UNIVERSITÀ DI PISA

ITALIAN NATIONAL AGENCY FOR NEW TECHNOLOGIES, ENERGY AND SUSTAINABLE ECONOMIC DEVELOPMENT

Study of magnetic islands rotation in tokamaks

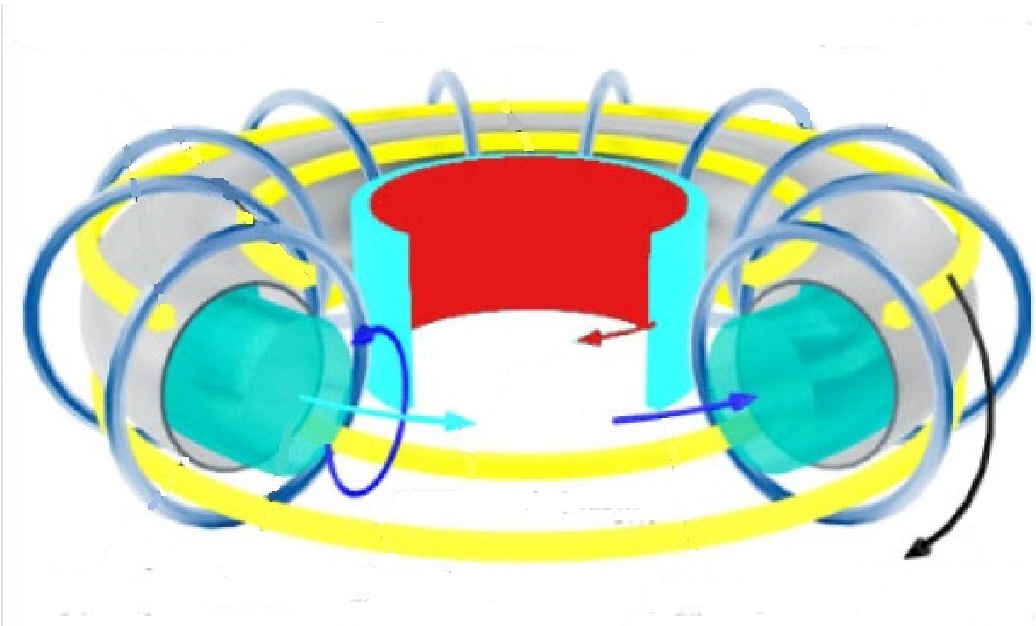
Andrea Casolari*† & Paolo Buratti**

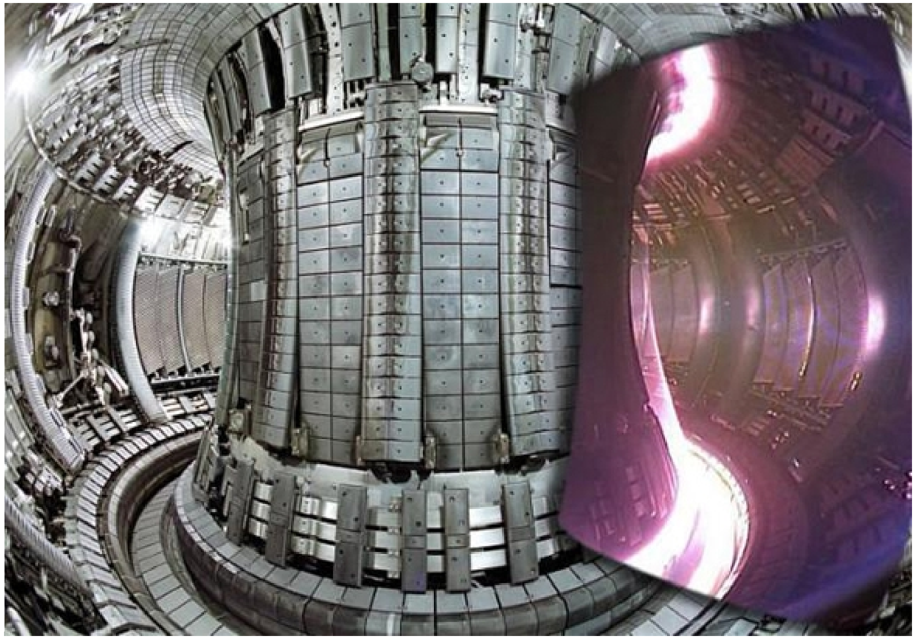
*Dipartimento di Fisica Enrico Fermi, Università di Pisa
†ENEA guest
**ENEA-FSN-FUSPHY-TSM, C. R. Frascati, Frascati, Italy

Motivation of the study

- Plasma equilibrium is effected by instabilities
- Magnetic islands are the result of tearing mode
- They cause the breaking of magnetic surfaces
- This causes increased heat and particles flux
- The study of islands rotation has important diagnostic applications
- A solid theory of islands rotation is still missing

Magnetic confinement

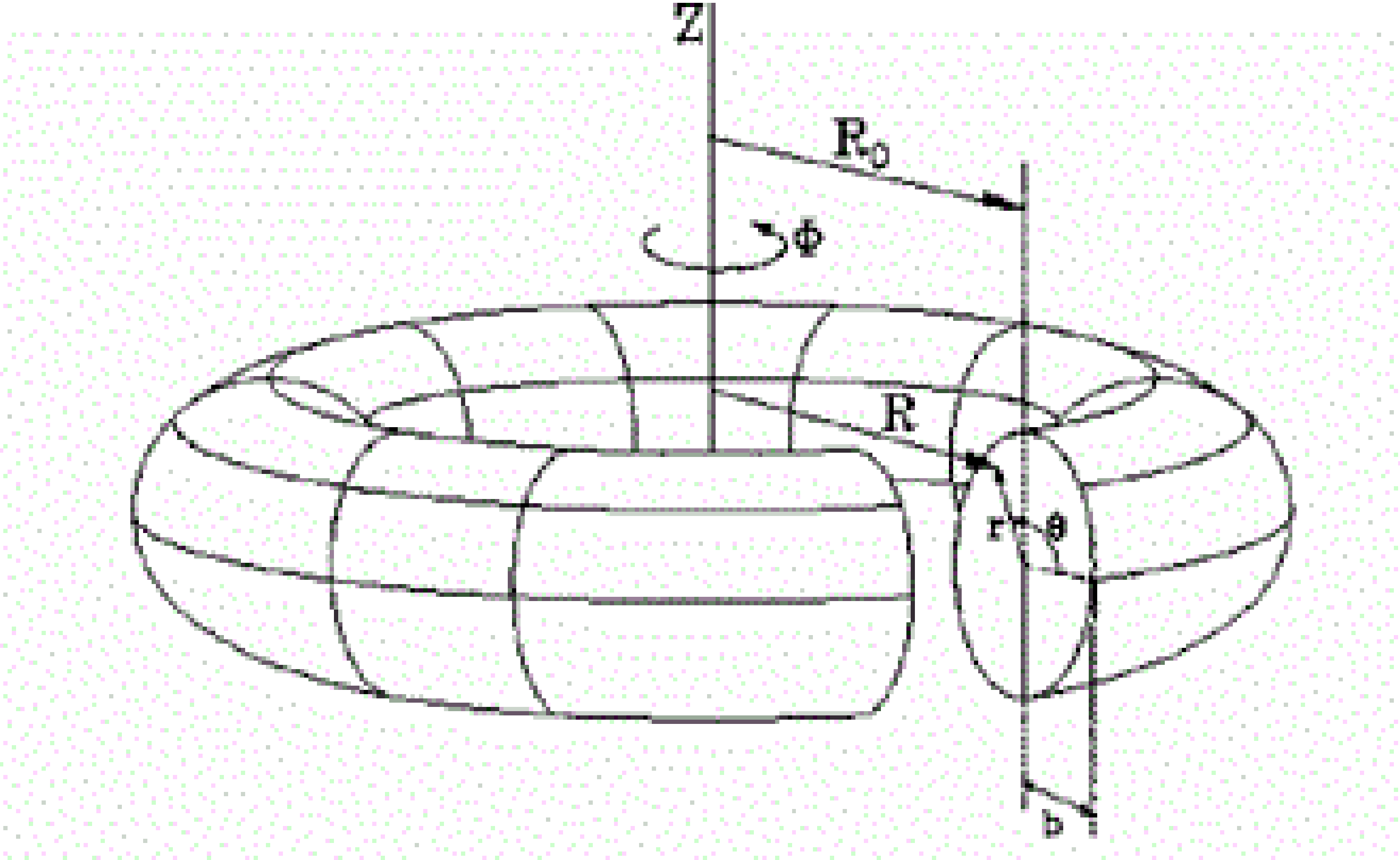




Tokamak structure and view of the JET inside

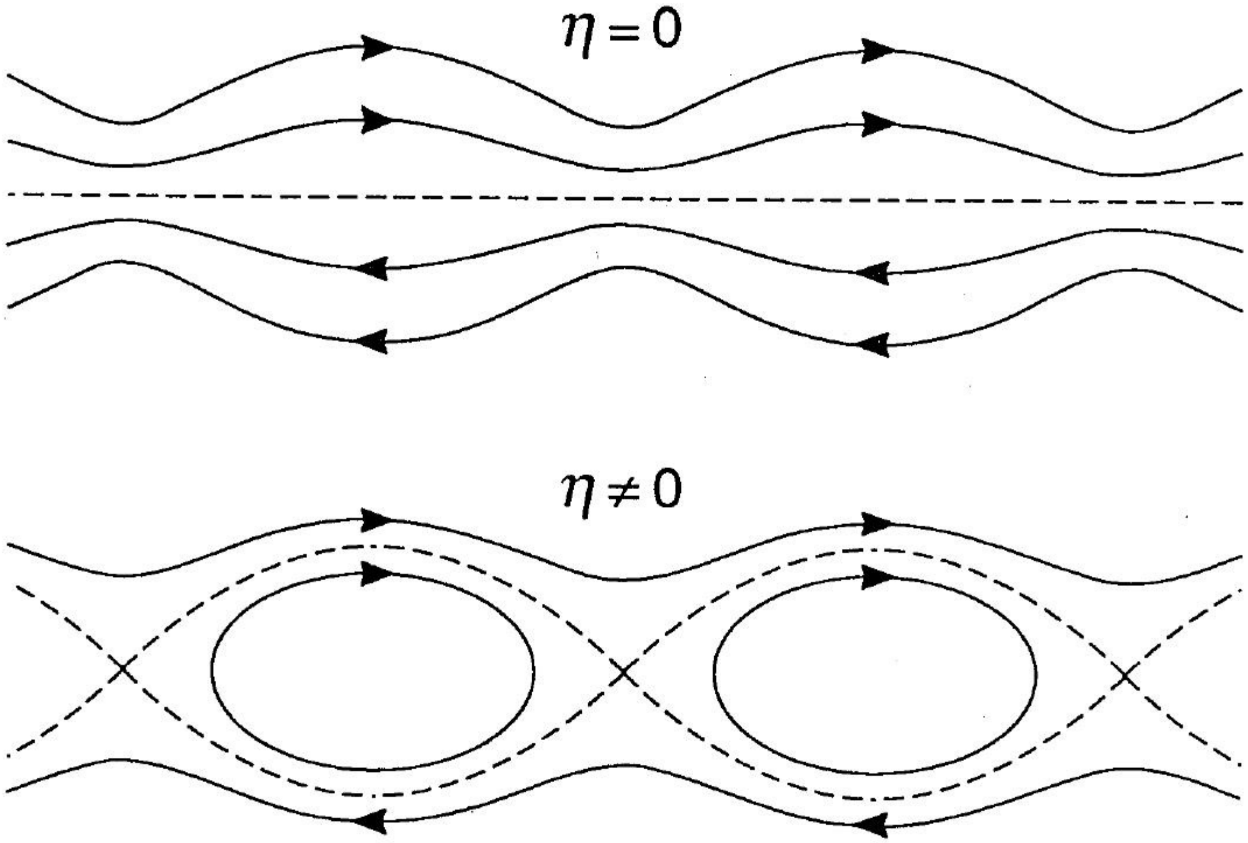
JET dimensions: $R_0 \approx 3$ m, a=1-2 m (elongated)

Tokamak geometry

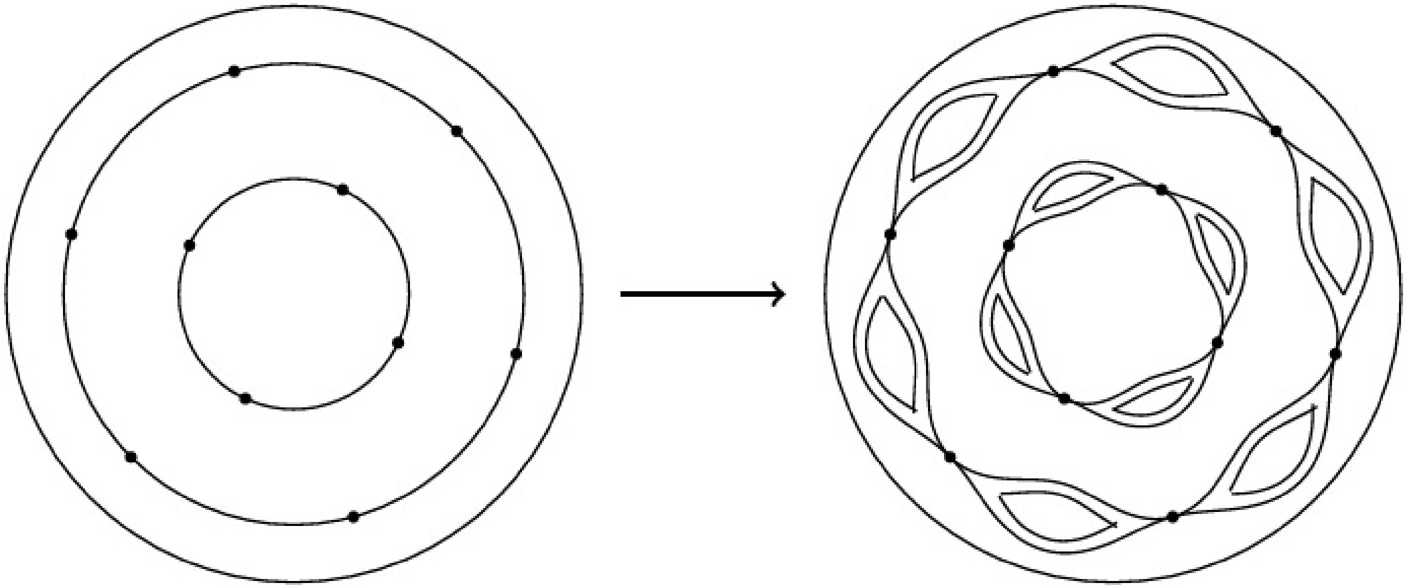


Onset of magnetic islands

$$\frac{\partial B}{\partial t} = \nabla \wedge (v \wedge B) + \frac{c^2}{4\pi} \eta \nabla^2 B$$



Onset of magnetic islands

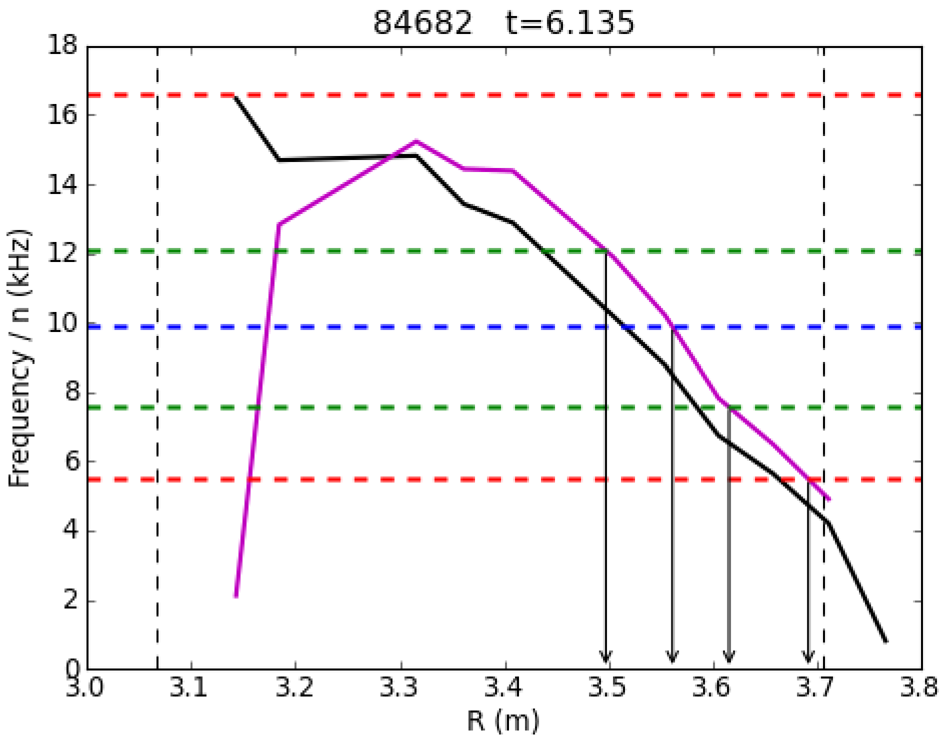


A resonant perturbation causes the magnetic surfaces to break:

- Local effect: radial transport increases
- Global effect: it may cause disruption

Diagnostic application

Magnetic islands occur on the rational surfaces
We can use their signal to reconstruct the q-profile



P. Buratti et al, "Diagnostic applications of magnetic islands rotation in JET", manuscript in preparation (2015)

Dispersion relation

The following set of reduced-MHD equations

$$\partial_t \psi + [\phi - n, \psi] = \eta J$$

$$\partial_t n + [\phi, n] = 0$$

$$\partial_t \partial_x^2 \phi + \partial_x [\phi + \tau n, \partial_x \phi] = [\partial_x^2 \psi, \psi]$$

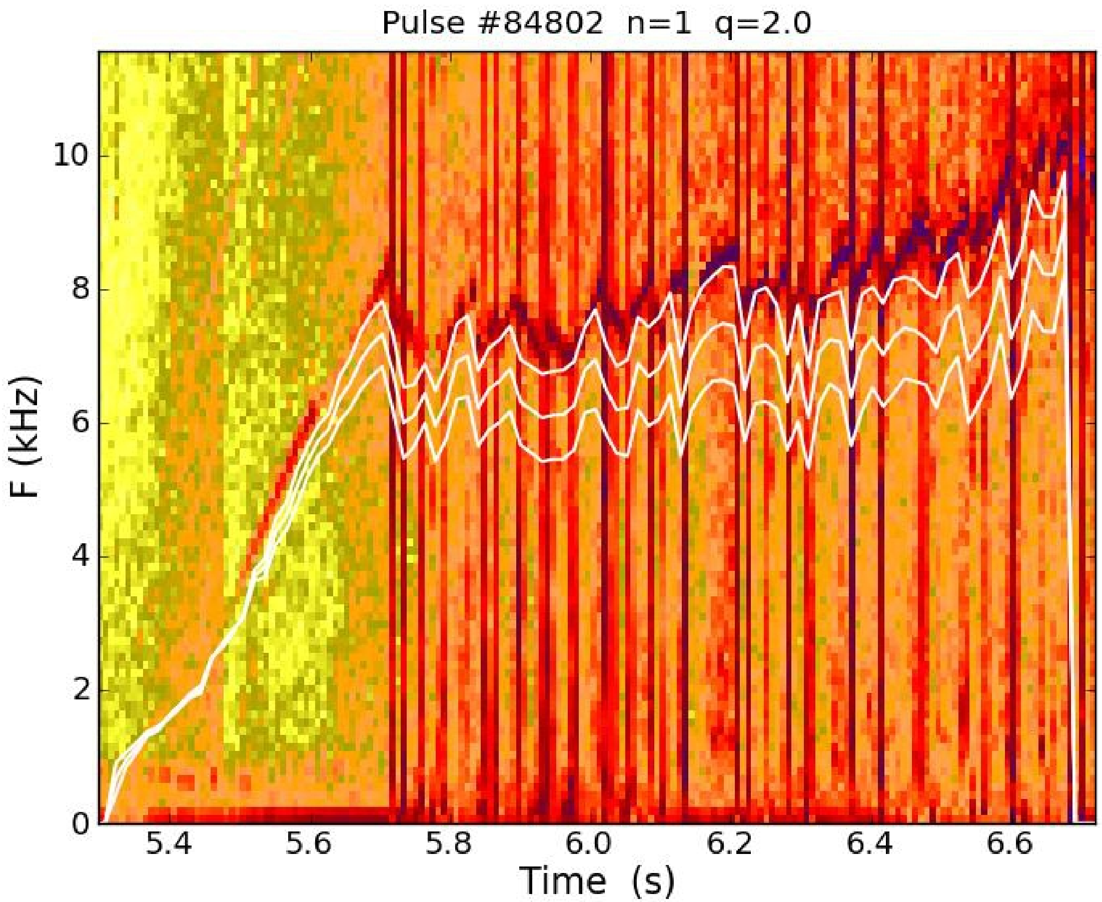
gives the dispersion relation of drift-tearing modes

$$(\omega - \omega_{*e})^3 \omega (\omega - \omega_{*i}) = i \gamma^5$$

The only unstable mode propagates with a frequency close to the electron diamagnetic one

G. Ara, B. Basu, B. Coppi et al, Annals of Physics **112** 443 (1978)

Experimental observations

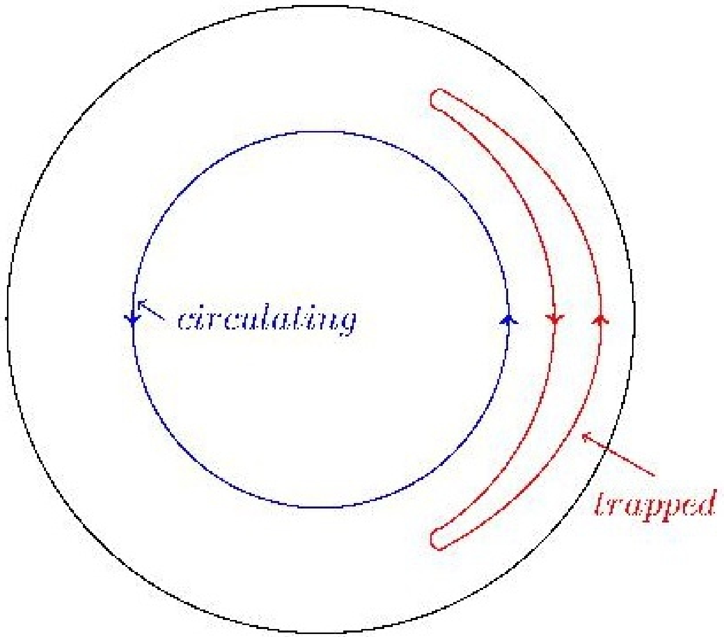


Signal from a magnetic coil inside JET

The upper white line is the ion fluid rotation frequency

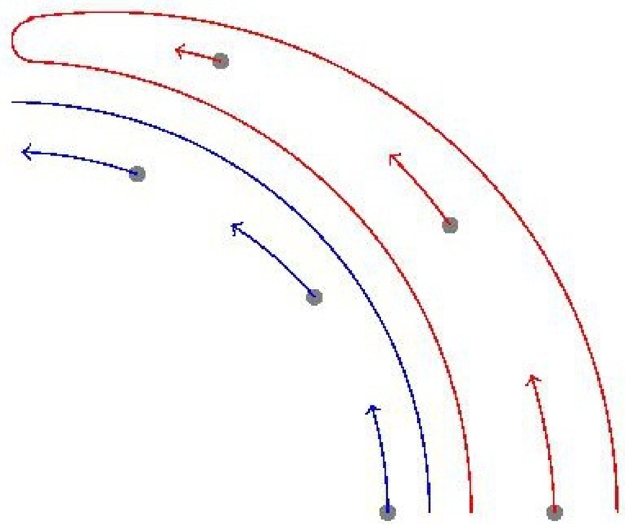
P.Buratti et al, "Magnetic islands rotation in JET", Proceedings of the 41st EPS Conference on Plasma Physics, 2014

Neoclassical effects



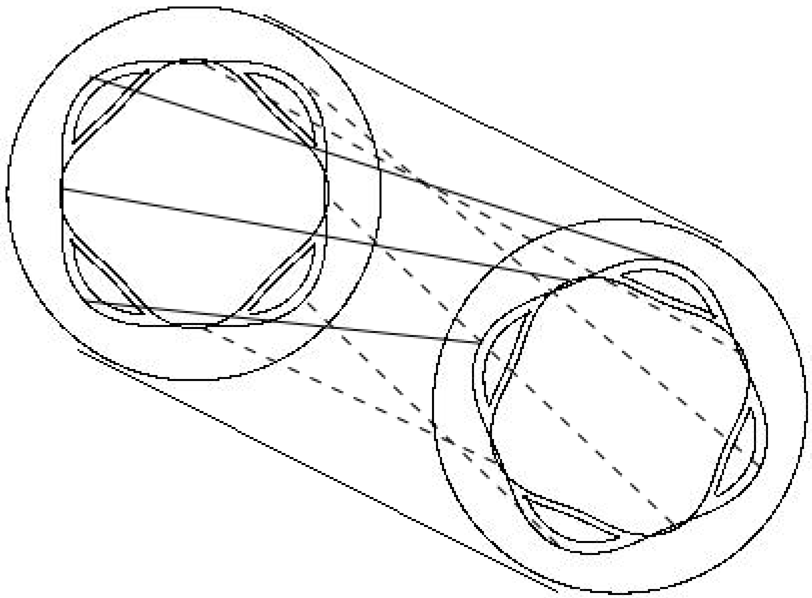
Caused by non-homogeneous magnetic field and low collisionality

Particles with $\sin^2\theta = \frac{v_\perp^2}{v_\perp^2 + v_\parallel^2} > \frac{B_{min}}{B_{max}}$ are trapped in the banana orbits



Momentum exchange between circulating and trapped particles causes poloidal viscosity

The broken axisymmetry of the tokamak due to the magnetic island causes toroidal viscosity



Neoclassical viscosities

Neoclassical poloidal flow damping

$$B \cdot \nabla \cdot \pi_{i\|} = m_i n_i \left\{ \mu_1 \left[B v_\| + \frac{T_i}{e} \left(\frac{n'}{n} + \frac{e\phi'}{T_i} \right) \right] + \mu_2 \frac{T_i}{e} \frac{T'_i}{T_i} \right\}$$

Neoclassical toroidal flow damping

$$B_\varphi \cdot \nabla \cdot \pi_{i\|} = m_i n_i \frac{(\delta B)^2}{B^2} \left\{ \lambda_1 \frac{T_i}{e} \left(\frac{n'}{n} + \frac{e\phi'}{T_i} \right) + \lambda_2 \frac{T_i}{e} \frac{T'_i}{T_i} \right\}$$

δB is the amplitude of the non-axisymmetric perturbation causing the toroidal flow damping

S.P. Hirshman & J.D. Sigmar, Nucl. Fusion **21** 1079 (1981)
K.C. Shaing & J.D. Callen, Phys. Fluids **26** 3315 (1983)

Neoclassical offset velocities

From parallel force balance flux-surface averaged:

$$\langle B \cdot \nabla \cdot \pi_{i\|} \rangle = 0 \qquad U_{i\theta} = c_p \frac{I}{e\langle B_0^2 \rangle} \frac{dT_{i0}}{d\psi}$$

The plasma tends to rotate to a fixed poloidal velocity

From toroidal force balance flux surface averaged:

$$\langle B_\varphi \cdot \nabla \cdot \pi_{i\|} \rangle = 0 \qquad U_{i\varphi} = (c_p + c_t) \frac{I}{e\langle B_0^2 \rangle} \frac{dT_{i0}}{d\psi}$$

The plasma tends to reach a fixed toroidal velocity

J.D. Callen, A.J. Cole & C.C. Hegna, Phys. Plasmas **16** 082504 (2009)

EM torques

EM torque exerted on the magnetic islands

$$T_{EM} = \int_{-\pi}^{\pi} d\xi \int_{-\infty}^{+\infty} dx\, \hat{e}_{\varphi} \cdot J \wedge B$$

Manipulating a little this expression and assuming zero external EM forces ($T_{EM} = 0$)

$$\int_{-\pi}^{\pi} d\xi \int_{-\infty}^{+\infty} dx\, J_{\|} \sin\xi = 0$$

Integrating by parts we can write the torque balance equation for a freely-rotating island as

$$\int_{-\pi}^{\pi} d\xi \int_{-\infty}^{+\infty} dx\, \nabla_{\|} J_{\|} = 0$$

Closure equations

The torque balance $\int_{-\pi}^{\pi} d\xi \int_{-\infty}^{+\infty} dx \, \nabla_{\parallel} J_{\parallel} = 0$
determines the island rotation frequency

$\nabla_{\parallel} J_{\parallel}$ can be deduced form the vorticity equation

$$\frac{B^2}{v_A^2} \frac{d}{dt} \nabla^2 \left(\phi + \frac{P_i}{e n_{i0}}\right) = B^2 \nabla_{\parallel} J_{\parallel} + B \cdot \nabla \wedge \nabla \cdot \pi_{i\parallel} + \mu B^2 \nabla^4 \left(\phi + \frac{P_i}{e n_{i0}}\right)$$

which is obtained by taking the curl of the ion momentum equation. To close the system we have to find equations for the other fields

Fluid equations

$$E+v_e\wedge B+\frac{1}{en}\nabla P_e=\eta J$$

$$\left(\frac{\partial}{\partial t}+v_i\cdot\nabla\right)n+n\nabla\cdot v_i=0$$

$$m_i n\left(\frac{\partial}{\partial t}+v_i\cdot\nabla\right)v_i=J\wedge B-\nabla P-\nabla\cdot\pi_i+\mu\nabla^2 v_i$$

The equations are the Ohm law, the continuity equation and the ion momentum equation

Together with the vorticity equation, they form a closed system for the electromagnetic fields, the ion velocity and the density

Four-field model

$$[\phi - n, \psi] + \delta_e \eta J = 0$$

$$[\phi, n] + [\frac{L_n}{L_s}\frac{q}{\epsilon} V + \rho^2 J, \psi] + D\partial_x^2 n = 0$$

$$[\phi, V] + \frac{L_s}{L_n}\frac{\epsilon}{q}\alpha^2[n, \psi] + \mu\partial_x^2 V - \nu_\theta\left(\frac{\epsilon}{q}\right)^2\{V - \partial_x(\phi + \tau n(1 - c_\theta^{NC}))\} = 0$$

$$\partial_x[\phi + \tau n, \psi] + [J, \psi] + \mu\partial_x^4(\phi + \tau n) + \nu_\theta\partial_x\{V - \partial_x(\phi + \tau n(1 - c_\theta^{NC}))\}$$
$$+ \nu_\perp\partial_x\{-\partial_x(\phi + \tau n(1 - c_\perp^{NC}))\} = 0$$

$$\partial_x^2\psi = -1 + \delta_e J$$

Poisson parenthesis formalism: $[A, B] = \hat{e}_\varphi \cdot \nabla A \wedge \nabla B$

Four fields: ψ, V, n, ϕ

R.D.Hazeltine, M.Kotschenreuther & P.J.Morrison, Phys. Fluids **28** 2466 (1985)
R. Fitzpatrick & F.L. Waelbroeck, Phys. Plasmas **16** 072507 (2009)
R. Fitzpatrick & F.L. Waelbroeck, Phys. Plasmas **19** 112501 (2012)

Theoretical predictions

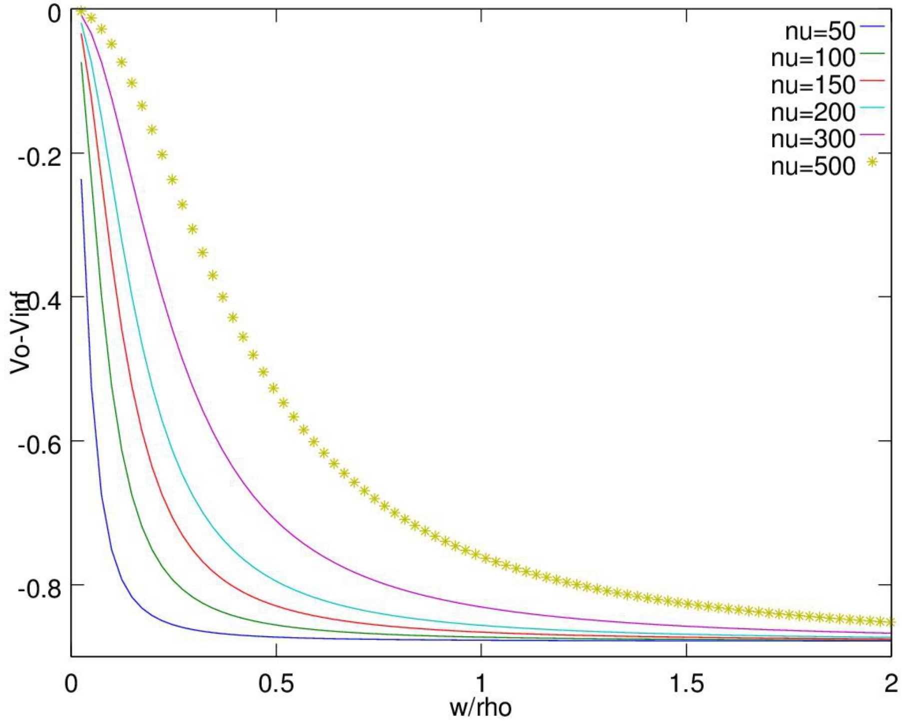
According to the theory, the plasma flow tends to the neoclassical velocity far from the islands

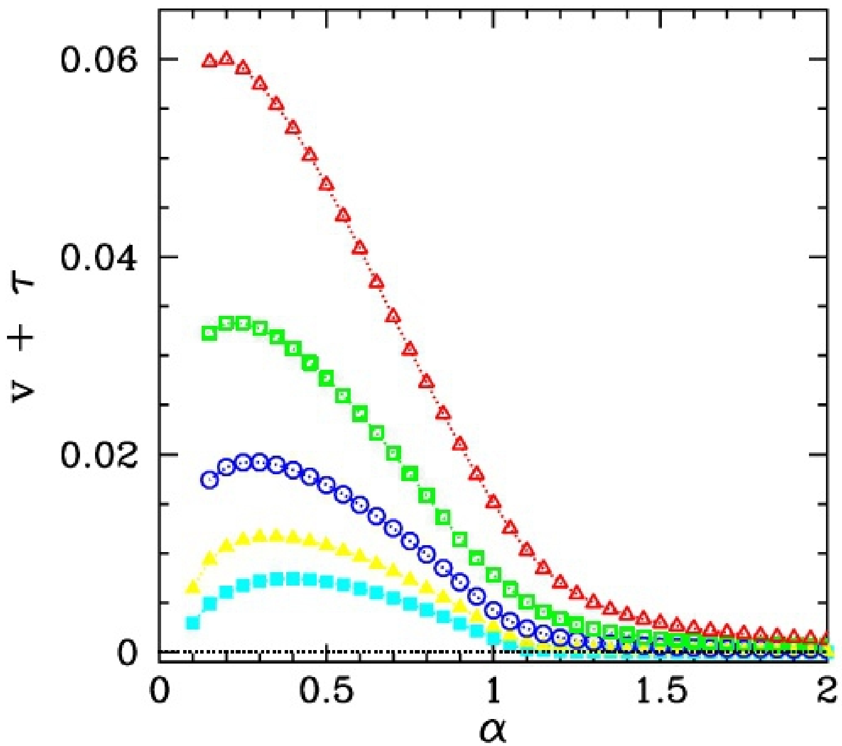
Islands such that $W/\rho_s \gg 1$, rotate with the velocity of the ion flow (ion diamagnetic velocity)

Islands such that $W/\rho_s \ll 1$, rotate with the velocity of the electron flow (electron diamagnetic velocity)

Results

We could ask how rapidly does the velocity decrease outside the separatrix depending on the island width and collisionality





The ion fluid velocity tends to the neoclassical value for large enough islands

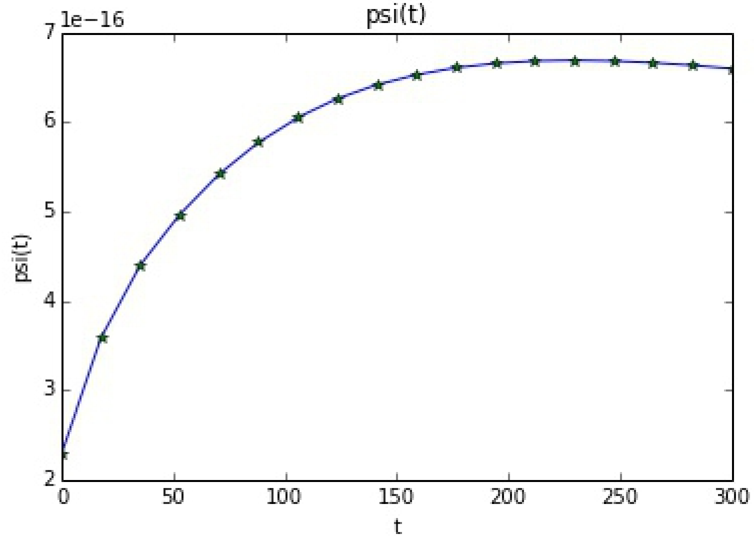
Flaws of the model

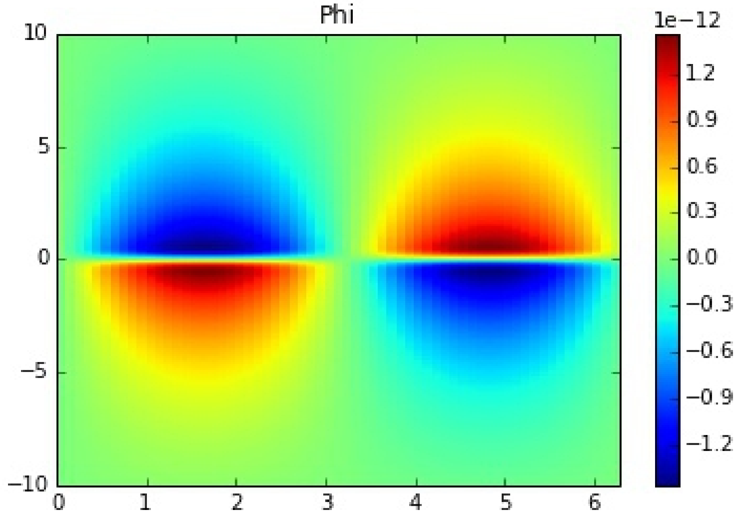
- The system of equations solved in this way is valid only in the regime of large islands $W/\rho_s \gg 1$
- Finite-Larmor-radius effects are neglected
- Small islands emit drift-acoustic waves, which require a kinetic approach
- The equations are stationary, so the evolution of the island is not taken into account

Numerical integration

A 2D slab finite-difference code can be used to integrate the time-dependent four-field equations

We can try to see the passage from the linear to the non-linear behavior as the island grows in size





D. Grasso, D. Borgogno & E. Tassi, Communications in Nonlinear Science and Numerical Simulation **17** 2085 (2012)

Finite Larmor radius

To include finite-Larmor-radius (FLR) effects we can consider using gyrofluid equations

- Not limited to small Larmor radius
- Still fluid equations (easier to solve both analytically and numerically)
- Differential operators become nonlinear
- Padé approximation makes things easier

R.D. Sydora, "Grokinetic and gyrofluid theory and simulation of magnetized plasmas", "Computational many particle physics", "Lecture notes on physics" 739, Springer (2008)

Nonlinear differential operator

Electrostatic and magnetic potentials become

$$\Phi = \Gamma_0^{1/2}\phi \qquad \Psi = \Gamma_0^{1/2}\psi$$

where we introduced the operator

$$\Gamma_0 = I_0\left(-\rho_i^2\nabla^2\right)e^{\rho_i^2\nabla^2}$$

Using the Padé approximation, formally

$$\Gamma_0 \approx \frac{1}{1-\rho_i^2\nabla^2}$$

Expanding in this way the operator and multiplying by the denominator, the nonlinearity disappears

Conclusions

- Magnetic islands are deleterious for the confinement
- Knowing the islands rotation frequency is important for theoretical reasons and for diagnostic applications
- The rotation frequency is determined by the torque balance
- The torque balance involves unknown fields
- The four-field model allows to reconstruct the fields profiles
- Islands rotation frequency is influenced by the neoclassical effects and by the islands width
- There's a good agreement between the theoretical predictions for large islands and the experimental results
- More efforts are needed to understand the transition between small islands and large islands

Thanks for the attention

Reduction procedure

The aim is to simplify the equations and to exclude the fast dynamics which brings to the equilibrium

$$P = P_0 + P_1 \qquad B = B_0 \hat{z} + b_z \hat{z} + \hat{z} \wedge \nabla \psi \qquad J = J_0 + J_1$$

$$E = -\nabla \phi - \frac{\partial \psi}{\partial t} \hat{z}$$

The zero-order quantities give the equilibrium

$$\nabla P_0 = J_0 \wedge B_0$$

The first-order quantities describe the slow evolution of the system $\phi, \psi, P_1 = T_0 n_1 \approx -b_z$

Gyro-average operation

FLR effects are taken into account by taking the agerage of the fields over one Larmor rotation

$$\langle \phi \rangle_\alpha = \frac{1}{2\pi} \int_0^{2\pi} \phi(R+\rho(\alpha), t)\, d\alpha$$

Using a Fourier expansion for ϕ , the average operation brings about the Bessel function

$$J_0(x) = \frac{1}{2\pi} \int_0^{2\pi} e^{ix\cos\alpha}\, d\alpha$$

Successively integrating in the velocities (to obtain the moments of the kinetic equation):

$$\phi(r) = \sum_k \phi_k I_0(k_\perp^2 \rho_i^2)\, e^{-k_\perp^2 \rho_i^2}\, e^{ik \cdot r}$$