Pretesi di Dottorato

"Time-evolution in open quantum systems on a lattice"

Dipartimento di Fisica "Enrico Fermi"

Corso di Dottorato in Fisica XXXI Ciclo

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Introduction to open quantum systems

Many-Body Physics with Ultracold Gases & Interacting Photons

Progress in Optics:

- Cool neutral atoms, molecules and ions (fully quantum dynamics)
- Design the system geometry (d-dimensional lattices...)
- Control the many-body physics (Feshbach resonances & active media)

External fields $\Leftrightarrow$ **engineer Hamiltonians**

Experimental results:

- BEC (Anderson *et al.*, 1995), Fermi degeneracy (DeMarco and Jin, 1999)
- obs. interference in overlapping condensates (Andrews *et al.*, 1997), obs. of long range phase coherence (Bloch *et al.*, 2000)
- Mott-insulator phase transition (Greiner *et al.*, 2002)
- obs. Tonks-Girardeau gas (Paredes *et al.*,2004), Kosterlitz-Thouless crossover (Hadzibabic *et al.*, 2006)
- BCS-BEC crossover (Bartenstein *et al.*, 2004)...

Isolated systems & Hamiltonian dynamics

Physical systems: Neutral atoms on optical lattices, trapped dilute gases...

Isolated systems	$\Leftrightarrow$	Hamiltonian dynamics

Standard Quantum Theory

- $S \leftrightarrow$ separable Hilbert Space $\mathbb{H}_S$
- $|\psi_S\rangle \leftrightarrow \rho_S = |\psi_S\rangle\langle\psi_S| \in P(\mathbb{H}_S)$
- observables $\leftrightarrow$ Hermitian operators
- amplitudes $\leftrightarrow \langle O\rangle = \mathrm{Tr}\,[O\rho_S]$

Time-Evolution (von Neumann equation):

$$i\frac{d}{dt}\rho_S = [\mathcal{H}_S,\, \rho_S]$$

Non-equilibrium systems & Non-Hamiltonian dynamics

Physical systems: Quantum cavities, Trapped ions, Rydberg atoms...
(systems affected by decay, decoherence and dissipation)

Fundamental d.o.f. finite lifetime $\Leftrightarrow$ **Non − equilibrium dynamics**

⇓

Non − Hamiltonian dynamics

⇓

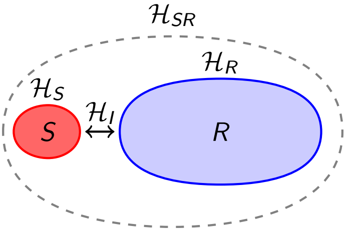
We cannot use Standard Quantum Theory

Open Quantum Systems: LGKS Equation

Total system $\leftrightarrow$ Unitary Evolution:

$\mathcal{H}_{SR} = \mathcal{H}_S + \mathcal{H}_R + \mathcal{H}_I$,

- $\mathcal{H}_S \leftrightarrow$ subsystem of interest
- $\mathcal{H}_R \leftrightarrow$ reservoir or environment
- $\mathcal{H}_I \leftrightarrow$ subsys.-res. interaction



Time-Evolution of the subsystem S:

[Lindblad (1976); Gorini, Kossakowski & Sudarshan (1976)]

$$i\frac{d}{dt}\rho_S = \mathcal{L}[\rho_S] = [\mathcal{H}_S,\, \rho_S] + i\sum_j \gamma_j \left[A_j \rho_S A_j^\dagger - \frac{1}{2}\left(A_j^\dagger A_j \rho_S + \rho_S A_j^\dagger A_j\right)\right]$$

where $\rho_S = \mathrm{Tr}_R[\rho_{SR}]$; $\mathcal{L}[\cdot]$ is the *Liouvillian superoperator*; γ_j are the positive rates; A_j are the Lindblad operators.

Open quantum systems on a lattice

Steady-States & Lattice Systems

Closed systems

$\mathcal{H}(\mathbf{g}),\ \rho_{gs}(\mathbf{g})$

Ground-State Phase Diagram

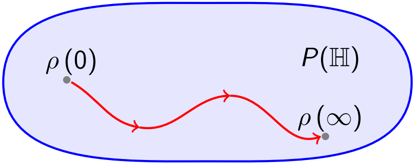
Open systems

$\mathcal{L}(\mathbf{g}),\ \rho_{ss}(\mathbf{g})$

Steady-State Phase Diagram

Steady-states:

$$\lim_{t\to+\infty} \rho(t) \in \{\rho_{ss}^{(n)}\}$$



Lattice systems with standard dissipator have a unique steady-state:

[S.G. Schirmer and X. Wang 2010]

$$\mathcal{D}[\rho] \equiv \sum_j \left[A_j \rho_S A_j^\dagger - \frac{1}{2}\left(A_j^\dagger A_j \rho_S + \rho_S A_j^\dagger A_j \right) \right], \quad A_j = S_j^-,\ b_j,\ c_j$$

Prototypical Lattice Models & Spin to Boson Mapping

Spin systems: XYZ model $\leftrightarrow$ Rydberg Atoms + Blockade Mechanism
[T.E. Lee, H. Häffner, M.C. Cross (2011)]

$$\mathcal{H}^{xyz} = \frac{1}{z} \sum_{\langle i,j \rangle} \left\{ J_x\, S_i^x\, S_j^x + J_y\, S_i^y\, S_j^y + J_z\, S_i^z\, S_j^z \right\}$$

Boson systems: Bose-Hubbard model $\leftrightarrow$ Arrays of Quantum Cavities
[A.A. Houck, H.E. Türeci and J. Koch (2012)]

$$\mathcal{H}^{BH} = -\frac{w}{z} \sum_{\langle i,j \rangle} \left(b_i^\dagger\, b_j + b_j^\dagger\, b_i \right) + \frac{U}{2} \sum_i n_i (n_i - 1)$$

Holstein-Primakoff Transformation (1940)

$$\boxed{S^- = b^\dagger \sqrt{2S - b^\dagger b}, \quad S^+ = \sqrt{2S - b^\dagger b}\, b, \quad S^z = S - b^\dagger b}$$

Q1: Spin model $\rightarrow$ Driven-Dissipative Bose-Hubbard model?
Q2: Steady-State Diagram for S $\gg$ 1?

Non-Linear Spin Model

Preliminary analysis:

$$\mathcal{H}_{NL} = \mathcal{H}^{xyz} + \sum_i \sum_{k=x,y,z} \alpha_k (S_i^k)^2 + \sum_i \sum_{k=x,y,z} B_k S_i^k$$

- $\mathcal{H}^{xyz} \Rightarrow$ First-Neighbor Interaction
- $\alpha_{x,y},\ B_{x,y} \Rightarrow$ Source Terms
- $\alpha_z \Rightarrow$ On-Site Interaction
- $B_z \Rightarrow$ Energy Shifts

Decay Channels:

$$\mathcal{D}_1[\rho] = \sum_j \left[S_j^- \rho S_j^+ - \frac{1}{2} \left\{ S_j^+ S_j^-,\, \rho \right\} \right]$$

$$\mathcal{D}_2[\rho] = \sum_j \left[(S_j^-)^2 \rho (S_j^+)^2 - \frac{1}{2} \left\{ (S_j^+)^2 (S_j^-)^2,\, \rho \right\} \right] \quad (S > 1/2)$$

Numerical Results for the NL Spin Model

Mean-Field Approximation

Time-Evolution:

$$i\frac{d}{dt}\rho = [\mathcal{H}_{NL},\,\rho] + i\gamma\mathcal{D}_1[\rho] + i\eta\mathcal{D}_2[\rho], \qquad \rho = \text{total lattice}$$

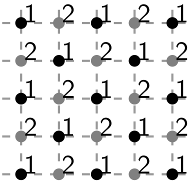
Mean-Field Approximation : Single Diff. Eq. $\rightarrow$ Set of Coupled Diff. Eqs

$$\rho \mapsto \rho^{MF} = \bigotimes_{n\in latt.\,sites} \rho_n \Rightarrow i\frac{d}{dt}\rho_n = \left[\mathcal{H}_{NL}^{MF\,(n)},\,\rho_n\right] + i\gamma\mathcal{D}_1[\rho_n] + i\eta\mathcal{D}_2[\rho_n],$$

- ρ_n = lattice site n
- $\mathcal{H}_{NL}^{MF\,(n)} = \frac{1}{z}\sum_{j(n),\,\beta}\left[J_\beta\, S^\beta\,\langle S_j^\beta\rangle\right] + \cdots\,,$ $\quad \boxed{\langle S_j^\beta\rangle = \mathrm{Tr}[S^\beta \rho_{j(n)}]\ \text{MF Amplitudes}}$

Mean-Field Dynamics on a Bipartite lattice:

$$\begin{cases} \dfrac{d}{dt}\rho_1 = -i\left[\mathcal{H}_{NL}^{MF\,(1)},\,\rho_1\right] + \gamma\mathcal{D}_1\left[\rho_1\right] + \eta\mathcal{D}_2\left[\rho_1\right] \\ \dfrac{d}{dt}\rho_2 = -i\left[\mathcal{H}_{NL}^{MF\,(2)},\,\rho_2\right] + \gamma\mathcal{D}_1\left[\rho_2\right] + \eta\mathcal{D}_2\left[\rho_2\right] \end{cases}$$



Dissipative XYZ model

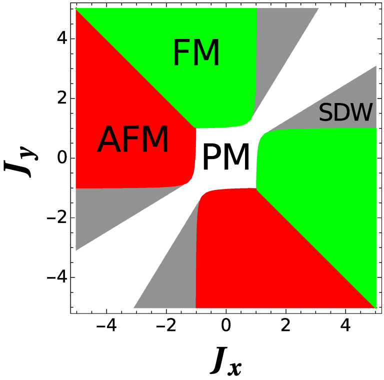
T.E. Lee, S. Gopalakrishnan, M.D. Lukin (2013)

(spin-1/2,$\gamma = J_z = 1$,$\eta = 0$)

- Eq. of motion for $\langle S^\alpha \rangle$;
- Linear Stability Analysis;
- MF-Steady-State Phase Diagram (on the right)

Phases $\leftrightarrow$ **Ordering in the x-y plane**

- **PM**: $\langle S_j^z \rangle = -1/2$, $\langle S_j^{x,y} \rangle = 0$;
- **FM**: $\langle S_j^{x,y} \rangle = \langle S_{j+1}^{x,y} \rangle$;
- **AFM**: $\langle S_j^{x,y} \rangle = -\langle S_{j+1}^{x,y} \rangle$;
- **SDW**: spatially modulated state with period greater than two lattice spacing in at least one direction .

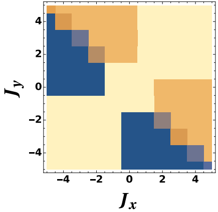


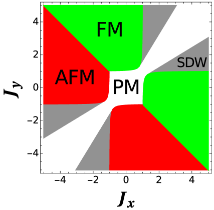
Dissipative XYZ model & Bipartite Lattice: Phases

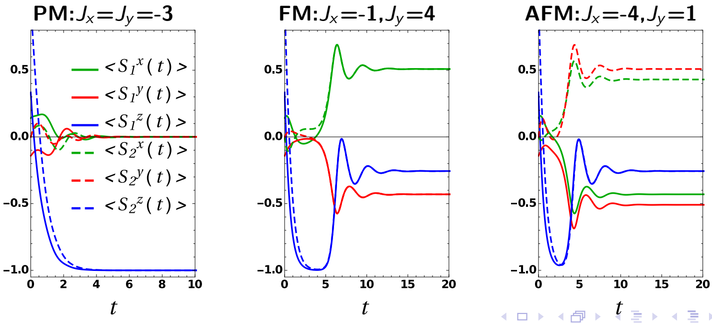
Phase Diagram:

- **PM**: (yellow);
- **FM**: (orange);
- **AFM**: (blue);

Example of time evolution (S=1):







Beyond the XYZ model on a bipartite Lattice...

Large number of parameters:

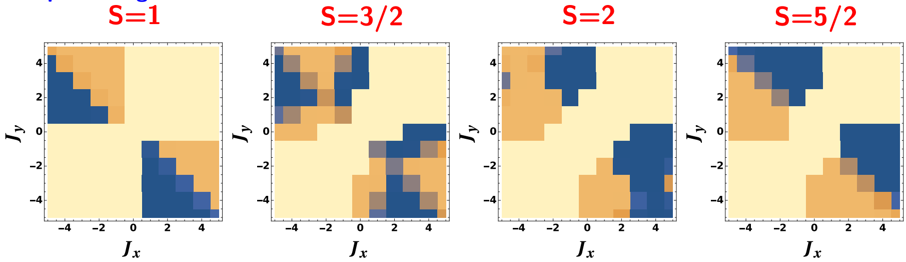
- **Non-Linear Spin Model:** $\{J_\beta\}$, $\{\alpha_\beta\}$, $\{B_\beta\}$ $\Rightarrow$ **9 parameters**;
- **Dissipators:** γ,η $\Rightarrow$ **2 parameters**.

Preliminary Results (Steady-State Phase Diagrams):

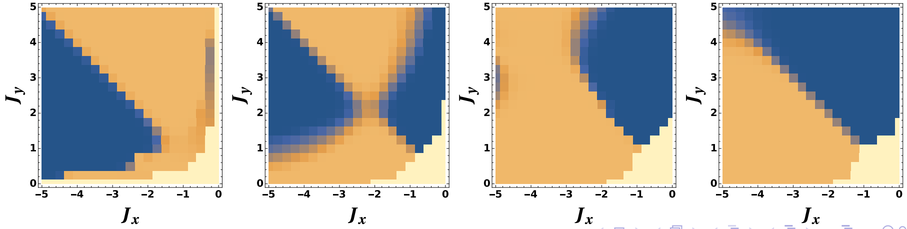
- $\gamma = 1, J_z = 1, \alpha_z = 0, \vec{B} = \vec{0}$
- $\{\alpha_x, \alpha_y\} = \{\pm 1, \pm 1\}$
- $\eta = 0, 1$
- **S** $\in \{$**1, 3/2, 2, 5/2** $\}$

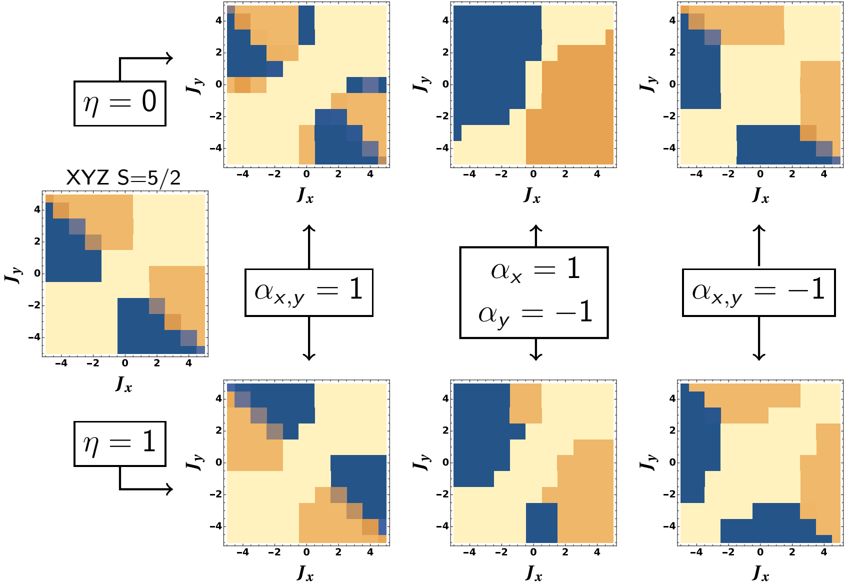
Phase Diagram at increasing S: $\alpha_x = \alpha_y = 1, \eta = 1$

Complete Diagram:



Top Left Corner:




η = 0
XYZ S=5/2
α_{x,y} = 1
α_x = 1
α_y = −1
α_{x,y} = −1
η = 1
J_x
J_y
−4 −2 0 2 4

Conclusions & Perspectives

- **Exotic Spin-Model $\leftrightarrow$ Driven-Dissipative Bose-Hubbard**
- **MF Steady-State Phase Diagrams on a Bipartite Lattice**

Main Goal for the Next Year:

- **Determine the complete Phase Diagrams $\leftrightarrow$ SDW Instability**
- **Include non-linearities in the z direction and magnetic field**
- **Scaling at increasing S $\leftrightarrow$ Steady-State Phase Diagram Bose-Hubbard**
- **Go beyond the Mean-Field Approximation (Cluster MF, Corner Space RG...)**