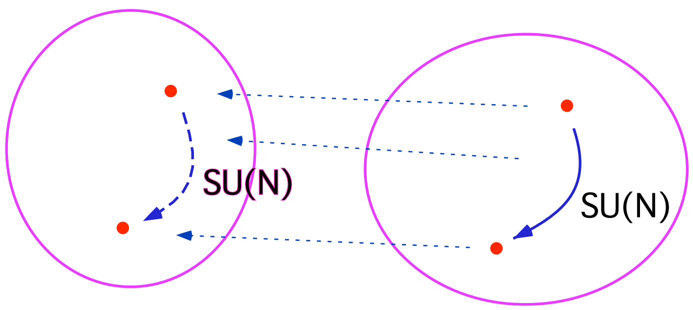
Monopole Moduli → Vortex Moduli $\sim CP^{N-1}$

- k=1 vortices transform as $\underline{N}$

$\Rightarrow$ Monopoles in $SU(N+1)/U(N)$

$\sim$ $\underline{N}$ of $SU(N)_{C+F} \equiv \widetilde{SU}(N)$ ✔

Explicit form of semiclassical monopoles not used



$\Pi_2(G/H) \sim \Pi_1(H)$

$SO(2N+1) \Rightarrow U(r) \times U(1)^{N-r} \Rightarrow \emptyset$

- r < N: k=2 vortices in $\underline{r}$ $\Rightarrow$ Monopoles ~ $\underline{r}$ of SU(r) ✔

- r = N: $SO(5) \Rightarrow U(2) \Rightarrow \emptyset$: k=2 vortices are in $\underline{3} + \underline{1}$ of SU(2) $\Rightarrow$ Monopoles (E.Weinberg) ~ $\underline{3}$ or $\underline{1}$ of SU(2)!

Higher-winding nonabelian vorices
Eto et. al. 06
ASY, HT 06

✔ = consistent with the full quantum theory

Remarks:

◇ M-V-$\bar{\text{M}}$ configurations not topologically stable

{ Vortex breaks via M $\bar{\text{M}}$ pair production (i)
Vortex become shorter and M and $\bar{\text{M}}$ annihilate (ii)

but { (ii) Can be stabilized dynamically (rotation)
(i) Resonances: only the lowest composite stable

cfr. QCD

q-S-$\bar{\text{q}}$ mesons are analogous (endpoints rotating with the light velocity: only the lightest mesons are stable) composite

◇ Monopoles (G/H) and vortices (H) both *almost* BPS.

∃ Limit in which they become exactly BPS (flux matching)

Softly broken N=2 supersymmetric SU, SO, USp

Concrete model

G ⇒ H ⇒ ∅

G=SU(N+1); H=U(N)

$$\mathcal{L} = \frac{1}{8\pi}\mathrm{Im}\ S_{cl}\left[\int d^4\theta\,\Phi^\dagger e^V \Phi + \int d^2\theta\,\frac{1}{2}WW\right] + \mathcal{L}^{(\mathrm{quarks})} + \int d^2\theta\,\mu\,\mathrm{Tr}\,\Phi^2;$$

$$\mathcal{L}^{(\mathrm{quarks})} = \sum_i\left[\int d^4\theta\,\{Q_i^\dagger e^V Q_i + \tilde{Q}_i e^{-V}\tilde{Q}_i^\dagger\} + \int d^2\theta\,\{\sqrt{2}\tilde{Q}_i\Phi Q^i + m_i\,\tilde{Q}_i\,Q^i\}\right]$$

Bosonic Lagrangean

semi-classical

$$\mathcal{L} = \frac{1}{4g^2}F_{\mu\nu}^2 + \frac{1}{g^2}|\mathcal{D}_\mu\Phi|^2 + |\mathcal{D}_\mu Q|^2 + \left|\mathcal{D}_\mu\bar{\tilde{Q}}\right|^2 - V_1 - V_2,$$

m ≫ μ ≫ Λ

$$\begin{aligned} V_2 \;&=\; g^2|\mu\,\Phi^A + \sqrt{2}\,\tilde{Q}\,t^A Q|^2 + \tilde{Q}\,[m+\sqrt{2}\Phi]\,[m+\sqrt{2}\Phi]^\dagger\,\tilde{Q}^\dagger \\ &+\; Q^\dagger\,[m+\sqrt{2}\Phi]^\dagger\,[m+\sqrt{2}\Phi]\,Q. \end{aligned}$$

☆

$v_1 = m$

$v_2 = \sqrt{\mu m}$

$$\langle\Phi\rangle = -\frac{1}{\sqrt{2}}\begin{pmatrix} m & 0 & 0 & 0 \\ 0 & \ddots & \vdots & \vdots \\ 0 & \dots & m & 0 \\ 0 & \dots & 0 & -N\,m \end{pmatrix};$$

$$Q = \tilde{Q}^\dagger = \begin{pmatrix} d & 0 & 0 & 0 & 0 & \dots \\ 0 & \ddots & 0 & \vdots & \vdots & \dots \\ 0 & 0 & d & 0 & 0 & \dots \\ 0 & \dots & 0 & -N\,d & 0 & \dots \end{pmatrix}, \qquad d = \sqrt{(N+1)\,\mu\,m} \ll m.$$

SU(N+1) ⇒ U(N)

Low-energy U(N) theory

Auzzi et al., Hanany-Tong, Shifman-Yung, Eto, Nitta, Ohashi, Mizoguchi, ... , Sakai

$$\mathcal{L} = \frac{1}{4g_N^2}(F_{\mu\nu}^a)^2 + \frac{1}{4e^2}(\tilde{F}_{\mu\nu})^2 + |\mathcal{D}_\mu q|^2 - \frac{e^2}{2}\,|\,q^\dagger\,q - c\,1\,|^2 - \frac{1}{2}\,g_N^2\,|\,q^\dagger\,t^a q\,|^2,$$

Nonabelian vortex Bogomolnyi equations (HT, ABEKY '03)

$$(\mathcal{D}_1 + i\mathcal{D}_2)\;q = 0,$$

$$F_{12}^{(0)} + \frac{e^2}{2}\,(c\,1_N - q\,q^\dagger) = 0; \qquad F_{12}^{(a)} + \frac{g_N^2}{2}\,q_i^\dagger\,t^a\,q_i = 0.$$

Solution

$$q = S^{-1}(z,\bar{z})\,H_0(z), \quad A_1 + i\,A_2 = -2\,i\,S^{-1}(z,\bar{z})\,\bar{\partial}_z S(z,\bar{z}),$$

S is an invertible matrix

$$\partial_z\,(\Omega^{-1}\partial_{\bar{z}}\,\Omega) = -\frac{g_N^2}{2}\,\mathrm{Tr}\,\left(\,t^a\,\Omega^{-1}\,H_0\,H_0^\dagger\right)t^a - \frac{e^2}{4N}\,\mathrm{Tr}\,\left(\,\Omega^{-1}H_0\,H_0^\dagger - 1\right).$$

$$\Omega = S\,S^\dagger$$

Master equation

$H_0(z)$: moduli matrix

- holomorphic in z = x + i y
- Det H_0 ~ z^k : k = vortex winding number

Eto, Nitta, Ohashi, Mizoguchi, ... , Sakai '05, '06

- H_0 contains all the moduli parameters;
- H_0 ⇒ Moduli space/ transformation of the vortices under G_{C+F}
- H_0 defined up to reparametrizations H_0 ⇒ V(z) H_0; S ⇒ V S

Detailed study of k=2 (axially symmetric) vortices of U(N) ⇒ ∅ theory

k=2 vortex moduli and transformations

$$H_0^{(1,0)}(z) = \begin{pmatrix} z - z_0 & 0 \\ -b' & 1 \end{pmatrix}, \qquad H_0^{(0,1)}(z) = \begin{pmatrix} 1 & -b \\ 0 & z - z_0 \end{pmatrix}$$

$$b = \frac{1}{b'} \qquad CP^1 \approx \underline{2}$$

k=1 vortex

SU(2)

moduli matrix (z = x + i y)

$$H_0^{(2,0)} = \begin{pmatrix} z^2 & 0 \\ -a'\,z - b' & 1 \end{pmatrix}, \; H_0^{(1,1)} = \begin{pmatrix} z-\phi & -\eta \\ -\tilde{\eta} & z+\phi \end{pmatrix}, \; H_0^{(0,2)} = \begin{pmatrix} 1 & -a\,z - b \\ 0 & z^2 \end{pmatrix}$$

$$X\,Y \equiv -\phi, \quad X^2 \equiv \eta, \quad Y^2 \equiv -\tilde{\eta}. \qquad \phi^2 + \eta\,\tilde{\eta} = 0. \qquad \det H = z^2$$

co-axial

$$\begin{pmatrix} a' \\ 1 \\ b' \end{pmatrix} \sim \begin{pmatrix} 1 \\ X \\ Y \end{pmatrix} \sim \begin{pmatrix} -a \\ b \\ 1 \end{pmatrix}$$

$$\tilde{\mathcal{M}}_{N=2,k=2} \simeq WCP^2_{(2,1,1)} \simeq$$
$$\simeq CP^2/Z_2 \; \simeq \; CP^2$$

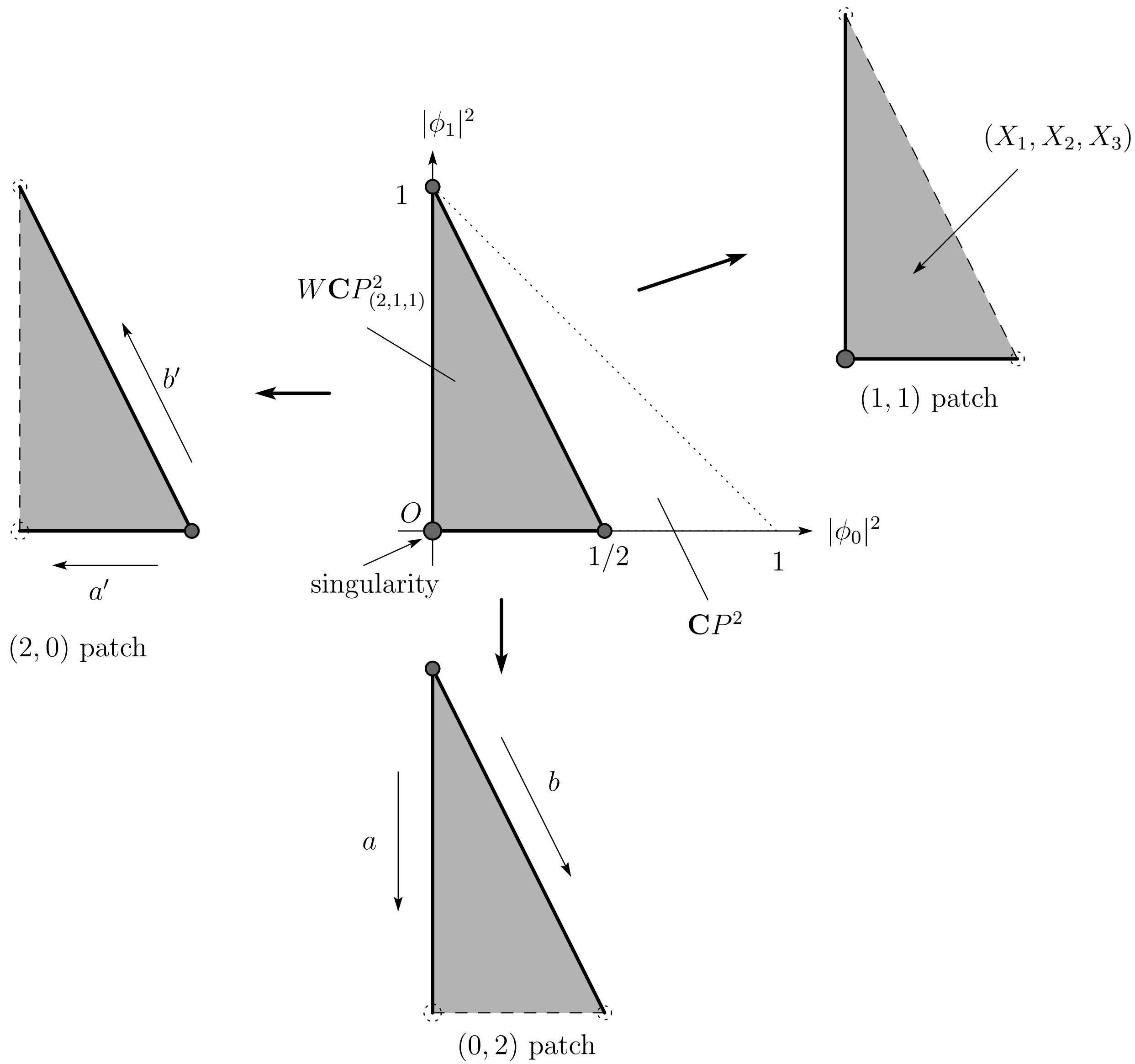
$$\vec{X}\cdot\vec{\sigma} \to U^\dagger \left(\vec{X}\cdot\vec{\sigma}\right) U, \qquad X_1^2 + X_2^2 + X_3^2 = 0$$

$$H_0^{(1,1)} = z\,1_2 - \vec{X}\cdot\vec{\sigma}$$

Summary: k=2 (axially symmetric) vortices transform as

$\underline{3} + \underline{1}$

Eto, Konishi, Marmorini, Nitta, Ohashi, Vinci, Yokoi '06



$|\phi_1|^2$
1
$W\mathbf{C}P^2_{(2,1,1)}$
O
singularity
1/2
1
$|\phi_0|^2$
$\mathbf{C}P^2$
(X_1, X_2, X_3)
(1, 1) patch
b'
a'
(2, 0) patch
a
b
(0, 2) patch

Eto, Ferretti, Konishi, Marmorini, Nitta, Ohashi, Vinci, Yokoi '07

SO(5) ⇒ U(2) ⇒ ∅

$$\langle \Phi \rangle = \begin{pmatrix} 0 & i\,v & 0 & 0 & 0 \\ -i\,v & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i\,v & 0 \\ 0 & 0 & -i\,v & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \simeq T_3^+$$

These monopoles are confined by k=2 vortices

~ transform as 3 + 1 of $SU(2)_{C+F}$

cfr. E. Weinberg

$$T_1^{\pm} = -\frac{i}{2}(\Sigma_{23} \pm \Sigma_{41}), \quad T_2^{\pm} = -\frac{i}{2}(\Sigma_{31} \pm \Sigma_{42}), \quad T_3^{\pm} = -\frac{i}{2}(\Sigma_{12} \pm \Sigma_{43}),$$

$SO(4)_{1234} \sim SU(2) \times SU(2) \rightarrow SU_-(2) \times U_+(1)$, $SO(3)_{125}$, $SO(3)_{345}$

SO(7) ⇒ U(2) × U(1) ⇒ ∅

$$\langle \phi_1 \rangle = \begin{pmatrix} 0 & iv_0 & 0 & 0 & 0 & 0 & 0 \\ -iv_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & iv_0 & 0 & 0 & 0 \\ 0 & 0 & -iv_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & iv_1 & 0 \\ 0 & 0 & 0 & 0 & -iv_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

→ Semiclassical monopoles in

2 + 3 + 1

2

from the breaking of $SO(4)_{1256}$ and $SO(4)_{3456}$

→ Carry (2 , 1) charges

These are confined by k=2 (π_1 (G)= Z_2) vortex, but with flux in

Min [π_1(U(2)] AND Min [π_1(U(1))] ~ transform as 2 of $SU(2)_{C+F}$

$SO(2N+1), SO(2N) \Rightarrow U(r) \times U(1) \times U(1) \times \cdots \Rightarrow \emptyset$

Qualitative difference for r=N (maximum rank) and r<N :

- ◆ H= U(N) monopoles in □□ or in □ over □ of SU(N)

RG: AF of G: $N_f < 2N-1$ (2N-2)
IF of H: $N_f > 2N$
} Not compatible

- ◆ H= $U(r) \times \cdots$ monopoles in □ of SU(r): OK for $r < N_f/2$

☀ Only light monopoles in □ of SU(r), $r < N_f/2$ in the full quantum analysis

(Carlino, KK, Kumar, Murayama '01)

$USp(2N) \Rightarrow U(r) \times U(1) \times U(1) \times \cdots, \quad r \leq N$

G simply connected:
Monopoles in r both {
from k=1 vortices of $\pi_1(U(r))$
fully quantum analysis (Carlino, KK, Murayama)

Fully quantum mechanical analysis at μ ~ m ~ Λ

(Carlino, K.K. , Kumar, Murayama, 2000, 2001; Hanany-Oz, 1998)

QM

- Semiclassical analysis at μ , m ≫ Λ;
- Decoupling analysis at μ ⇒ ∞, Λ ,m fixed;
- Quantum analysis at μ ⇒ ∞ Λ(N=1) m fixed. N=1 ADS instanton superpotential
- Fully quantum mechanical analysis at μ , m ∼ Λ; Seiberg-Witten curves

$$y^2 = \prod_{k=1}^{n_c} (x - \phi_k)^2 + 4\Lambda^{2n_c - n_f} \prod_{j=1}^{n_f} (x + m_j)|_{m_i=0}$$

- Low-energy effective actions at μ , m ∼ Λ;
- m perturbation of the conformal invariant points (Eguchi et. al)
- Vacuum counting

L_{eff}

	$SU(r)$	$U(1)_0$	$U(1)_1$	$\dots$	$U(1)_{n_c-r-1}$	$U(1)_B$
$n_f \times q$	$\underline{\mathbf{r}}$	$\mathbf{1}$	$\mathbf{0}$	$\dots$	$\mathbf{0}$	$\mathbf{0}$
e_1	$\underline{\mathbf{1}}$	0	1	$\dots$	$\mathbf{0}$	$\mathbf{0}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
e_{n_c-r-1}	$\underline{\mathbf{1}}$	0	0	$\dots$	1	0

Phases of Softly Broken $\mathcal{N}=2$ Gauge Theories

label (r)	Deg.Freed.	Eff. Gauge Group	Phase	Global Symmetry
0	monopoles	$U(1)^{n_c-1}$	Confinement	$U(n_f)$
1	monopoles	$U(1)^{n_c-1}$	Confinement	$U(n_f-1)\times U(1)$
$\leq [\frac{n_f-1}{2}]$	NA monopoles	$SU(r)\times U(1)^{n_c-r}$	Confinement	$U(n_f-r)\times U(r)$
$n_f/2$	rel. nonloc.	-	Confinement	$U(n_f/2)\times U(n_f/2)$
BR	NA monopoles	$SU(\tilde{n}_c)\times U(1)^{n_c-\tilde{n}_c}$	Free Magnetic	$U(n_f)$

Table 1: Phases of $SU(n_c)$ gauge theory with n_f flavors. $\tilde{n}_c \equiv n_f - n_c$.

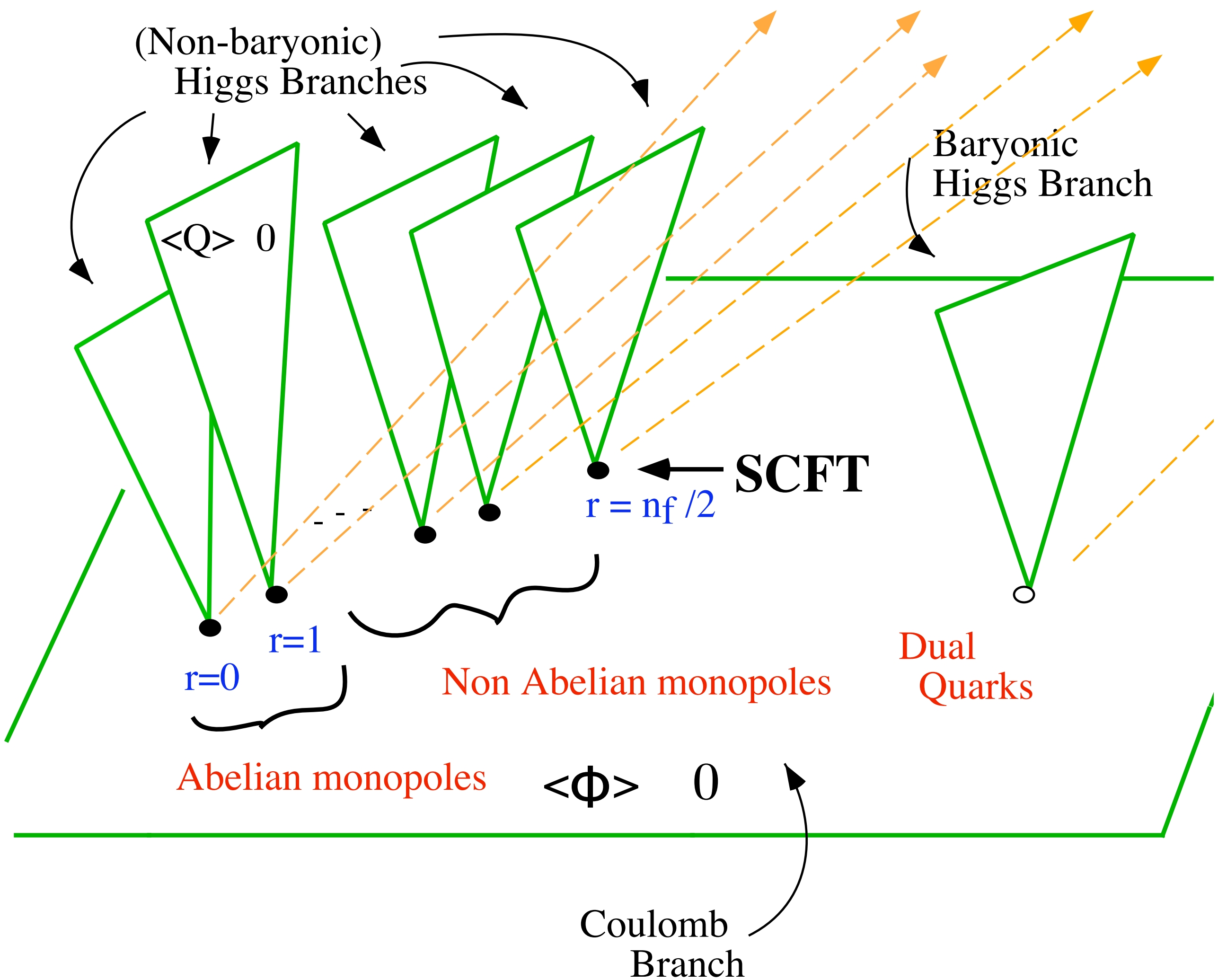
	Deg.Freed.	Eff. Gauge Group	Phase	Global Symmetry
1st Group	rel. nonloc.	-	Confinement	$U(n_f)$
2nd Group	dual quarks	$USp(2\tilde{n}_c)\times U(1)^{n_c-\tilde{n}_c}$	Free Magnetic	$SO(2n_f)$

Table 2: Phases of $USp(2n_c)$ gauge theory with n_f flavors with $m_i \to 0$. $\tilde{n}_c \equiv n_f - n_c - 2$.

$$\mathcal{W}(\phi, Q, \tilde{Q}) = \mu\, \mathrm{Tr}\Phi^2 + m_i \tilde{Q}_i Q^i, \qquad m_i \to 0$$

Dual qualks of r vacua are GNO monopoles

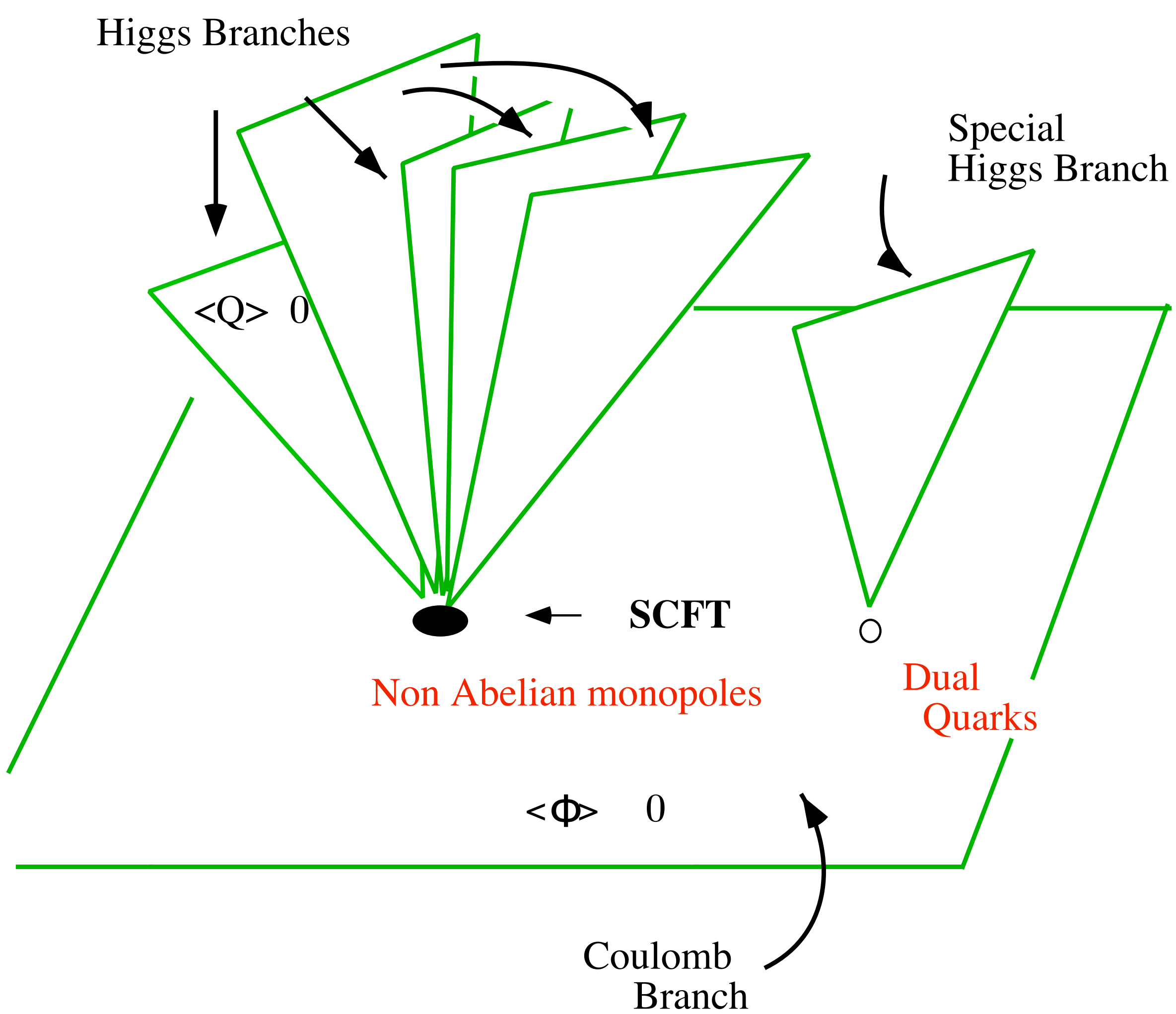
QMS of N=2 SQCD (SU(n) with n_f quarks)



● N=1 Confining vacua (with $\mu\phi^2$ perturbation)

○ N=1 vacua (with $\mu\phi^2$ perturbation) in free magnetic pha

QMS of N=2 USp(2n) Theory with n_f Quarks



● N=1 Confining vacua (with $\mu\Phi^2$ perturbation)

○ N=1 vacua (with $\mu\Phi^2$ perturbation) in free magnetic pha

Various observations

Marmorini, Yokoi, KK '05
Carlino, Murayama, KK '00

- GNOW monopoles not always the correct IR degrees of freedom

  e.g. $USp(2r) \Leftrightarrow SO(2r+1)$ in N=2 models: wrong symmetry

  $SU(N) \Leftrightarrow SU(N_f - N)$ Seiberg duality

- Nonabelian monopoles $\sim$ baryonic components of Abelian monopoles

$$A \sim \epsilon^{a_1 \dots a_r} q^{i_1}_{a_1} q^{i_2}_{a_2} \cdots q^{i_r}_{a_r}$$

$\longrightarrow$ $SU(N_f)$ flavor

$\longrightarrow$ $SU(r)$ color

(Neccesary in order not to violate Nambu-Goldstone theorem)

QCD: if Abelian monopoles $M_i^{\,j} \sim (N_f, N_f)$ of $SU(N_f)_L \times SU(N_f)_R$

$\langle M \rangle \neq 0 \quad \Rightarrow$ Confinement & chiral symm. breaking

but too many NG bosons

- SCFT vacua in USp : Global symm ($SO(2N_f)$) not realized by local fields
- Seiberg duality $\Leftarrow$ vacuum counting; matching classical and quantum r vacua

SCFT

Strongly interacting SCFT and confinement

- r= N_f /2 vacua of SU(N) theory; all confining vacua of SO and USp theories with μ~Λ, m⇒0, are nontrivial SCFT

- Relatively nonlocal dyons ⇒ no local effective action

- SU(2)x U(1) vacua of SU(3), N_f =4 theory, G_f = U(4): 4 $\underline{2}$ monopoles + 2 $\underline{2}$ dyons Confinement and DSB due to ⟨ ϵ M M ⟩ ≠ 0. U(4) ⇒ U(2) x U(2) Auzzi-Grena-KK

- SU(2)x U(1) vacua of USp(4), N_f =4 theory, G_f = SO(8): 4 $\underline{2}$ monopoles + 2 $\underline{2}$ dyons + $\underline{2}$ quarks. Confinement and DSB due to ⟨ M $\tilde{M}$ ⟩ ≠ 0. SO(8) ⇒ U(4) Auzzi-Grena

Non abelian Argyres-Douglas vacua

QCD?

Particles	Charge
M_1, M_2	$(\pm 1, 1, 0, 0)^4$
D_1, D_2	$(\pm 2, -2, \pm 1, 0)$
E_1, E_2	$(0, 2, \pm 1, 0)$
C_1, C_2	$(\pm 2, 0, \pm 1, 0)$

Particles	$(g_1, g_2; q_1, q_2)$
M_1, M_2	$(\pm 1, 1; 0, 0)^4$
D_1, D_2	$(\pm 2, -2; \pm 1, 0)$
E_1, E_2	$(0, 2; \pm 1, 0)$

Conjecture

- QCD : a conjecture

Non-Abelian monopoles of dual

$SU(3) \longrightarrow U(2) \longrightarrow X$

$M_L \sim (N_f, 1), \; M_R \sim (1, N_f)$ of $SU_L(N_f) \times SU_R(N_f)$

$$\langle M_L M_R \rangle \sim \Lambda^2 \delta_{ij} \qquad (\ast)$$

($\ast$) leads to confinement / XSB

$\langle \psi_L \overline{\psi}_R \rangle \sim \Lambda^3 \delta_{ij}$ induced by ($\ast$)

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Summary:

① Many fully quantum-mechanical properties about the nonabelian monopoles in G= SU, SO, USp theories are known (their existence, their q.n., etc.) ('94 - '06)

Nonabelian dual gauge groups occur only in theories with flavor

◇ New results on the vortex moduli in H= U(N) theories $H \Rightarrow \emptyset$ ('03 - '06) ⇒

◇ New results on the vortex-monopole complex in $SO(N+2) \Rightarrow SO(N) \times U(1)$

Ferretti, Konishi, '07

② Monopole properties from $\tilde{H} \sim H_{C+F}$, as seen in $G \Rightarrow H \Rightarrow \emptyset$

♡ Agreement between ① and ②

③ Model of confinement (nonabelian) : natural generalization of Nambu, 't Hooft, Mandelstam picture