UNIVERSITÀ DI PISA

Resolution Modelling in PET

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PET: definition

Positron Emission Tomography (PET) is molecular imaging technique used for “the visual representation, characterization and quantification of biological processes that take place in a living being at the cellular and sub-cellular level”.

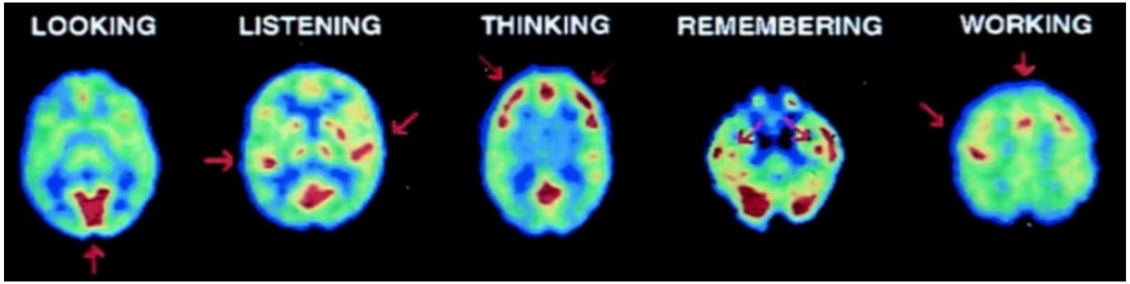
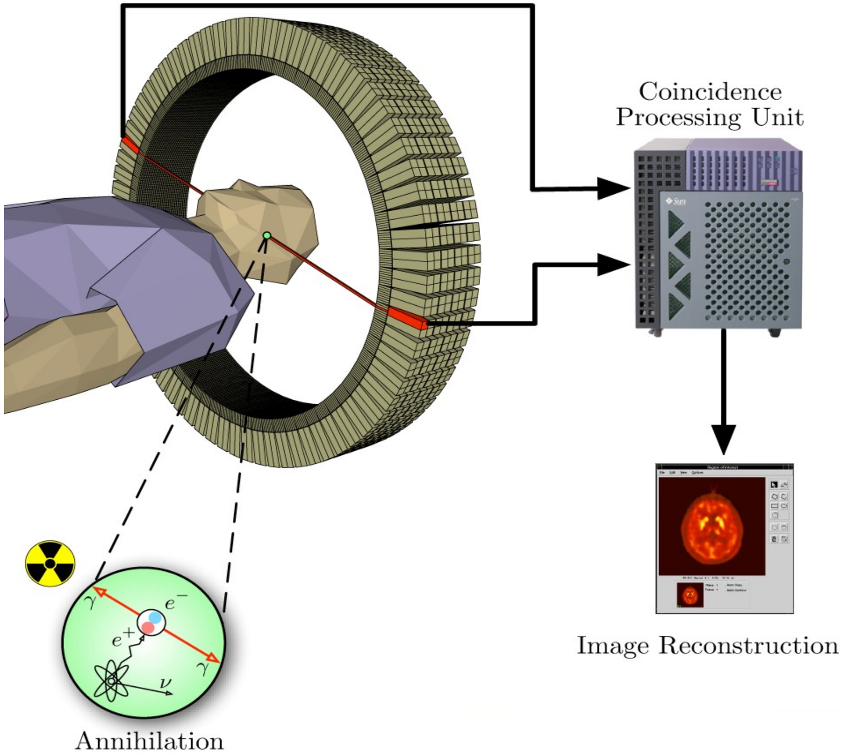


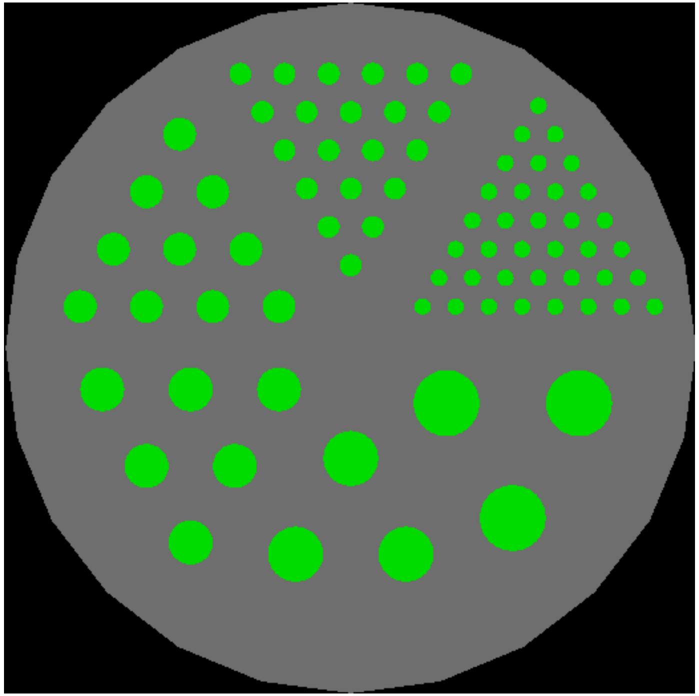
Figure: PET studies of glucose metabolism (by means of 18 F-FDG) to map human brain’s response in performing different tasks. Highest metabolic rates are in red, with lower values from yellow to blue (Drs. Michael Phelps and John Mazziotta, UCLA School of Medicine).

PET: imaging process

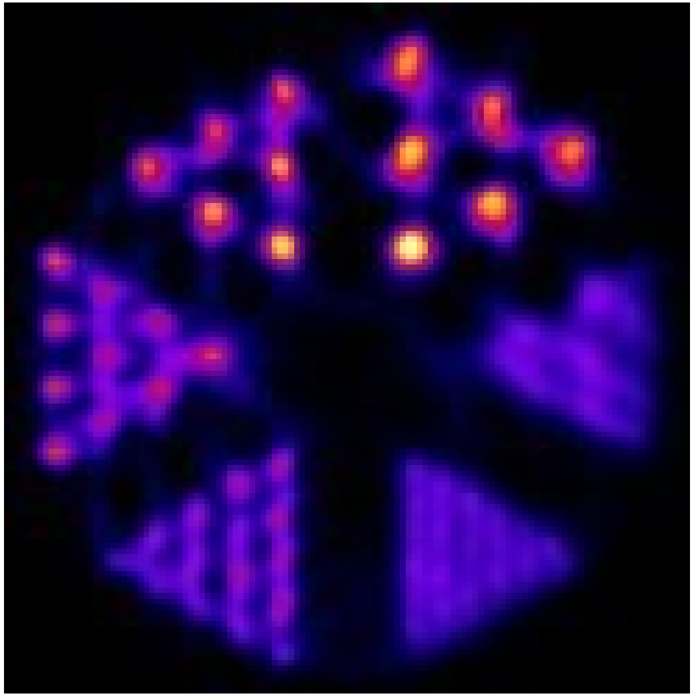
- Radiotracer administration to the patient
- Radiotracer diffusion within the body
- Positron emission and annihilation
- Photon coincidence detection
- Data processing
- Image reconstruction



PET: example

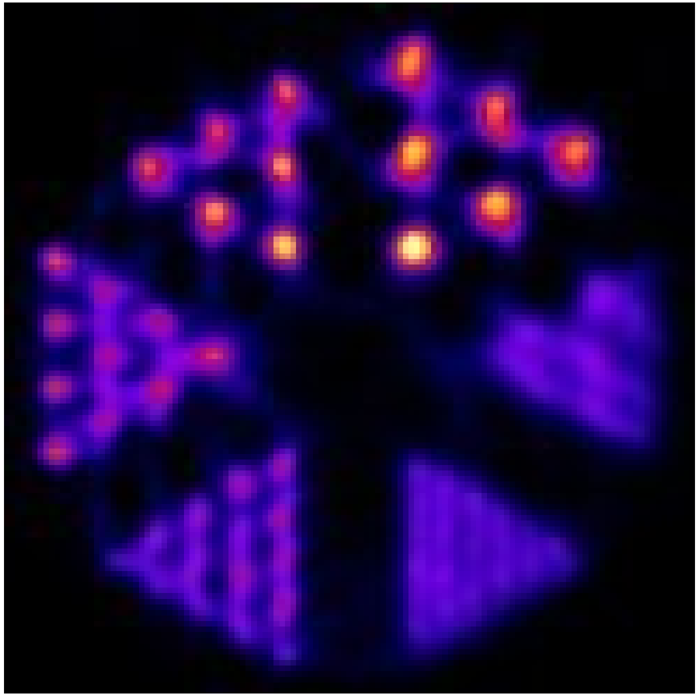


Derenzo Phantom

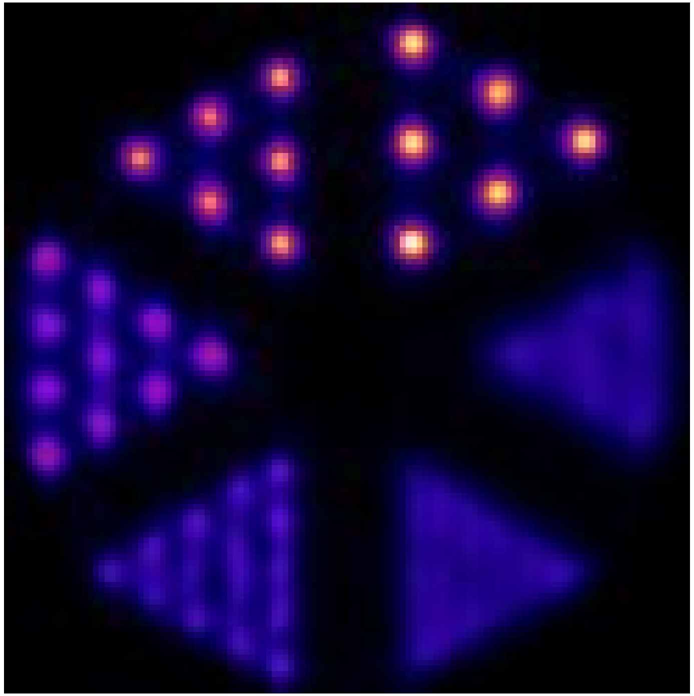


Standard image reconstruction

PET: example



Standard image reconstruction



Improved image reconstruction

Motivations

The main goal of this work is the study of a methodology to perform accurate image reconstruction for PET and to develop a fast image reconstruction software, with particular focus to the development of solutions for the preclinical and brain scanners under development at the Physics Department of the University of Pisa.

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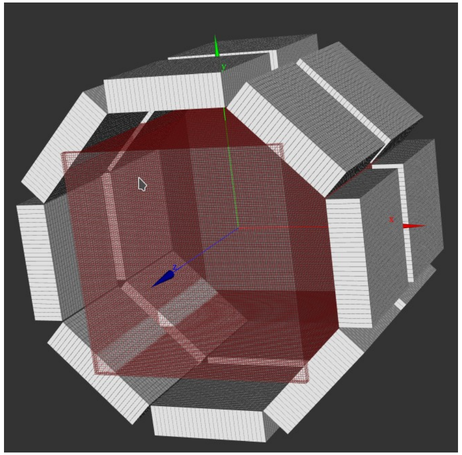
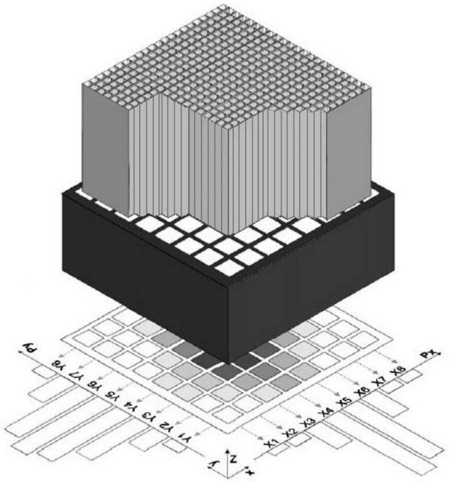
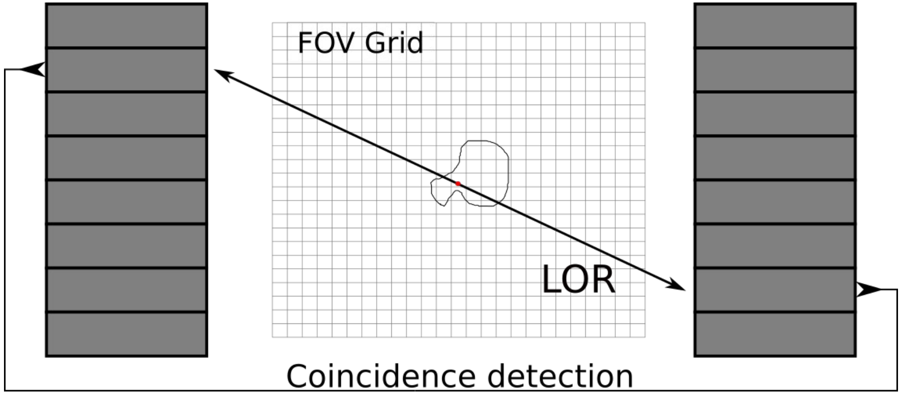
… thanks to a better modelling of the physics of the system. This is achieved using an image reconstruction framework known as resolution modelling.

Image formation

Simplified view







Scanner: planar detectors arranged within one or more rings

Module: scintillation crystals coupled to PMT or SiPM

Line of Response: it’s the line connecting two crystals of facing detectors.

Image reconstruction

Definition of the problem

$$\eta = S\rho$$

- η : LOR counts vector
- ρ : Voxels activity vector
- S : System model
- ML-EM algorithm with prior poisson distribution for data

$$\rho_i^{n+1} = \frac{\rho_i^n}{\sum_x S_{xi}} \sum_j S_{ji} \frac{\eta_j}{\sum_k S_{jk} \rho_k^n}$$

System model: resolution modelling

$$S = NDAGR$$

- N : normalization factors
- D : detector response
- A : attenuation factors
- G : geometric component
- R : positron range

Factorized System Model

$$S = NDG$$

$$N \in \mathbb{R}_{nLoRs,nLoRs}$$

$$D \in \mathbb{R}_{nLoRs,nLoFs}$$

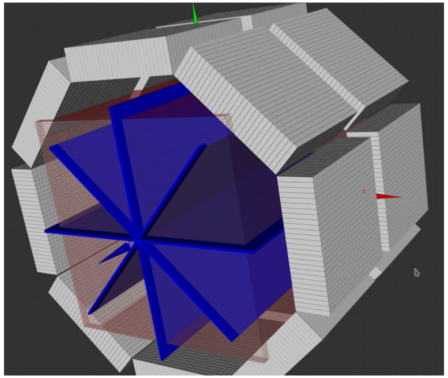
$$G \in \mathbb{R}_{nLoFs,nVoxels}$$

- The element $g_{ij} \in G$ represents the probability that a photon-pair emitted in the j-th voxel reaches the i-th LoF
- The element $d_{ki} \in D$ represents the probability that a phton-pair travelling along the i-th LoF is assigned to the k-th LoR
- The element $n_{kk} \in N$ represents the sensitivity of the k-th LoR

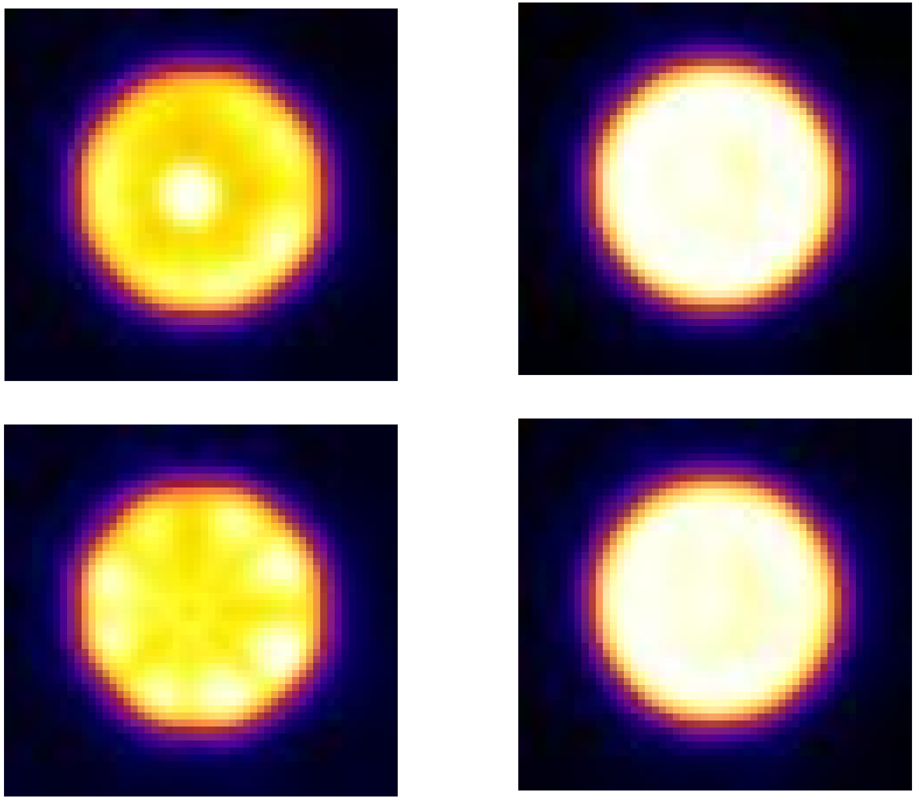
Every component is precomputed and stored to disk

Normalization component

- Direct normalization: a planar source is placed with different inclines inside the FOV
- The normalization coefficients are computed as the ratio between the measured value and the teoric values obtained using the other compoenets of the model.

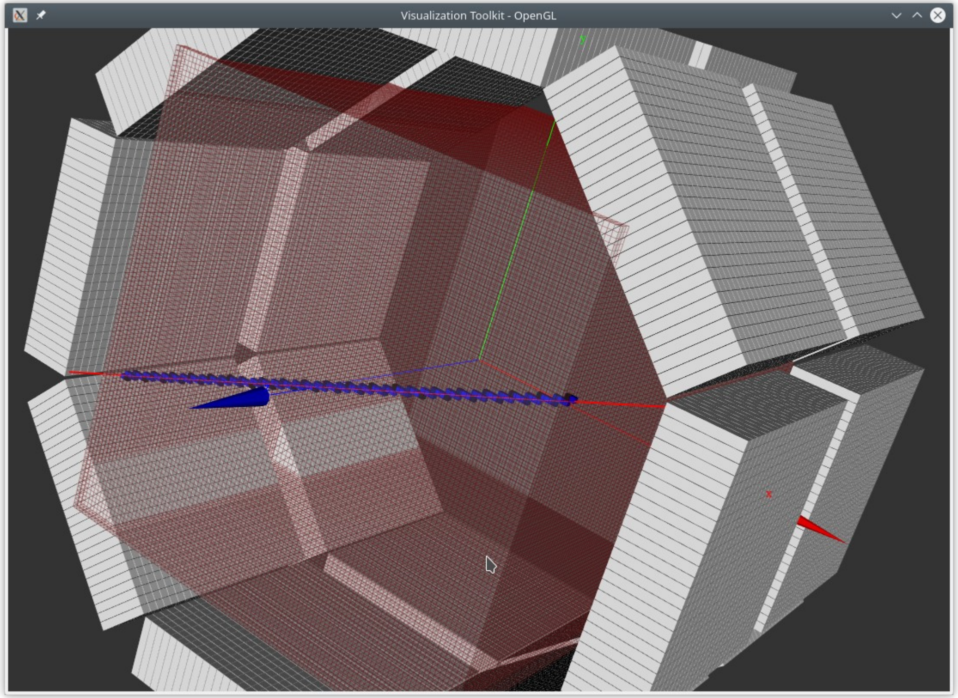


$$N = \frac{\eta}{DG\rho_{phantom}}$$



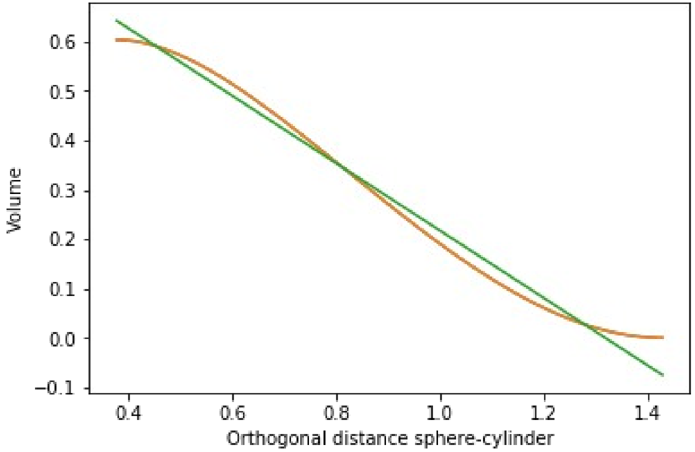
Geometric component

- The geometric component is usually computed using a ray-tracer “projector”
- The projector computes the volume of interception between the ToR and the FOV
- Most of the time it is computed on-the-fly during each reconstruction (we instead use a stored model)
- We are looking for a fast and accurate projector



Geometric component: projectors

- Siddon single ray:
  - Fastest method
  - Not suited to model a Tube of Respose
- Multiray
  - More accurate
  - Slowest method
- Orthogonal distance (OD)
  - Fast
  - Not fully accurate
- Sphere-Cylinder interc. (SC)
  - Fast
  - Not fully accurate, but better then OD



Geometric component: new proj.

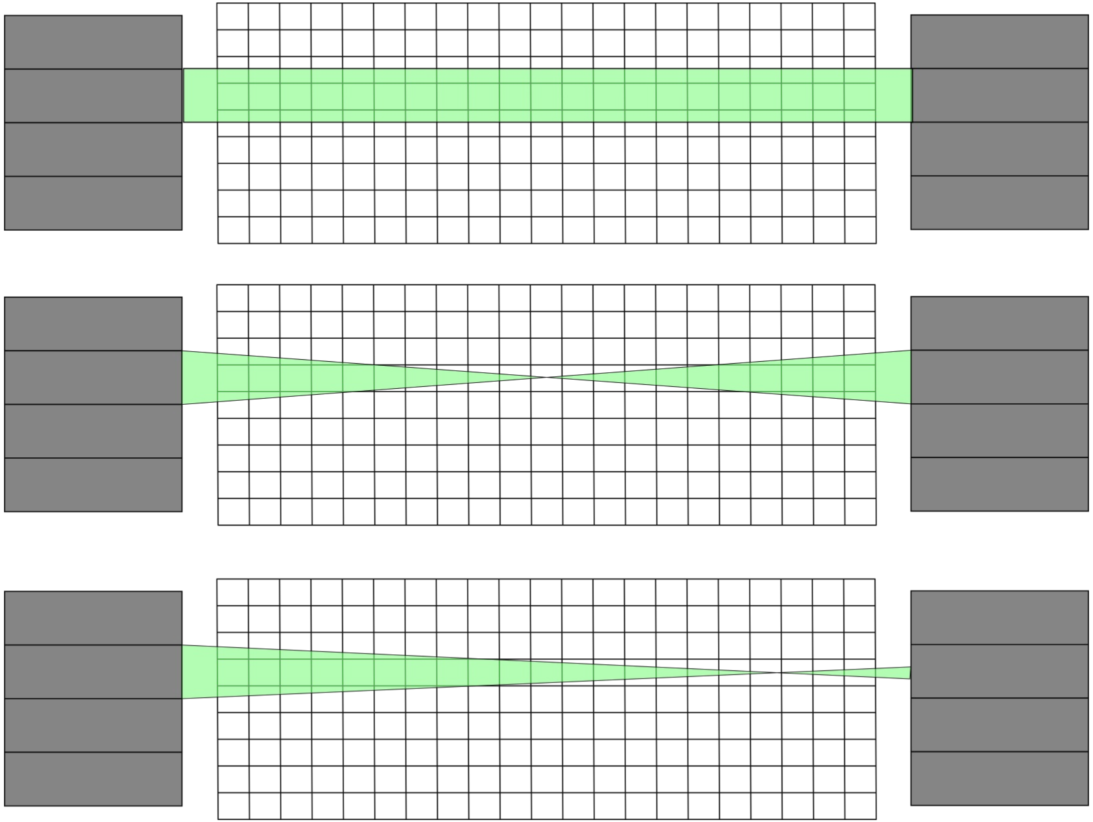
We developed a new projector by composing the previous algorithms:

- First we have a siddon single ray to determine the crossed voxels for each plane
- We search around every pixel found at the first step in a fixed area
- We use the sphere cylinder approximation to determine if a voxel is inside the Tube of Response.
- We compute the VOI for the partially crossed voxels with the use of the linear approximation instead of the elliptic integration

The computation time of the whole model is strongly reduced: from a couple of days to tens of seconds/minutes

Future: Solid Angle Fraction

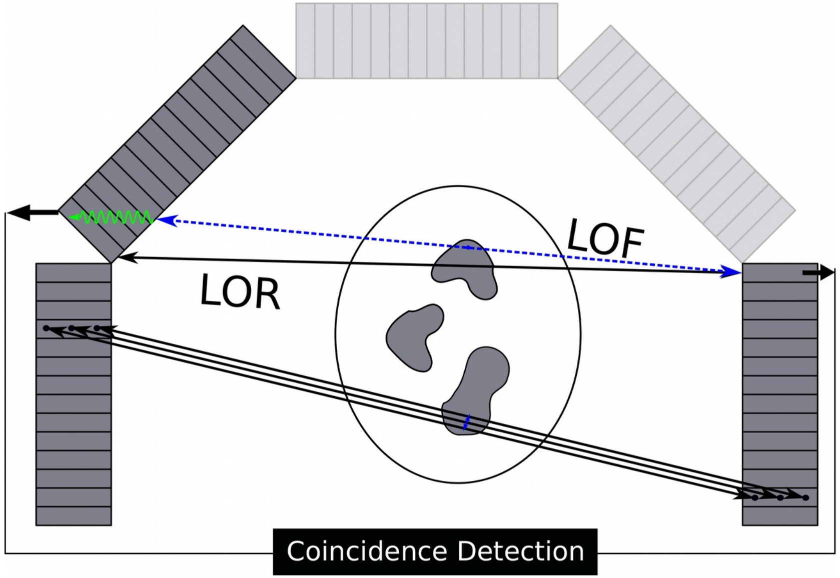
- Every line-tracer projector do not consider the solide angle subtended by the detector face with respect to the voxel
- For a standard projector the volume of interception is constant along the ToR (top image)
- The probability should be instead proportional to the subtended solid angle, which diminish while getting closer to the detector surface (middle and bottom images)
- A Monte Carlo geometric model can account for this problem



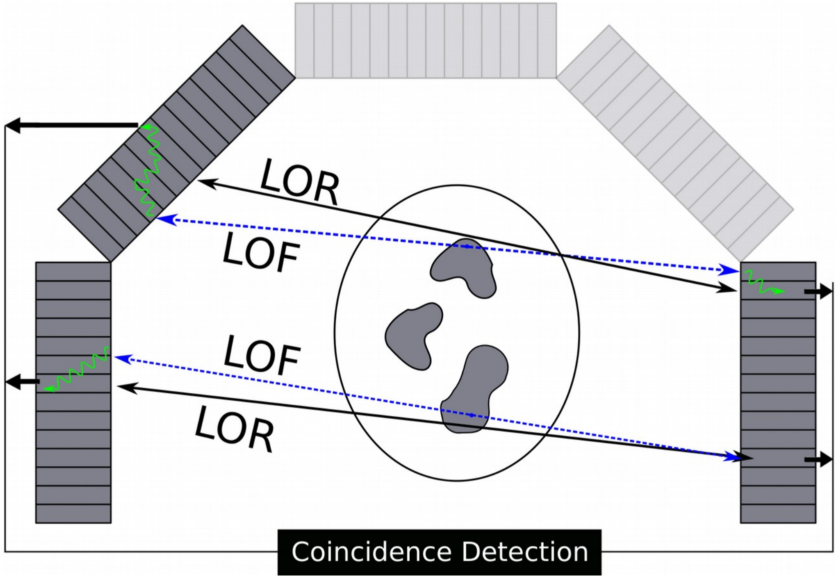
Detector component

The detector component models the penetration and inter-crystal scatter effecs.

Penetration effect

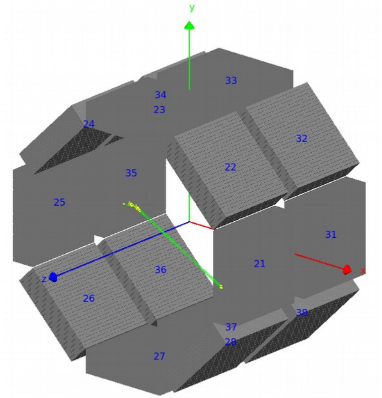


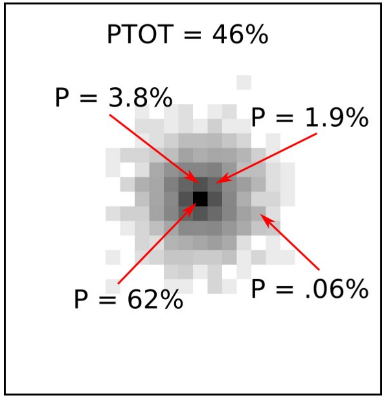
Inter-crystal scatter effect

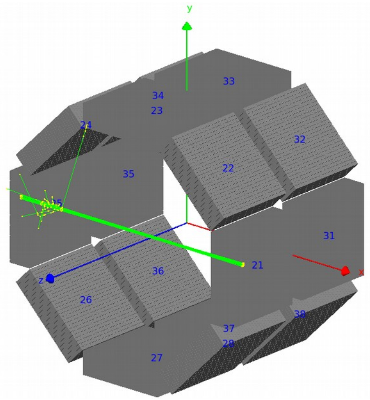


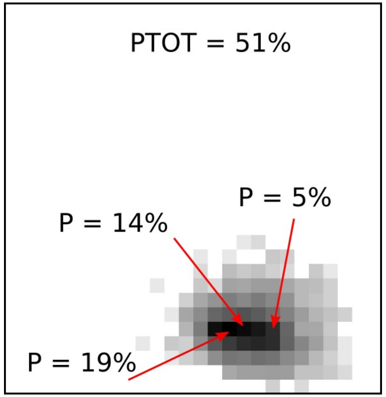
Detector component

- It is directly computed with a Monte Carlo simulation (GEANT4)
- It must be computed and stored to disk due to the high computation time required
- It is necessary to apply a threshold on the probabilities values to discard useless elements and reduce the size of the model









Using the detector component

$$\eta = NDG\rho = S\rho$$

$$\rho_i^{n+1} = \frac{\rho_i^n}{\sum_x S_{xi}} \sum_j S_{ji} \frac{\eta_j}{\sum_k S_{jk}\rho_k^n}$$

Full-product

It consists of computing the product between D and G to create the final model S

- ✔ Easy to use (no software change)
- ✔ Low noise
- ✗ Big model (need compression)
- ✗ Long reconstruction times

Pilleri et al, Physica Medica, Jan 2019

Two-step

It consists into splitting the fundamental linear system into two separate systems to be sovled in consecutive order

- ✔ Short reconstruction times
- ✗ High noise
- ✗ It requires further study to be used

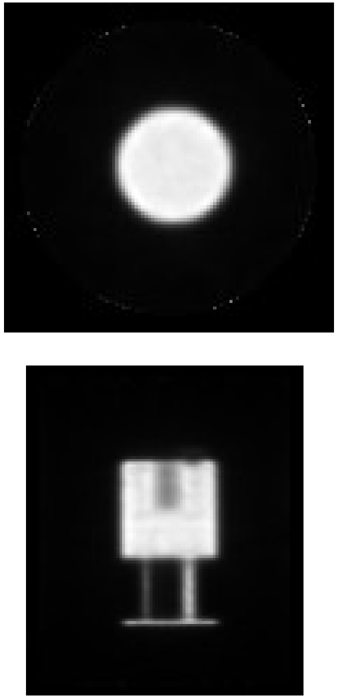
Pilleri et al, IEEE NSS MIC 2019, Oct 2019

Associative-product

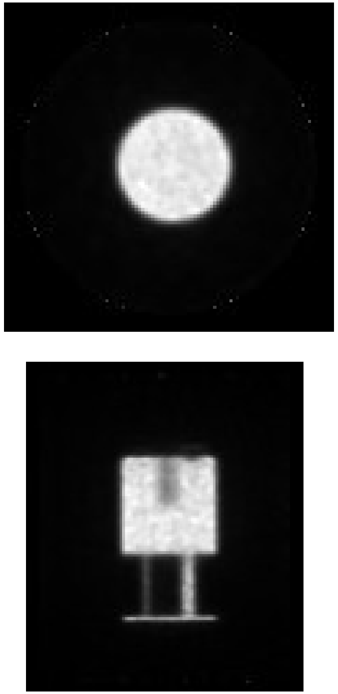
It exploits the associative property of the matrix multiplication operator to perform the projection and back- projection operations.

- ✔ Short reconstruction times
- ✔ Low noise
- ✗ Software change

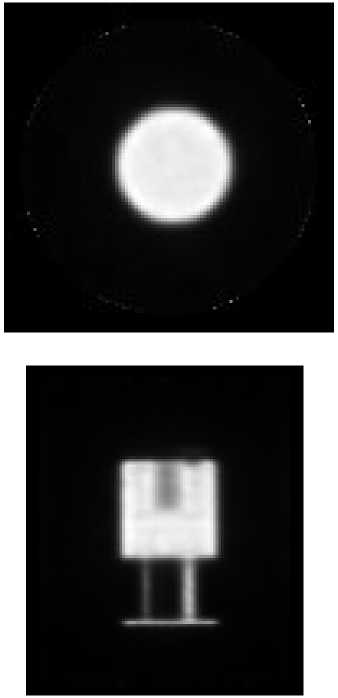
Detector component: comparison



Full-p

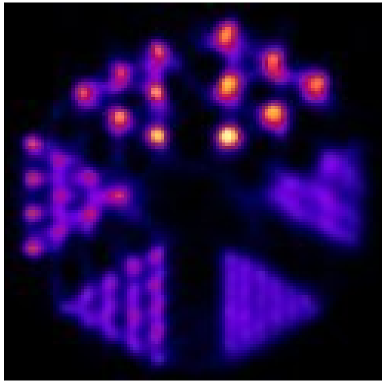


Two-step

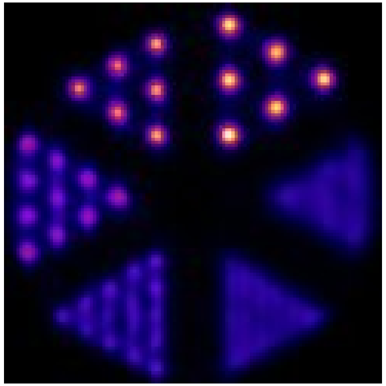


Associative-p

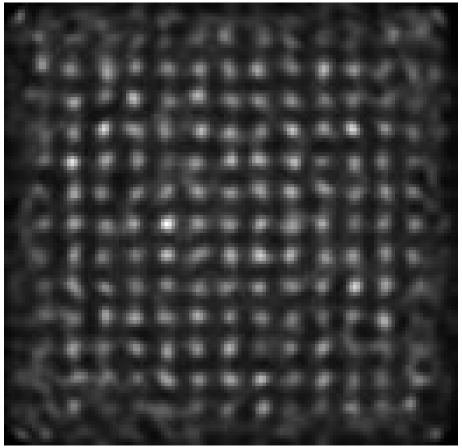
Detector component: example



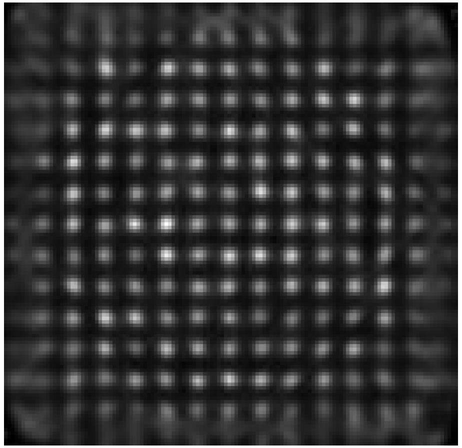
G model only



G and D models



G model only

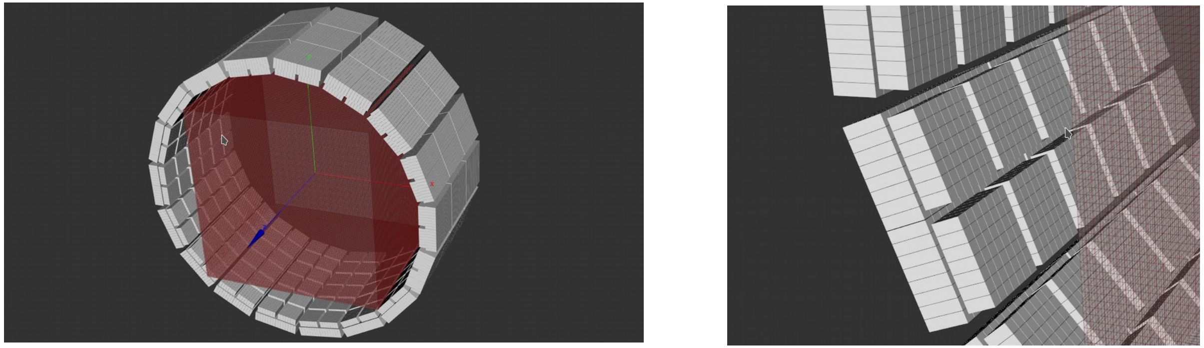


G and D models

Detector component (future)

Iris and Iris XL scanners: single-layer scintillation matrix

TRIMAGE scanner: dual-layer scintillation matrix

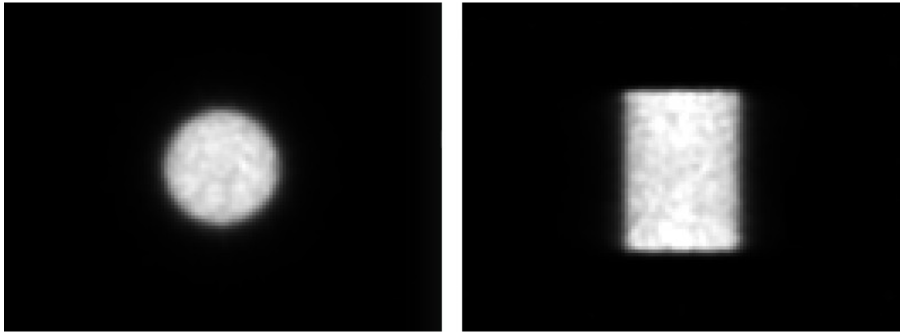


Developement of a methodology to compute the detector matrix for a multi-layer detector

New reconstruction software

The software requires a ~50lines xml file which describes the geometry of the scanner and it provides:

- Geometry and projectors visualization (debug purposes)
- Normalization coefficients computation starting from phantom acquisition
- Geometric model computation
- Detector model computation (single-layer only)
- Fast reconstruction using the three componenet N, D and G



TRIMAGE cylinder geometric reconstruction

Conclusions

- A new projector has been developed. The projector is accurate as most other projectors in literature and it allows fast matrix rebuild after any configuration change.
- A fast method to include the detector component has been developed for a detector with a single-layer scintillation matrix.

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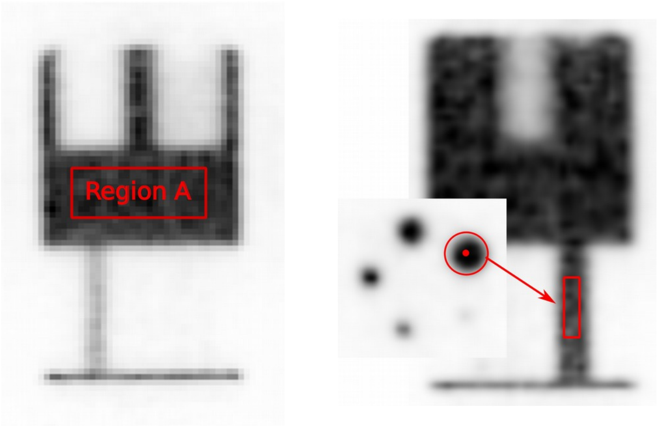
A new reconstruction software has been developed. It provides an easy method for the automatic computation of the system matrix components (N, G and D). The software also provides fast reconstruction using both geometric and detector matrix.

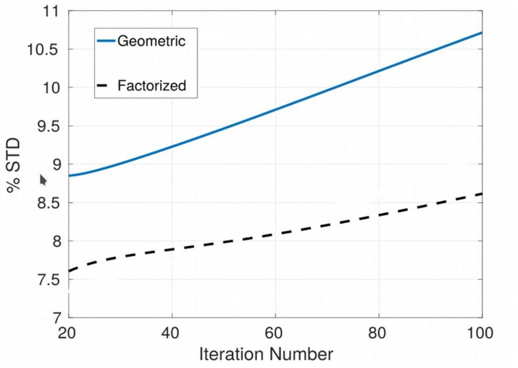
Future work

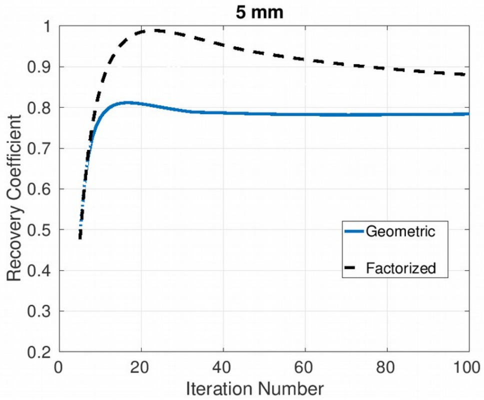
- Gometric component: understand the impact of the aplication of a correction associated to the Solid Angle Fraction.
- Detector component: development of the methodology to compute the matrix for a multi-layer detector
- Reconstruction software: provide a stable reconstruction framework for the scanners under development at the University of Pisa: IRIS, IRIS XL and TRIMAGE

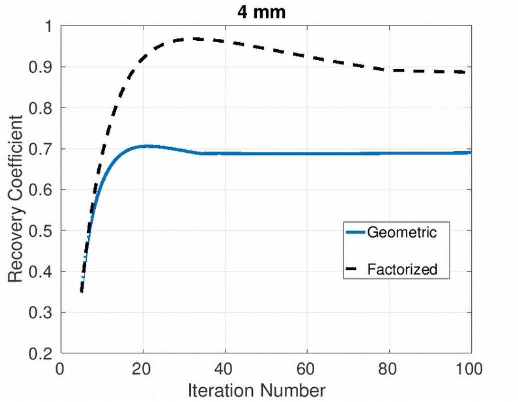
Backup

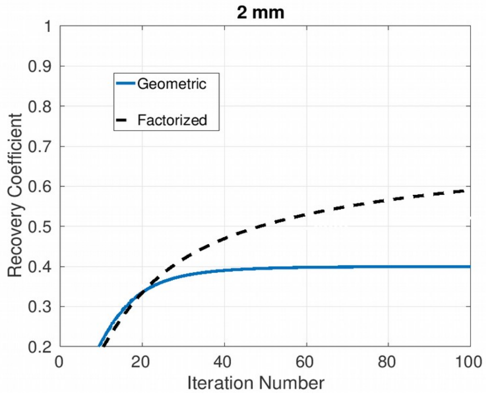
NEMA quality IRIS











Two-step reconstruction

Standard reconstruction

$$\eta = S\ \rho$$

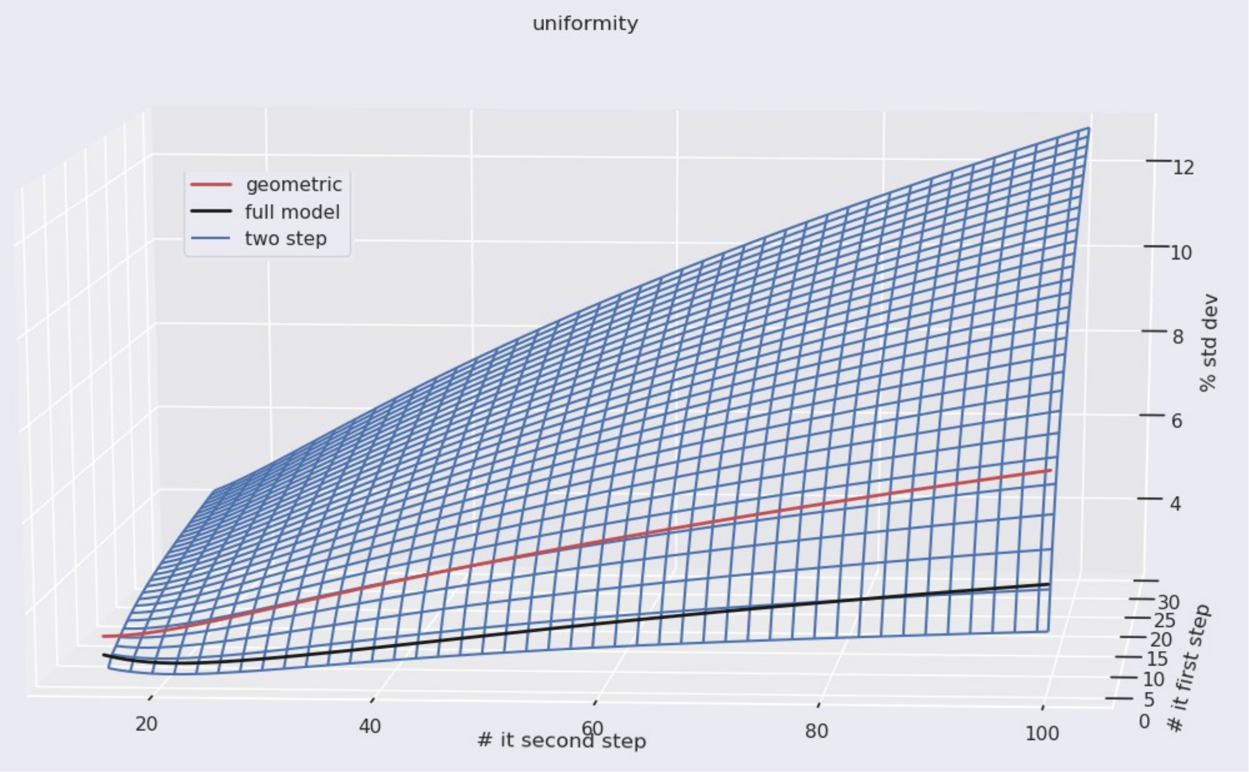
Two-step reconstruction

$$\eta^{LOR} = D\ \eta^{LOF}$$

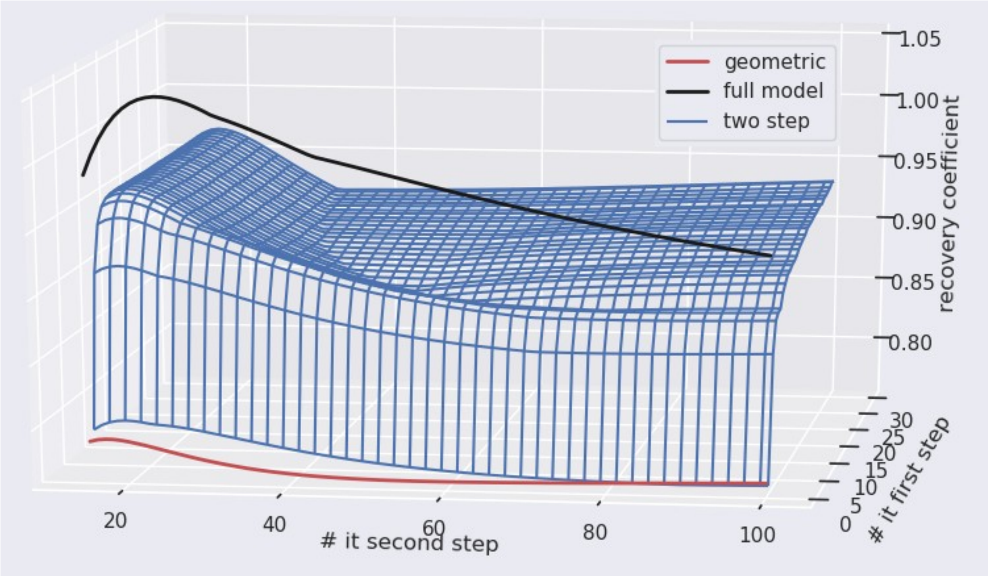
$$\eta^{LOF} = G\ \rho$$

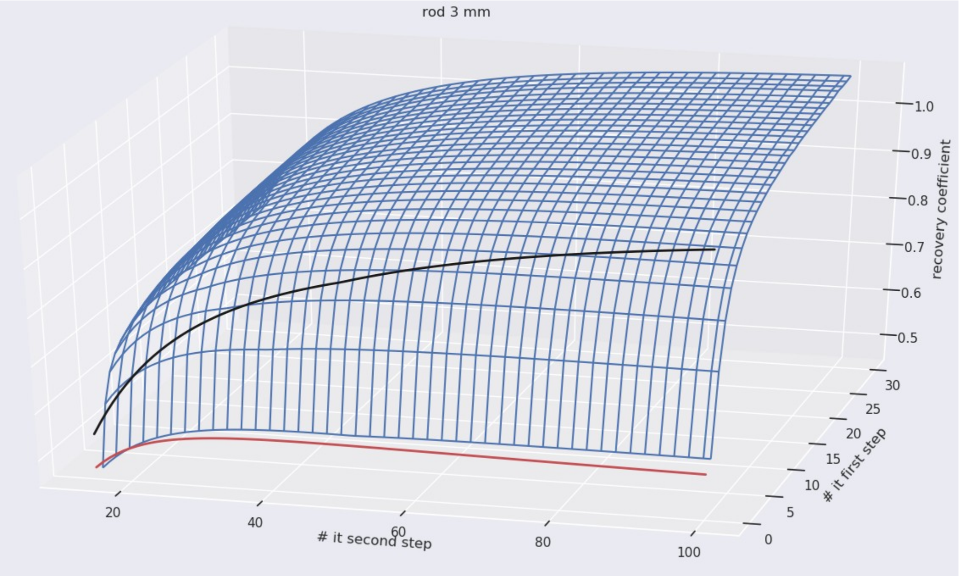
Two-step: noise

- The uniformity changes as a function of the number of iteration of the first step
- The noise is lower in the full-product model
- Does it exist a configuration (#it1, #it2) with same noise and recovery coefficients of the full-product model? The answer is: Not with this algorithm.



Two-step: recovery coefficients





If we consider a configuration (#it1, #it2) with same uniformity of the full-product reconstruction, the two-step solution has lower recovery coefficients.