Baryons as Holographic Solitons

Pre-Thesis Seminar

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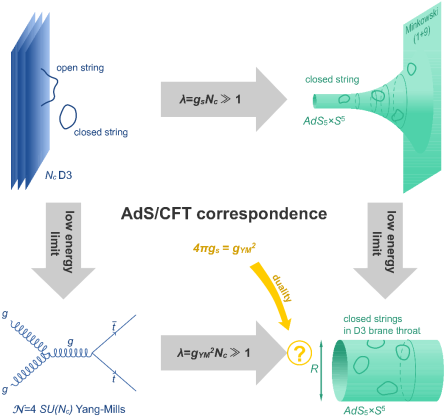
- Holographic QCD *(Review)*
- Mesons and Large λ Baryons *(Review)*
- Holographic Sextic Term *(L.B, S.Bolognesi, A.Proto 2017)*
- Small λ Baryons *(L.B, S.Bolognesi, A.Proto 2017)*
- Holographic θ: NEDM *(L.B, F.Bigazzi, S.Bolognesi, A.L.Cotrone, A.Manenti 2016)*
- NLO: EDM splitting *(L.B, S.Bolognesi work in progress...)*
- Conclusions

What is holography?

Conjecture that a duality exists

(Supersymmetric) QFTs in flat space $\rightleftarrows$ (Super)Gravity in higher dimension

Born from String-Theory



Why holography?

Strong/weak duality

$\Downarrow$

Perturbative QFT $\leftrightarrow$ Strongly coupled gravity

Strongly coupled QFT $\leftrightarrow$ Perturbative gravity

$\Downarrow$

Great for studying the rich non-perturbative sector of strongly coupled QFTs

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BARYONS in QCD

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BARYONS in QCD

Holographic QCD

1997: Maldacena introduces AdS/CFT correspondence

1998: Witten develops the confining background geometry

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Bulk geometry fixes a length scale dual to confinement scale

Holographic QCD models become feasible

WSS Model

Key elements:

- COLOR: Background 10d geometry from string theory
- FLAVOR: 5D effective $U(N_f)$ Yang-Mills/Chern-Simons theory

Compactify to 5d

Flavor gauge field:

$$\mathcal{A}_\alpha = \widehat{A}_\alpha \frac{\mathbb{I}}{N_f} + A^a_\alpha \frac{\tau^a}{2}$$

$$\alpha = 0, i, z$$

WSS Model

Key elements:

- COLOR: Background 10d geometry from string theory
- FLAVOR: 5D effective $U(N_f)$ Yang-Mills/Chern-Simons theory

$$S_{YM} = -\frac{N_c \lambda}{216\pi^3} \mathrm{tr} \int d^4x dz \left[k(z)\mathcal{F}_{\mu z}^2 + \frac{1}{2} h(z) \mathcal{F}_{\mu\nu}^2 \right]$$

$$S_{CS} = \frac{N_c}{384\pi^2} \epsilon_{\alpha_1 \cdots \alpha_5} \int d^4x dz \widehat{A}_{\alpha_1} \left[3F^a_{\alpha_2\alpha_3} F^a_{\alpha_4\alpha_5} + \widehat{F}_{\alpha_2\alpha_3} \widehat{F}_{\alpha_4\alpha_5} \right]$$

$$\boxed{k(z) = 1 + z^2 \; ; \; h(z) = k(z)^{-1/3}}$$

Mesons

We can move to the $\mathcal{A}_z = 0$ gauge (easier 4d interpretation)
Meson modes expansion:

$$\mathcal{A}_\mu = \mathcal{U}^{-1}\partial_\mu \mathcal{U}\psi_+(z) + \sum_1^{+\infty} B_\mu^{(n)}(x)\psi_{(n)}(z)$$

Holographic profile functions are eigenmodes of:

$$-h(z)\partial_z(k(z)\partial_z\psi_n) = \lambda_n\psi_n$$

$4d$ Effective theory of massless scalar and infinite massive vectors

The model automatically contains ALL the vector mesons

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Baryons

First remark: if we only include pions

$$\mathcal{A}_\mu = \mathcal{U}^{-1}\partial_\mu \mathcal{U}\psi_+(z)$$

Drop the CS term, integrate away bulk, we find that:

Skyrme model in the LOW ENERGY regime

$$f_\pi = \sqrt{\frac{\kappa}{\pi}} \quad ; \quad e \sim -\frac{1}{2.5\kappa}$$

Actual full description:

In the LARGE λ limit: solitonic configuration of mesonic fields

BPST instanton of small size $\rho \sim \lambda^{-1/2}$ located deep in the bulk

The Sextic term

A potential which is sextic in the derivatives can be added to the Skyrme model:

$$\mathcal{L} = \frac{\gamma^2}{24^2}\left[\epsilon^{\mu\nu_1\nu_2\nu_3}\left(R_{\nu_1}R_{\nu_2}R_{\nu_3}\right)\right]^2$$

With $R_\mu = \mathcal{U}^{-1}\partial_\mu\mathcal{U}$

A sextic term can be generated by extending the Skyrme model with ω-mesons and integrating them away

$\Downarrow$

In the holographic model we should not need to add anything: every mesonic mode we want to use is already there.

The Sextic term

Instead of setting vectors to zero:
Abelian ansatz and factorization for the vector meson part:

$$\mathcal{A}_\mu = \begin{cases} \widehat{A}_\mu = B_\mu(x)\chi(z) \\ A_\mu = \mathcal{U}^{-1}\partial_\mu \mathcal{U}\psi_+(z) \end{cases}$$

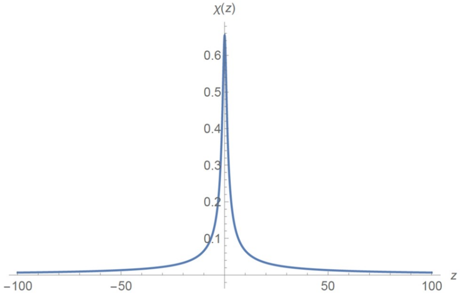
Equation of motion is nicely decoupled as

$$2z\chi' + k(z)\chi'' = \frac{N_c}{16\kappa\pi^2}\psi_+\psi_+'(i\psi_+ - 1)$$

$$B_\mu(x) = -\epsilon_\mu^{\ \ z\nu_1\nu_2\nu_3}\mathrm{Tr}\left(R_{\nu_1}\left[R_{\nu_2}, R_{\nu_3}\right]\right)$$

Holographic profile function

$$\chi = -\frac{N_c}{64\pi^3\kappa}\left(\frac{5\pi^2}{48} - \frac{1}{2}\arctan^2(z) + \frac{1}{3\pi^2}\arctan^4(z)\right)$$



Which modes are we integrating out?

$\chi(z)$ is even $\Rightarrow$ Vector mesons

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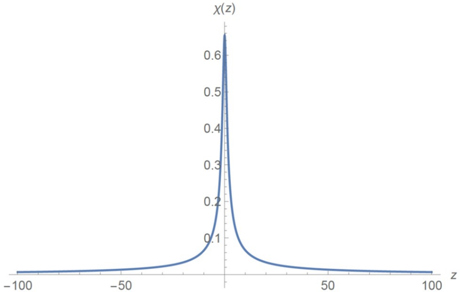

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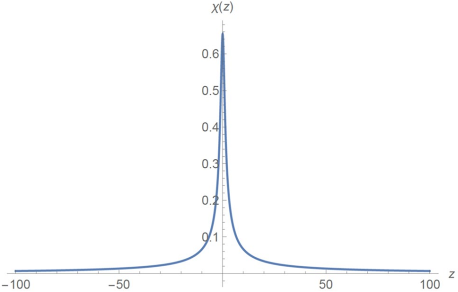

$$a_n \equiv |\langle \chi^{(norm)}, \psi_n \rangle|^2$$

$$a_1 = 0,988 \quad ; \quad a_3 = 0,0115 \quad ; \quad a_5 = 0,00029$$

Holographic profile function

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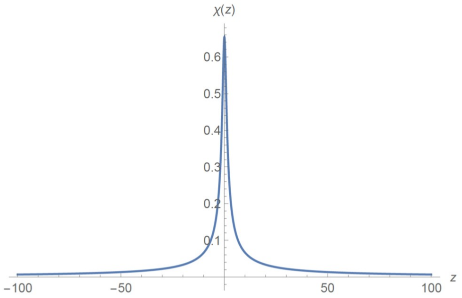

Mostly the ω meson (expected)

Really every vector meson with same quantum numbers

Resulting sextic term

Plug configuration in $S_{YM} + S_{CS}$ and integrate bulk direction

$$S_6 = \frac{51 N_c}{8960\lambda} \int d^4x \left[\epsilon^{\mu z \nu_1 \nu_2 \nu_3} \mathrm{Tr}\left(R_{\nu_1} R_{\nu_2} R_{\nu_3}\right)\right]^2$$

Quark mass produces a pion mass potential

$$S_0 = 4mc \int d^4x \left(\sigma - 1\right)$$

$$\mathcal{U} \equiv \sigma + i\vec{\pi} \cdot \vec{\tau}$$

The SSM "contains" a Generalized Skyrme Model

$$\mathcal{L}_{GSM} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \mathcal{L}_0$$

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The small λ limit

We already know the picture in the large λ regime:

BPS 5d instanton of size $\mathcal{O}(\lambda^{-1/2})$

We have derived a GSM as a low energy effective field theory: what happens to this picture in the new SMALL λ limit?

make use of Derrick's theorem

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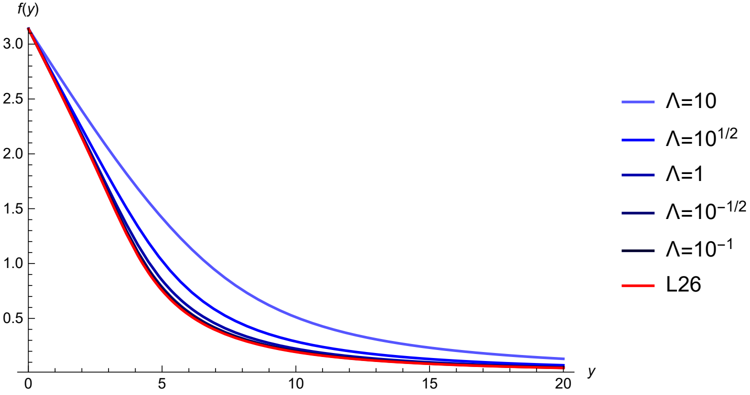
Different outcomes for massive or massless quarks

Massless quarks case

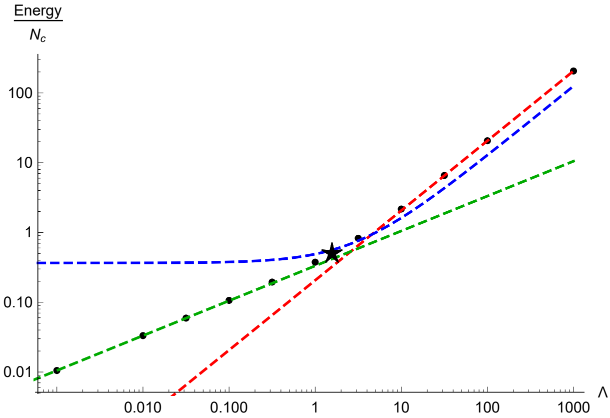
$\mathcal{L}_2$ and $\mathcal{L}_6$ are dominant



Skyrmion profiles for various values of λ rescaled to the same size. In red the solution of the model $\mathcal{L}_2 + \mathcal{L}_6$

Massless quarks case

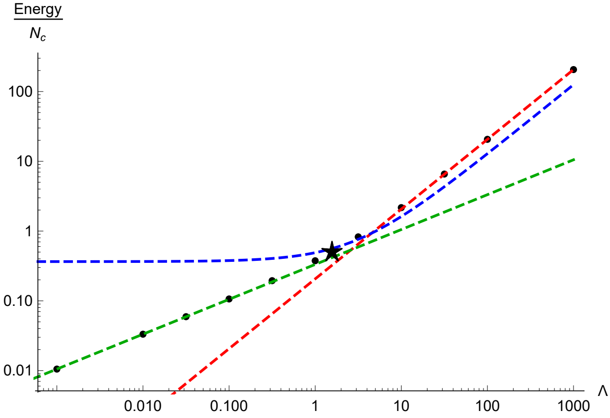
Energy as a function of λ



Green and red lines: asymptotic power laws (Derrick)
Blue line: correct behaviour for large λ
Black star: phenomenological λ

Massless quarks case

Energy as a function of λ



Phenomenological λ is exactly in the region where it cannot be regarded neither as SMALL or LARGE

Massive quarks case

$\boxed{\mathcal{L}_0 \text{ and } \mathcal{L}_6 \text{ are dominant}}$

$\mathcal{L}_0 + \mathcal{L}_6$ is the "BPS Skyrme Model"

$$\Downarrow$$

It admits an analytic solution of the compacton type

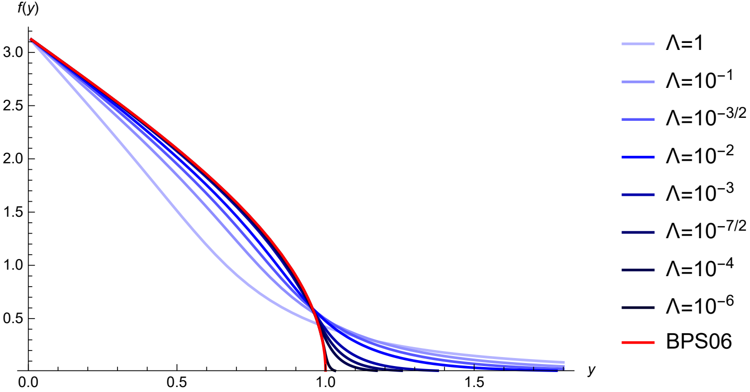
$$f(r) = \begin{cases} 2\arccos\left(Ar\right) & \text{for} \quad r \in \left[0, A^{-1}\right] \\ 0 & \text{for} \quad r \geq A^{-1} \end{cases}$$

$$A^{-1} = \sqrt[3]{\frac{4\sqrt{\alpha}}{\Lambda m_\pi}}$$

$$\alpha = 76,701 \quad ; \quad \Lambda = \frac{8\lambda}{27\pi}$$

Massive quarks case

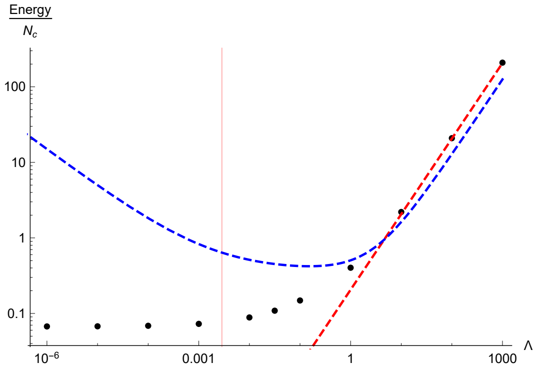
Rescale again with A^{-1}



In red the analytic compacton solution. Again, numerical solutions confirm the expected behaviour

Massive quarks case

Energy as a function of λ



Red dashed line: asymptotic linear law

Red vertical line: size corresponding to $R \sim m_\pi^{-1}$

Blue line: correct behaviour at large λ

Holographic θ term

$$L_\theta^{QCD} = -\theta \underbrace{\frac{1}{32\pi^2}\epsilon^{\mu_1\cdots\mu_4}\int d^4x \mathrm{tr}\left(G_{\mu_1\mu_2}G_{\mu_3\mu_4}\right)}_{\text{instanton number}}$$

θ dual to Ramond-Ramond 1-Form

$$\frac{1}{\ell_s}\int_{Cigar} F_2 = \theta$$

$$F_2 = dC_1 \xRightarrow{Stokes} \frac{1}{\ell_s}\int_{S^1|_{UV}} C_1 = \theta$$

Enters quark mass holographic action (as expected from QCD) via $\widehat{A}_z$ (anomaly inflow)

Neutron electric dipole moment

L_{θ}^{QCD} breaks CP: can give a nonvanishing NEDM

Outside the domain of perturbative techniques (instantons)

Lattice techniques plagued by sign problem

- Experimental upper bound $|\mathcal{D}_n^{exp}| < 3 \times 10^{-26} e{\cdot}\text{cm}$
- Theoretical estimates (eff. theories) $\mathcal{D}_n^{th} \sim 10^{-16} \theta e \cdot$ cm

$$\theta \lessapprox 10^{-10}$$

Strong CP problem

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Strong CP problem

Holgraphic NEDM in a slide

Perturb baryon at first order in θ, m

$$\mathcal{A} = \mathcal{A}^{bar} + m\theta\delta\mathcal{A}$$

- Solve perturbed e.o.m.$\Rightarrow \delta A_0 \propto W(r,z)\vec{x}\cdot\vec{\tau}$
- Insert in holographic EM density charge J^0_{EM}
- Insert in EDM operator $\mathcal{D}^i = \int d^3x x^i J^0_{EM}$
- Evaluate on unperturbed $|n\rangle$ $(\delta|n\rangle \sim \theta^2)$

$$\boxed{\mathcal{D}_n \simeq +2\times 10^{-16}\theta e\cdot cm}$$

Scaling: $\mathcal{D}_n \propto \lambda^{-3/2}(\ldots + N_c^{-1}\text{quantum corrections})$

Nucleon's EDM splitting?

$$\boxed{\mathcal{D}_p = -\mathcal{D}_n}$$

Common problem for solitonic nucleons: same object, just opposite spinning direction

But charge operator breaks Isospin, so we expect a splitting

How to produce it?

- $\mathcal{D}_p = -\mathcal{D}_n$ because $\delta A_0(t) \propto I_3$
- Only possibility: $\vec{\tau}$ only vector in A^{bar}
- We should need a $\delta\mathcal{A} \propto \vec{J}$

Possible solution: take into account time dependence in $\mathcal{A}^{bar}$ (go to NLO)

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State of the problem

We constructed the most generic ansatz proportional to $\vec{J}$ and wrote down the relevant equations

We computed the scaling of the splitting with the parameters of the model: $\lambda^{-5/2}N_c^{-2}$

Problems we are dealing with are:

- 11 diff. equations in two variables for $\delta A_{z,i}$
- One more equation for $\delta\widehat{A}_0$ (this should not be an issue)
- The $\delta A_0 \simeq W\vec{x}\cdot\vec{\tau}$ may contribute once derived w.r.t. time: ansatz still working?

Summing up our results

- We obtained a Generalized Skyrme model within the holographic model of Sakai-Sugimoto.
- The mechanism with which it is obtained resembles of the old idea of integrating out the ω meson, extending it to the whole tower of states with the same quantum numbers.
- In the small λ regime a $4d$ BPS Skyrme model is obtained
- The SSM interpolates with λ between two BPS models
- Phenomenological λ is not in any of the two regimes: can small nuclear binding energies be thought as a consequence of (inevitable?) closeness to a BPS model?
- We identified the mechanism that splits EDM of nucleons
- Order of the splitting: $\lambda^{-5/2} N_c^{-2}$

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Thanks for your attention