Phase-coherence properties of three-dimensional Bose-Einstein condensed gases

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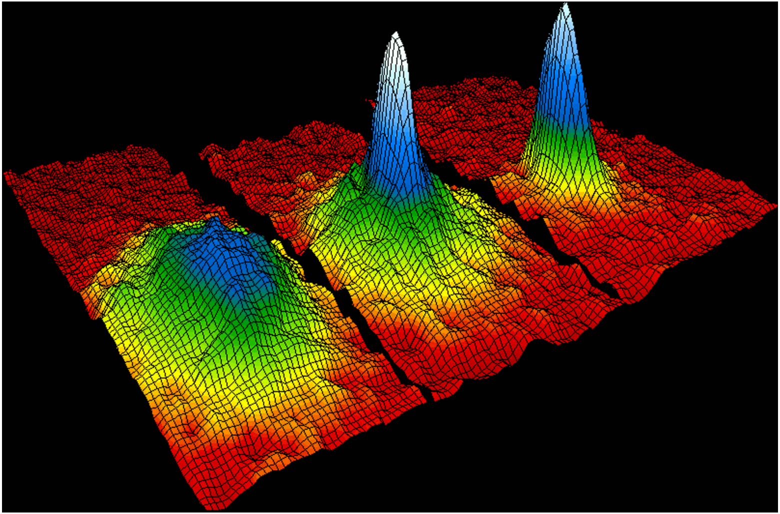
Plan of the presentation

1. Bose-Einstein condensation
2. Spin-wave theory
3. QMC simulations of the Bose-Hubbard model

Bose-Einstein condensation

Momentum distribution of a gas of Rb^{87} atoms before and after BEC



M.H. Anderson, J.R. Ensher, M.R. Matthews, C.E. Wiemann, E.A. Cornell, *Science* **269**, 5221, 198 (1995)

BEC phase

Wave function of the condensate

$$\Psi(\mathbf{x}) = \langle b(\mathbf{x})\rangle = |\Psi(\mathbf{x})| e^{i\theta(\mathbf{x})}$$

$$\int |\Psi(\mathbf{x})|^2 = n_0$$

where $b(\mathbf{x})$ is the bosonic field operator and n_0 is the condensate fraction

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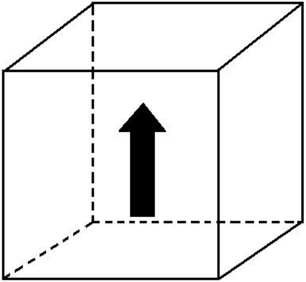
Superfluidity

Two fluid model at finite temperature: $\rho = \rho_s + \rho_n$

$$\mathbf{v}_s = \frac{\hbar}{m} \nabla\theta$$

Cubic-like systems

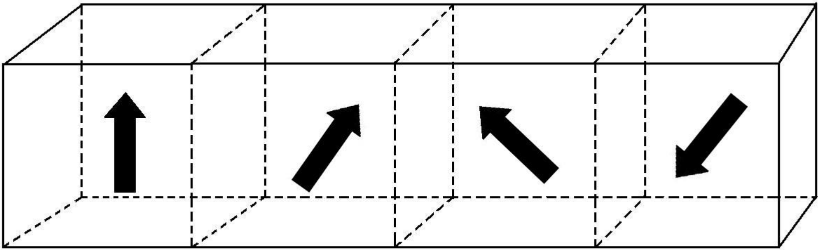


Homogeneous cubic-like systems

$$\Psi(\mathbf{x}) = \sqrt{n_0}$$

All the sizes of the system are of the same order

Anisotropic systems



Anisotropy parameter

$$\lambda = L_a/L^2$$

$L_a \equiv$ axial size, $L \equiv$ transverse size

$\lambda \to \infty$: Crossover from 3D behavior to effectively 1D behavior

Spin-wave theory

Quantitative informations on long-range phase correlations are obtained from

$$G(\mathbf{x}, \mathbf{y}) = \langle b^\dagger(\mathbf{x}) b(\mathbf{y}) \rangle$$

using the following macroscopic representation of the field operator

$$b(\mathbf{x}) = \sqrt{n_0} e^{i\hat{\theta}(\mathbf{x})}$$

Assumptions

1. Long distance fluctuations of the density are negligible
2. $$\mathcal{S}_{\rm sw} = \int d^3x \, \frac{\alpha}{2} (\partial_\mu \theta)^2 \qquad \alpha = \left(\frac{\hbar}{m}\right)^2 \frac{\rho_s}{T}$$

Correlation function

$$G_{\rm sw}(\mathbf{y}-\mathbf{x}) = \langle e^{-i\theta(\mathbf{x})}\, e^{i\theta(\mathbf{y})} \rangle$$

Anisotropic limit $\lambda \to \infty$ ($\lambda = L_a/L^2$)

$$G_{\rm sw}(0,0,z \gg 1) \sim e^{-z/\xi_a}$$

Axial correlation length $\qquad \xi_a = 2\alpha L^2$

Helicity modulus

Helicity modulus Y_μ: measure of the response of the system to a phase twisting ϕ along direction μ

$$Y_x \equiv -\frac{L_x}{L_y L_z} \left. \frac{\partial^2 \log \mathcal{Z}(\phi)}{\partial \phi^2} \right|_{\phi=0}$$

Anisotropic geometries

$$Y_t = Y_x = Y_y = \alpha$$

$$Y_a = Y_z = \alpha - \frac{4\pi^2\alpha^2}{\lambda} \frac{\sum_{n=-\infty}^{\infty} n^2 e^{-2\pi^2 n^2 \alpha/\lambda}}{\sum_{n=-\infty}^{\infty} e^{-2\pi^2 n^2 \alpha/\lambda}}$$

$Y_a = \alpha$ for $\lambda \to 0$; $\quad Y_a \to 0$ for $\lambda \to \infty$

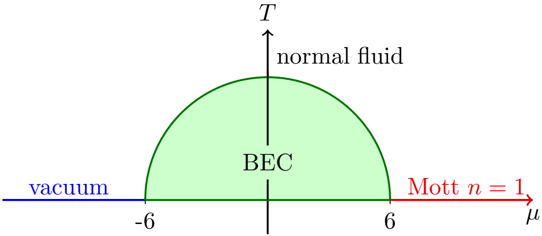
Bose-Hubbard model

Bose-Hubbard Hamiltonian

$$H = -t \sum_{\langle ij \rangle} (b_i^\dagger b_j + b_j^\dagger b_i) + \frac{U}{2} \sum_i n_i(n_i - 1) - \mu \sum_i n_i$$

b_i ($b_i^\dagger$) is a bosonic destruction (creation) operator

$n_i \equiv b_i^\dagger b_i$ is the particle density operator



Phase diagram, in units of the hopping parameter (t), of the 3D Bose-Hubbard model in the hard-core $U \to \infty$ limit

$T_c(\mu = 0) = 2.0160(1)$

Quantum Monte Carlo simulations

Algorithm: Quantum Monte Carlo directed operator-loop algorithm

Simulations' parameters

- Anisotropic $L^2 \times L_a$ lattices with periodic boundary conditions
- Hopping parameter: $t = 1$
- Chemical potential: $\mu = 0$
- Temperature: $T = 1.5$ and $T = 1.75$

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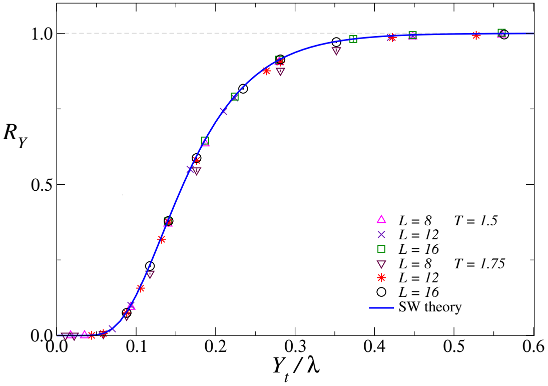
$\longrightarrow$ Fluctuations of the density are significantly different from zero only at one lattice spacing

Helicity modulus



$$R_Y \equiv Y_a/Y_t = f_\Upsilon(Y_t/\lambda)$$

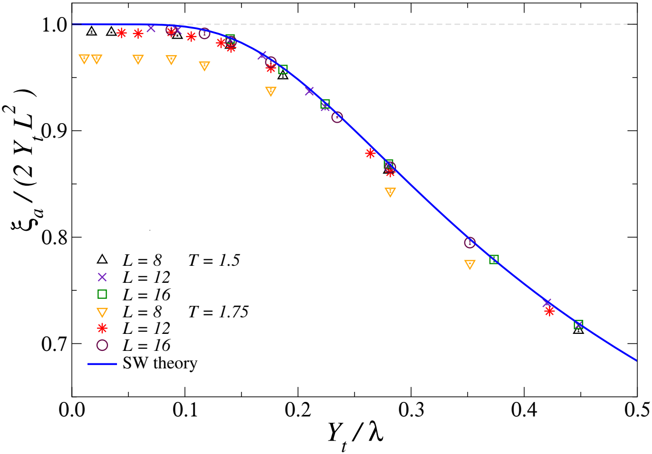
$$f_\Upsilon(x) = 1+2x\,\partial_x \ln \vartheta_3(0|i2\pi x)$$

$\vartheta_3(z|\tau)$ is the third elliptic theta function

QMC estimates of Y_t:
$Y_t = 0.280(1)$ at $T = 1.5$
$Y_t = 0.176(1)$ at $T = 1.75$

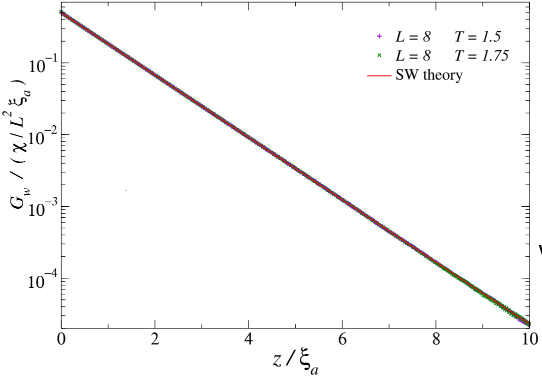
Axial correlation length

$$\xi_a(\lambda) = 2\, Y_t\, L^2\, f_\xi(Y_t/\lambda)$$



Universal behavior in the limit $L \to \infty$ with $\lambda = L_a/L^2$ fixed

Wall-wall correlation function



$$G_w(z) \equiv \frac{1}{L^2} \sum_{x,y} G(x,y,z)$$

$$G_w(z) = \frac{\chi}{L^2 \xi_a}\, f_w(z/\xi_a, Y_t/\lambda)$$

with

$$\chi = \sum_{\mathbf{x}} G(\mathbf{x})$$

$G_w(z)$ in the limit $\lambda \to \infty$

- Exponential decay
- Correlation length: $\xi_a = 2 Y_t L^2$

Conclusions

- The long-range phase-coherence properties of homogeneous 3D BEC systems exhibit a universal behavior

- Universal scaling functions are approached in the limit $L \to \infty$ with $\lambda \equiv L_a/L^2$ fixed

- Phase decoherence occurs in the limit of infinite axial-size.
  The axial coherence length ξ_a remains finite and proportional to the transverse area A_t

$$\xi_a \propto \frac{\rho_s}{T} A_t$$