Totally asymptotically free models

Giulio Maria Pelaggi

Università di Pisa

25 Settembre 2015

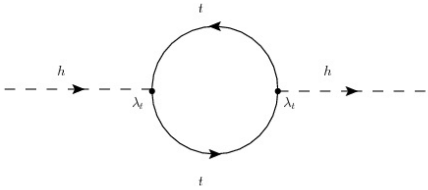
Index

A few questions

The Standard Model (SM) has been introduced in the '70.
Guide line $\rightarrow$ gauge symmetry.
Discovery of the Higgs, almost everything agrees with this theory.
And the neutrino masses? The DM? The hierarchy problem?
Is it an effective theory of something deeper?
But where is the new physics?

Hierarchy problem

- Why the mass term for the Higgs boson breaks the scale invariance of the SM lagrangian?
- Why this parameter gets this value? Is there a physical explanation?
- How can we deal with the quadratic divergences to the mass of the scalar boson?



Naturalness

New guide line for Beyond the SM physics → Naturalness.
Divergences are canceled by new physics at some energy scale Λ_{nat}.

$$\delta m_h^2(\Lambda_{\text{nat}}^2) \lesssim m_h^2.$$

Common solutions: SUSY and Higgs composite models.
Naturalness suggests new physics below the TeV but LHC didn't see anything.
Maybe it doesn't work in this way...

Finite naturalness

Reinterpretation: The naturalness principle must be respected only by the physical corrections.
The quadratic divergences arise because of the formulation of the theory, so they can be ignored (as we do in dimensional regularization computations).

Soft gravity

Gravity gives big high energy corrections to the parameters.
Brutally: we want it to stay **weak at all the scales**.
If there are more degrees of freedom these corrections may be suppressed and natural.

Pro: the gravitational sector of the model does not affect the low energy observable sector.
Con: we must study models also at $\Lambda > M_{Pl}$ (beware of Landau poles!)

What is TAF

We want to build a model for the observable sector as a 4 dimensional QFT, s.t. the gauge, Yukawa, scalar **couplings are dimensionless** and it stays in a **perturbative regime** at energy higher than a given scale.

How to find a TAF model

The general form of the RGEs for all the couplings is known [Machacek & Vaughn, Nuc. Phys. B **222** (1983), 83 and subsequent].

One can solve them, finding the asymptotic behaviour. Simple.

BUT

If we have tens of couplings? Is there a general way to deal with it?

How to find a TAF model

We rescale the couplings: given $t = \ln(\mu^2/\mu_0^2)/(4\pi^2)$

$$g_i^2(t) = \frac{\tilde{g}_i^2(t)}{t} \qquad y_a^2(t) = \frac{\tilde{y}_a^2(t)}{t} \qquad \lambda_m(t) = \frac{\tilde{\lambda}_m(t)}{t}$$

The one-loop RGEs become

$$\frac{d\tilde{g}_i}{d\ln t} = \frac{\tilde{g}_i}{2} + \beta_{g_i}(\tilde{g}); \ \frac{d\tilde{y}_a}{d\ln t} = \frac{\tilde{y}_a}{2} + \beta_{y_a}(\tilde{g}, \tilde{y}); \ \frac{d\tilde{\lambda}_m}{d\ln t} = \tilde{\lambda}_m + \beta_{\tilde{\lambda}_m}(\tilde{g}, \tilde{y}, \tilde{\lambda})$$

In one line: $\frac{dx_I}{d\ln t} = V_I(x)$ where $x_I = \{\tilde{g}_i, \tilde{y}_a, \tilde{\lambda}_m\}$

Solving the **algebraic** system $\frac{dx_I}{d\ln t} = V_I = 0$, we find the **fixed points** x_∞: the couplings $\longrightarrow 0$ with fixed ratios.

Features of a TAF model

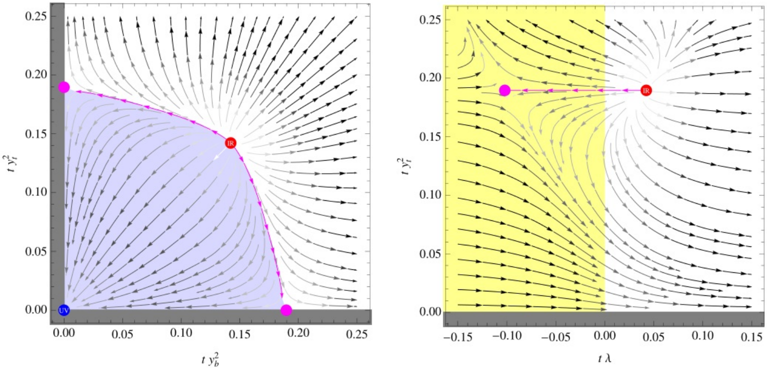
We linearize these equations around x^I_∞:

$$V_I \simeq \sum_J M_{IJ}(x^J - x^J_\infty)$$

where M_{IJ} is block triangular.
The running of $\Delta_I = x^I - x^I_\infty$ is given by $\frac{d\Delta_I}{d\ln t} = \sum_J M_{IJ}\Delta_J$.

All positive eigenvalues $\longrightarrow$ "fully IR-attractive" fixed point (and vice-versa).
A fully IR-attractive fixed point predicts a complete set of low energy values of the parameters.
In general we get a low energy constrain on parameters for each positive eigenvalue.

Features of a TAF model



	$\tilde{y}^2_{t\infty}$	$\tilde{y}^2_{b\infty}$	$\tilde{y}^2_{\tau\infty}$	$\tilde{y}^2_{\nu\infty}$	Eigenvalues
Solution 1	227/1197	0	0	0	$+-++$
Solution 2	0	227/1197	0	0	$-+++$
Solution 3	227/1596	227/1596	0	0	$++++$
Solution 4	0	0	0	0	$--++$

	$\tilde{\lambda}_\infty$	Eig
Sol. 1	$\dfrac{-143+\sqrt{119402}}{4788} \approx +0.0423$	$+$
Sol. 2	$\dfrac{-143-\sqrt{119402}}{4788} \approx -0.1020$	$-$

[Giudice, Isidori, Salvio, Strumia, arXiv:1412.2769]

The code: TAFfatron

A program does the dirty job:

1. builds the generators;
2. checks the invariance of the Lagrangian;
3. computes the RGEs for all the couplings;

Exploiting the form of the β functions, it

4. finds fixed points for gauge couplings and plugs the results into the Yukawa RGEs
5. For each Yukawa fixed point, it searches for scalar fixed points.

If there is at least a fixed point also for scalar couplings it is TAF.

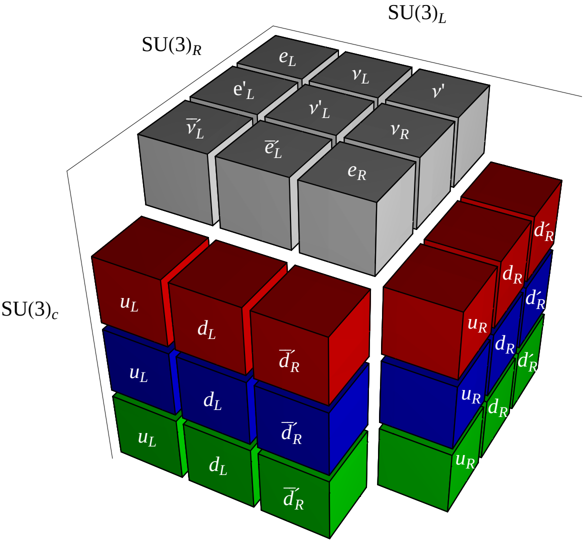
Question: is phenomenology consistent with the reality?

Standard Model

First thought: is SM TAF? No.
The coupling g_Y hits a Landau pole (a problem of all the abelian groups). We have to embed it in a non-Abelian group.
We chose Trinification GUT models, whose gauge group is

$$\mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R \otimes \mathrm{SU}(3)_c$$

Minimal Trinification



Symmetry breaking pattern

The Higgses have a non-zero VEV:

$$\langle H_n \rangle = \begin{pmatrix} v_{un} & 0 & 0 \\ 0 & v_{dn} & v_{Ln} \\ 0 & V_{Rn} & V_n \end{pmatrix}$$

In this way

$$\mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R \otimes \mathrm{SU}(3)_c \xrightarrow{V_n} \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{B-L} \otimes \mathrm{SU}(3)_c$$

$$\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{B-L} \otimes \mathrm{SU}(3)_c \xrightarrow{V_{Rn}} \mathrm{U}(1)_Y \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(3)_c$$

$$\mathrm{U}(1)_Y \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(3)_c \xrightarrow{v_n} \mathrm{U}(1)_{em} \otimes \mathrm{SU}(3)_c$$

Bad news

- With 1 Higgs it can't reproduce the fermion masses;
- with 2 Higgses the new fermions get a mass $M \sim yV$ while the SM quarks get a mass $m \sim yv$.
  SM quark masses are reproduced if couplings $y \lesssim 10^{-5}$.
  But if $V \approx$ few TeV we have $M \lesssim 0.1\text{GeV}$ in contradiction with data. Fine tuning?
- 3 Higgses can reproduce "naturally" the fermion masses, but no TAF solutions.

We have to introduce other particles.

How to expand?

New particles modify RGEs $\longrightarrow$ finite number of good models candidates to TAF.

	name	representation	Δb_i			Yukawas	
unstable	1	$(1,1,1)$	0	0	0	$1LH^*$	–
	8_L	$(8,1,1)$	2	0	0	$8_L LH^*$	–
	8_R	$(1,8,1)$	0	2	0	$8_R LH^*$	–
	$L' \oplus \bar{L}'$	$(3,\bar{3},1) \oplus (\bar{3},3,1)$	2	2	0	$L'LH$	$L'L'H + \bar{L}'\bar{L}'H^*$
	$Q'_L \oplus \bar{Q}'_L$	$(\bar{3},1,3) \oplus (3,1,\bar{3})$	2	0	2	$Q'_L Q_R H$	–
	$Q'_R \oplus \bar{Q}'_R$	$(1,3,\bar{3}) \oplus (1,\bar{3},3)$	0	2	2	$Q'_R Q_L H$	–
stable	$3_L \oplus \bar{3}_L$	$(3,1,1) \oplus (\bar{3},1,1)$	$\frac{2}{3}$	0	0	–	–
	$3_R \oplus \bar{3}_R$	$(1,3,1) \oplus (1,\bar{3},1)$	0	$\frac{2}{3}$	0	–	–
	$3_c \oplus \bar{3}_c$	$(1,1,3) \oplus (1,1,\bar{3})$	0	0	$\frac{2}{3}$	–	–
	8_c	$(1,1,8)$	0	0	2	–	–
	$6_L \oplus \bar{6}_L$	$(6,1,1) \oplus (\bar{6},1,1)$	$\frac{10}{3}$	0	0	–	–
	$6_R \oplus \bar{6}_R$	$(1,6,1) \oplus (1,\bar{6},1)$	0	$\frac{10}{3}$	0	–	–
	$6_c \oplus \bar{6}_c$	$(1,1,6) \oplus (1,1,\bar{6})$	0	0	$\frac{10}{3}$	–	–
	$\tilde{L} \oplus \bar{\tilde{L}}$	$(3,3,1) \oplus (\bar{3},\bar{3},1)$	2	2	0	–	–
	$\tilde{Q}_L \oplus \bar{\tilde{Q}}_L$	$(3,1,3) \oplus (\bar{3},1,\bar{3})$	2	0	2	–	–
	$\tilde{Q}_R \oplus \bar{\tilde{Q}}_R$	$(1,3,3) \oplus (1,\bar{3},\bar{3})$	0	2	2	–	–

A Trinification TAF model

A phenomenologically interesting model:
Minimal Trinification (with Q_L, Q_R, L and 3 Higgses)
plus a vector-like quark family $\tilde{Q}_L \oplus \bar{\tilde{Q}}_L$ and $\tilde{Q}_R \oplus \bar{\tilde{Q}}_R$.

$$\mathscr{L} \ni M_L^{i'} Q_{Li'} \bar{\tilde{Q}}_L + M_R^{j'} Q_{Rj'} \bar{\tilde{Q}}_R + y_Q^{ni'j'} \; Q_{Li'} Q_{Rj'} H_n$$
$$+ y_{\bar{Q}}^n \; \bar{\tilde{Q}}_L \bar{\tilde{Q}}_R H_n^* + \frac{y_L^{nij}}{2} L_i L_j H_n + h.c.$$

We will choose $M_L^i = M_R^j = 0$ and write
$Q_{Li} = (u_{Li}, d_{Li}, \bar{d}'_{Ri}),\; Q_{L4} = (U_L, D_L, \bar{D}'_R),\; \bar{Q}_{L4} = (\bar{U}_L, \bar{D}_L, D'_L),$
$Q_{Rj} = (u_{Rj}, d_{Rj}, d'_{Rj}),\; Q_{R4} = (U_R, D_R, D'_R),\; \bar{Q}_{R4} = (\bar{U}_R, \bar{D}_R, \bar{D}'_L),$

Quark masses

Up quarks:

$$
\begin{array}{c} \\ u_{Li} \\ U_L \\ \bar{U}_R \end{array}
\begin{array}{c}
\begin{array}{ccc} u_{Rj} & U_R & \bar{U}_L \end{array} \\
\begin{pmatrix}
v_{un} y_Q^{nij} & v_{un} y_Q^{ni4} & 0 \\
v_{un} y_Q^{n4j} & v_{un} y_Q^{n44} & M_L \\
0 & M_R & v_{un} y_{\bar{Q}}^{n}
\end{pmatrix}
\end{array}
$$

Down quarks:

$$
\begin{array}{c} \\ d_L^i \\ \bar{d}_R^{i'} \\ \bar{D}'_R \\ D_L \\ D'_L \\ \bar{D}_R \end{array}
\begin{array}{c}
\begin{array}{cccccc} d_R^j & d_R^{j'} & D'_R & D_R & \bar{D}'_L & \bar{D}_L \end{array} \\
\begin{pmatrix}
v_{dn} y_Q^{nij} & v_L y_Q^{2ij} & v_L y_Q^{2i4} & v_{dn} y_Q^{ni4} & 0 & 0 \\
V_R y_Q^{2ij} & V_n y_Q^{nij} & V_n y_Q^{ni4} & V_R y_Q^{2i4} & 0 & 0 \\
V_R y_Q^{24j} & V_n y_Q^{n4j} & V_n y_Q^{n44} & V_R y_Q^{244} & M_L & 0 \\
v_{dn} y_Q^{n4j} & v_L y_Q^{24j} & v_L y_Q^{24j} & v_{dn} y_Q^{n44} & 0 & M_L \\
0 & 0 & M_R & 0 & v_{un} y_{\bar{Q}}^{n} & 0 \\
0 & 0 & 0 & M_R & 0 & v_{un} y_{\bar{Q}}^{n}
\end{pmatrix}
\end{array}
$$

Lepton masses

Charged leptons:

$$\begin{array}{c c} & \begin{array}{cc} e_R & \bar{e}'_L \end{array} \\ \begin{array}{c} e_L \\ e'_L \end{array} & \begin{pmatrix} -v_{dn}y_{Ln} & V_{Rn}y_{Ln} \\ v_{Ln}y_{Ln} & -V_n y_{Ln} \end{pmatrix} \end{array}$$

Neutral leptons:

$$\begin{array}{c c} & \begin{array}{ccccc} \nu_L & \nu_R & \nu'_L & \bar{\nu}'_L & \nu' \end{array} \\ \begin{array}{c} \nu_L \\ \nu_R \\ \nu'_L \\ \bar{\nu}'_L \\ \nu' \end{array} & \begin{pmatrix} 0 & -v_{un}y_{Ln} & 0 & -V_{Rn}y_{Ln} & 0 \\ & 0 & 0 & -v_{Ln}y_{Nn} & 0 \\ & & 0 & V_n y_{Ln} & v_{un}y_{Ln} \\ & & & 0 & v_{dn}y_{Ln} \\ & & & & 0 \end{pmatrix} \end{array}$$

Vector masses

Let's take only V_n and V_{Rn}: SM group still unbroken.

Defining $V^2 \equiv \sum_n (V_n^2 + V_{Rn}^2)$, $\alpha \equiv \sum_n V_{Rn}^2 / V^2$ and $\beta \equiv \sum_n V_n V_{Rn} / V^2$, the gauge bosons are:

- a $SU(2)_L$ doublet with 4 components: $M_{H_L} = g_L V/\sqrt{2}$;
- two charged fields $H_R^\pm$ with mass $M_{H_R^\pm} = g_R V/\sqrt{2}$
- two neutral fields H_R^0 with mass

$$M_{H_R^0}^2 = \frac{g_R^2 V^2}{4}\left[1 + \sqrt{(1-2\alpha)^2 + 4\beta^2}\right].$$

Vector masses

- the right-handed $W_R^\pm$ vectors (the lightest) with mass

$$M^2_{W_R^\pm} = \frac{g_R^2 V^2}{4}\left[1 - \sqrt{(1-2\alpha)^2 + 4\beta^2}\right]$$

- the Z_R and the Z_{B-L} vectors, that mix together. In the limit $V_{Rn} \ll V_n$ the mass eigenvalues are

$$M_{Z'} = V\sqrt{\frac{2}{3}(g_L^2 + g_R^2)}, \quad M_{Z''} \simeq |\beta| g_R V_R \sqrt{\frac{g_R^2/2 + 2g_L^2}{g_R^2 + g_L^2}}.$$

- The 12 SM vectors remain massless.

Conclusions

We performed a systematic search of TAF models such that:

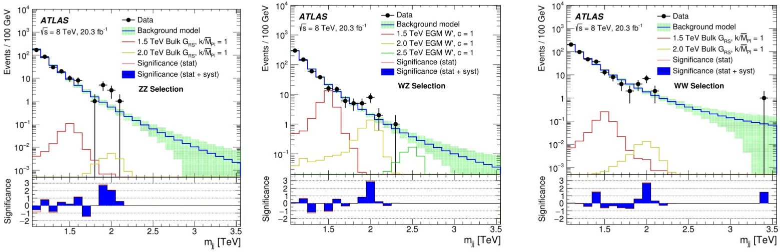
- gravity is softened, it doesn't spoil the observable sector predictions
- the perturbative regime holds up to infinite energy
- the phenomenology is consistent with the data

Among Trinification GUT models we found that:

- minimal Trinification has no TAF solutions
- the most interesting Trinification TAF model includes $\tilde{Q}_L \oplus \bar{\tilde{Q}}_L$ and $\tilde{Q}_R \oplus \bar{\tilde{Q}}_R$
- to get the right phenomenology naturally, 3 Higgs are needed.

What's next?

The experimentalists at LHC have seen excesses ($\sim 3\sigma$) in some channels at an energy $\simeq 2$ TeV.
The lightest vector W_R may be responsible of these observations?



[ATLAS Collaboration, arXiv:1506.00962]

Minimal Trinification

Field	spin	$SU(3)_L$	$SU(3)_R$	$SU(3)_c$
$Q_R = \begin{pmatrix} u_R^1 & u_R^2 & u_R^3 \\ d_R^1 & d_R^2 & d_R^3 \\ d_R'^1 & d_R'^2 & d_R'^3 \end{pmatrix}$	$1/2$	1	3	$\bar{3}$
$Q_L = \begin{pmatrix} u_L^1 & d_L^1 & \bar{d}_R'^1 \\ u_L^2 & d_L^2 & \bar{d}_R'^2 \\ u_L^3 & d_L^3 & \bar{d}_R'^3 \end{pmatrix}$	$1/2$	3	1	$\bar{3}$
$L = \begin{pmatrix} \bar{\nu}_L' & e_L' & e_L \\ \bar{e}_L' & \nu_L' & \nu_L \\ e_R & \nu_R & \nu' \end{pmatrix}$	$1/2$	3	$\bar{3}$	1
H	0	3	$\bar{3}$	1

Vector masses

The vev V_1 alone breaks
$G_{333} \to SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \otimes SU(3)_c$.
A $SU(2)_L$ doublet H_L, a $SU(2)_R$ doublet H_R and a Z' singlet acquire mass:

$$M_{H_L} = \frac{g_L}{\sqrt{2}} V_1, \qquad M_{H_R} = \frac{g_R}{\sqrt{2}} V_1, \qquad M_{Z'} = V_1 \sqrt{\frac{2}{3}(g_L^2 + g_R^2)}.$$

Quark masses with 3 Higgses

Let's assume that H_1 breaks G_{333} but preserves $G_{\rm SM}$ (i.e. $V_1 \neq 0$ and $v_{d1} = v_{u1} = v_{L1} = 0$).
The Yukawa couplings y_{Q1} and y_{L1} allow to give large enough masses $M_{d'_R} = V_1 y_{Q1} \gtrsim 700\text{GeV}$ and $M_{e'_R} = V_1 y_{L1} \gtrsim 200\text{GeV}$ to the extra primed fermions, without also giving too large masses to the SM fermions.
H_2 and H_3 can have the small Yukawa couplings needed to reproduce the light SM fermion masses,

$$m_e \sim \sum_{n=2}^{3} v_{dn} y_{Ln}, \qquad m_u \sim \sum_{n=2}^{3} v_{un} y_{Qn}, \qquad m_d \sim \sum_{n=2}^{3} v_{dn} y_{Qn}.$$

An idea of the problem

For minimal Trinification with 3 Higgses

```
72 free scalar couplings {λ10a, λ10aI, λ10b, λ10bI, λ11a, λ11aI, λ11b, λ11bI, λ12a, λ12c, λ12d, λ12eI, λ12f, λ12fI, λ13a, λ13aI, λ13b, λ13bI, λ13c, λ13cI,
  λ13f, λ13fI, λ14a, λ14aI, λ14b, λ14bI, λ14c, λ14cI, λ14f, λ14fI, λ15a, λ15aI, λ15b, λ15bI, λ15c, λ15cI, λ15f, λ15fI, λ1a, λ1b, λ2a, λ2b, λ3a, λ3b, λ4a, λ4aI,
  λ4b, λ4bI, λ5a, λ5aI, λ5b, λ5bI, λ6a, λ6c, λ6d, λ6eI, λ6f, λ6fI, λ7a, λ7aI, λ7b, λ7bI, λ8a, λ8aI, λ8b, λ8bI, λ9a, λ9c, λ9d, λ9eI, λ9f, λ9fI}  in positions
 {{23, 23, 28, 46}, {22, 31, 31, 39}, {20, 21, 33, 50}, {25, 31, 35, 48}, {34, 39, 42, 53}, {27, 44, 50, 52}, {29, 47, 52, 52}, {27, 46, 53, 53}, {23, 23, 43, 43}, {34, 36, 40, 42},
  {21, 28, 41, 48}, {19, 36, 37, 53}, {22, 26, 37, 45}, {19, 35, 42, 49}, {13, 20, 40, 51}, {9, 34, 43, 49}, {14, 35, 37, 42}, {16, 32, 38, 39}, {16, 34, 44, 44}, {13, 32, 45, 45},
  {8, 34, 46, 50}, {5, 26, 37, 47}, {1, 8, 24, 47}, {2, 7, 28, 40}, {14, 17, 20, 41}, {14, 18, 26, 47}, {8, 8, 33, 51}, {15, 15, 20, 37}, {8, 16, 28, 50}, {1, 12, 25, 41}, {2, 22, 33, 49},
  {18, 25, 32, 48}, {11, 19, 24, 44}, {16, 20, 22, 49}, {7, 36, 36, 43}, {17, 28, 28, 54}, {17, 30, 32, 43}, {2, 24, 31, 54}, {10, 10, 17, 17}, {3, 6, 15, 18}, {29, 29, 31, 31},
  {21, 24, 33, 36}, {40, 40, 48, 48}, {40, 41, 51, 54}, {14, 22, 22, 32}, {14, 24, 24, 31}, {14, 19, 23, 36}, {7, 28, 32, 34}, {7, 13, 15, 27}, {6, 15, 17, 21}, {1, 10, 10, 19},
  {6, 6, 10, 27}, {10, 10, 24, 24}, {9, 11, 34, 36}, {10, 16, 29, 35}, {10, 18, 27, 36}, {6, 13, 20, 35}, {11, 15, 28, 35}, {15, 37, 37, 51}, {10, 42, 42, 45}, {3, 41, 52, 54},
  {11, 38, 42, 44}, {3, 13, 16, 38}, {9, 14, 16, 44}, {5, 5, 14, 50}, {6, 8, 8, 41}, {16, 16, 44, 44}, {2, 5, 43, 48}, {10, 16, 44, 50}, {5, 10, 41, 45}, {7, 16, 45, 50}, {9, 14, 44, 52}}

18 overlapped scalar couplings {λ12b, λ12e, λ13d, λ13dI, λ13e, λ13eI, λ14d, λ14dI, λ14e, λ14eI, λ15d, λ15dI, λ15e, λ15eI, λ6b, λ6e, λ9b, λ9e}  in positions
 {{20, 28, 37, 45}, {30, 31, 48, 49}, {1, 25, 37, 43}, {1, 34, 37, 51}, {17, 20, 37, 54}, {18, 26, 44, 53}, {12, 17, 30, 53}, {10, 10, 27, 46}, {10, 18, 27, 53},
  {7, 13, 31, 44}, {6, 20, 24, 38}, {13, 30, 31, 47}, {17, 28, 36, 45}, {15, 34, 36, 54}, {5, 18, 23, 36}, {5, 14, 24, 31}, {12, 13, 48, 49}, {6, 16, 41, 51}}

45 fermion couplings {yEL1R1a, yEL1R2a, yEL1R3a, yL1R1a, yL1R2a, yL1R3a, yL2R1a, yL3R1a, yEL2R2a, yEL2R3a, yL2R2a, yL2R3a, yL3R2a,
  yEL3R3a, yL3R3a, yEL1R1b, yEL1R2b, yEL1R3b, yL1R1b, yL1R2b, yL1R3b, yL2R1b, yL3R1b, yEL2R2b, yEL2R3b, yL2R2b, yL2R3b, yL3R2b, yEL3R3b, yL3R3b,
  yEL1R1c, yEL1R2c, yEL1R3c, yL1R1c, yL1R2c, yL1R3c, yL2R1c, yL3R1c, yEL2R2c, yEL2R3c, yL2R2c, yL2R3c, yL3R2c, yEL3R3c, yL3R3c}  in positions
 {{1, 5, 9}, {1, 5, 36}, {1, 5, 63}, {1, 10, 19}, {1, 10, 46}, {1, 10, 73}, {1, 19, 37}, {1, 19, 64}, {1, 32, 36}, {1, 32, 63}, {1, 37, 46}, {1, 37, 73},
  {1, 46, 64}, {1, 59, 63}, {1, 64, 73}, {19, 5, 9}, {19, 5, 36}, {19, 5, 63}, {19, 10, 19}, {19, 10, 46}, {19, 10, 73}, {19, 19, 37}, {19, 19, 64},
  {19, 32, 36}, {19, 32, 63}, {19, 37, 46}, {19, 37, 73}, {19, 46, 64}, {19, 59, 63}, {19, 64, 73}, {37, 5, 9}, {37, 5, 36}, {37, 5, 63}, {37, 10, 19},
  {37, 10, 46}, {37, 10, 73}, {37, 19, 37}, {37, 19, 64}, {37, 32, 36}, {37, 32, 63}, {37, 37, 46}, {37, 37, 73}, {37, 46, 64}, {37, 59, 63}, {37, 64, 73}}
```

An idea of the problem

For Trinification with 3 Higgses and the new quark family (TAF)

```
TrovaTAF[{0, 0, 0}] // Timing
```

Coefficienti RGE di gauge: {L,R,C} = $\left\{-\frac{3}{2}, -\frac{3}{2}, -1\right\}$

1

{Found TAF solution number , 1, at attempt , 1}

2

{Found TAF solution number , 2, at attempt , 1}

3

...down to 0.00366261

...down to 2.83773×10^{-28}

{Found TAF solution number , 3, at attempt , 3}

4

...down to 0.00321249

...down to 3.53552×10^{-28}

{Found TAF solution number , 4, at attempt , 2}

5

...down to 3.09757×10^{-28}

{Found TAF solution number , 5, at attempt , 1}

Found 5 TAF for 90 quartic couplings. (check: 0 = 1.06581×10^{-14} ?)

{133.562500, Null}