Entropy and thermodynamic cycles in Josephson Junctions

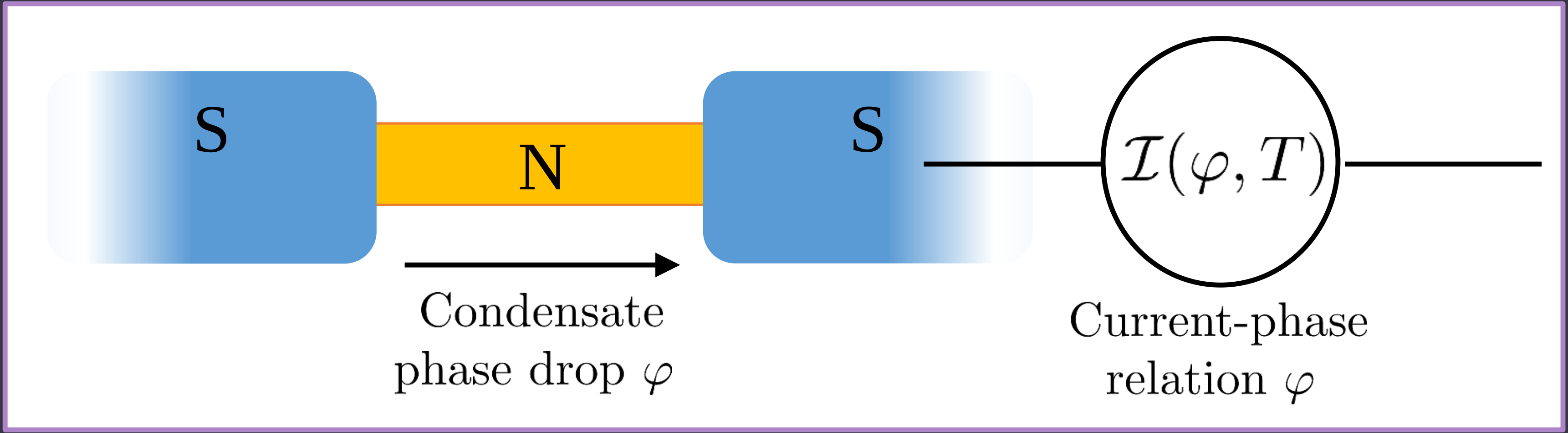
FRANCESCO VISCHI

Introduction

- Main topic: thermodynamics (TD) in Josephson junctions
- Starting point: inconsistency between expected TD and microscopical calculations
- Solution: inverse proximity effect
- Consequences/applications: heat capacity, TD processes and cycles

SNS Josephson Junction

- **SNS Josephson junction :** two terminal electrical device
  two superconducting part coupled by means of a normal metal
- **Stationary case :** supercurrent flowing, NO voltage , NO dissipation
- Flowing supercurrent determined by the **condensate phase difference** φ
- **Current-phase relation** $\mathcal{I}(\varphi, T)$



Thermodynamics of a Junction Maxwell relation

The junction must be treated as thermodynamic system, with free energy differential

$$dF(\varphi,T) = -S\mathrm{d}T + \mathrm{d}\mathcal{E} = -S\mathrm{d}T + \frac{eR_0}{2\pi}\mathcal{I}\mathrm{d}\varphi$$

It holds the following Maxwell relation connecting **entropy** and **current phase relation**:

$R_0 = \frac{h}{2e^2} = 13\,\mathrm{k}\Omega$

Quantum of resistance

Junction entropy

$$-\frac{eR_0}{2\pi}\frac{\partial \mathcal{I}(\varphi,T)}{\partial T} = \frac{\partial S(\varphi,T)}{\partial \varphi}$$

Temperature

Junction phase drop

Ref. 2, 3

Entropy and supercurrent, Microscopic mechanism

Quasi-particles

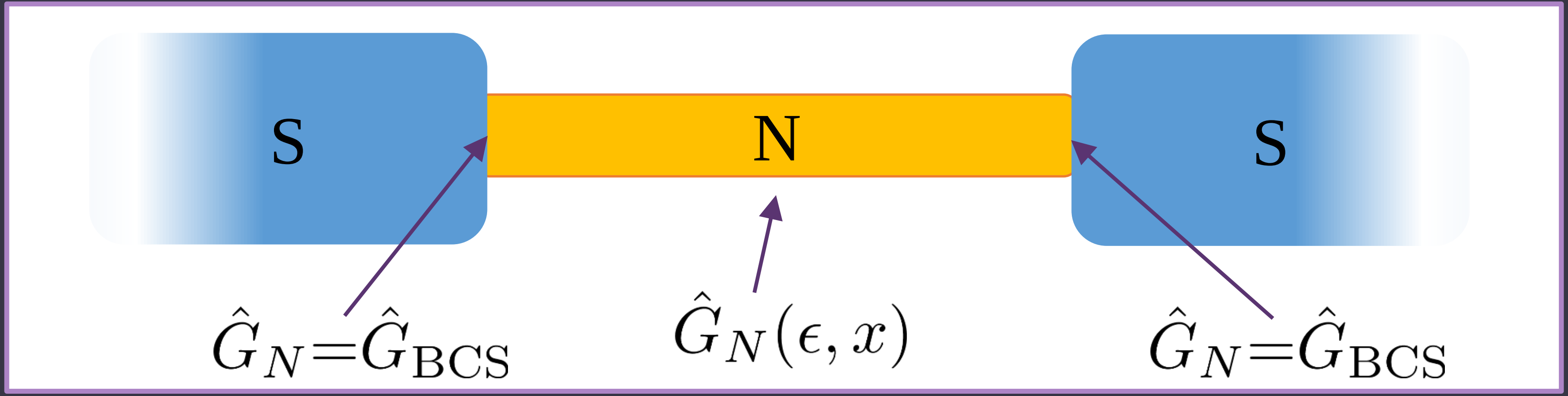
Condensate

	Quasi-particles	Condensate
Phase coherence	✗ No phase coherence	✓ Macroscopic phase φ
Heat and entropy	✓ Entropy and heat capacitance	✗ Fundamental state
Transport	Heat transport	Supercurrent $\mathcal{I}$ transport

Ref. 3, 4

SNS junction : microscopical treatment

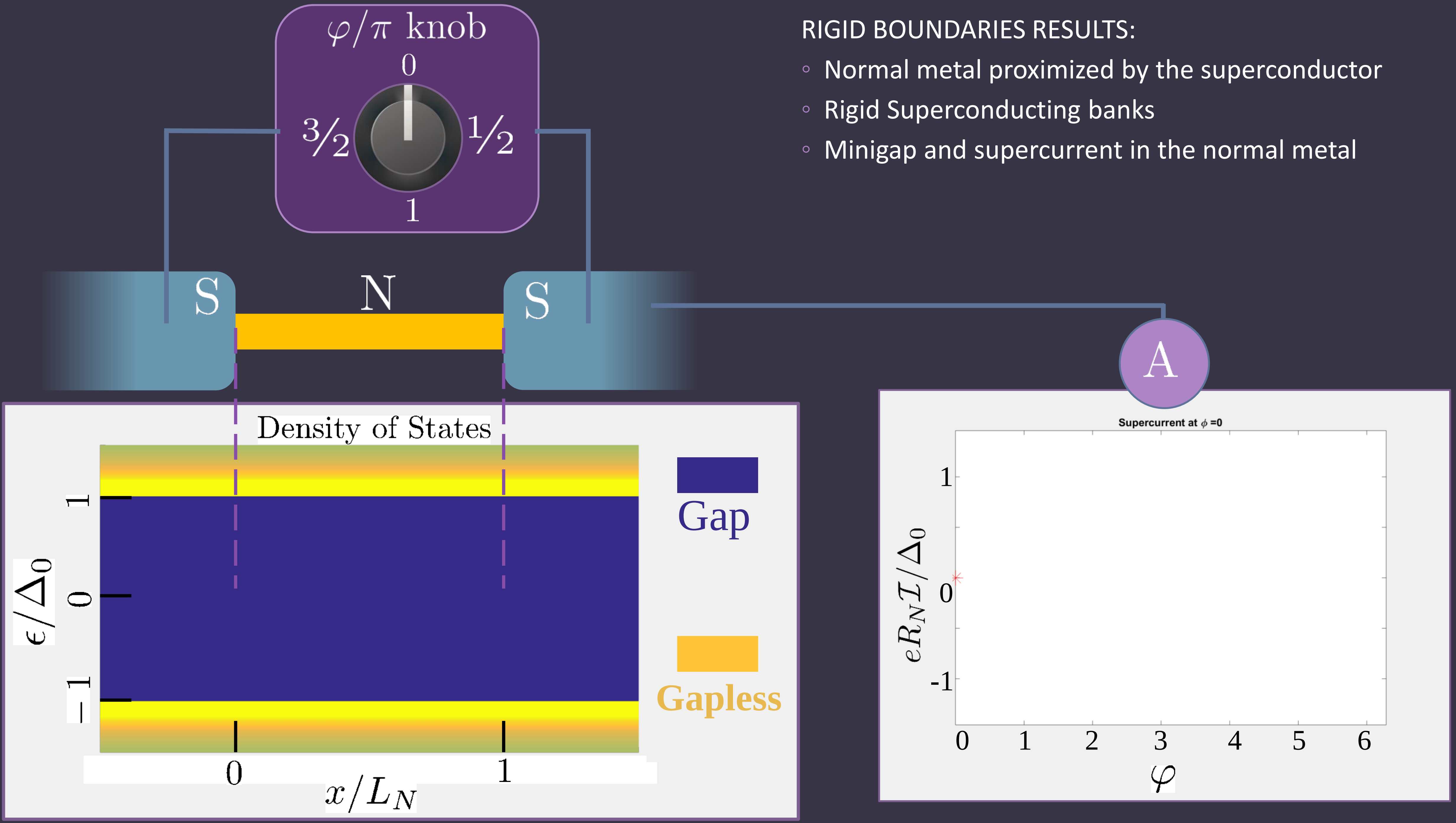
- In literature, for simplicity : quasi-classical theory and **rigid boundary condition**
- Green function in normal region and BCS boundaries at SN interfaces
- **Proximity effect :** the superconductor affects metal properties
- Qualitative results **: current phase relation** and **phase-dependent minigap**



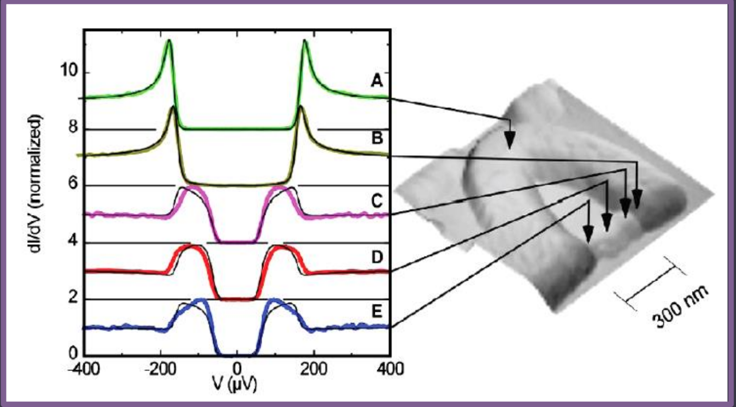
Ref. 4, 5

RIGID BOUNDARIES RESULTS:

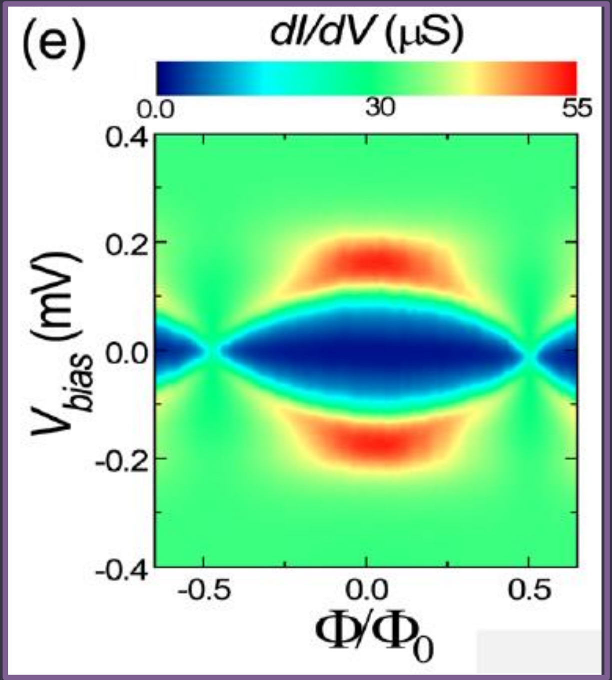
- Normal metal proximized by the superconductor
- Rigid Superconducting banks
- Minigap and supercurrent in the normal metal



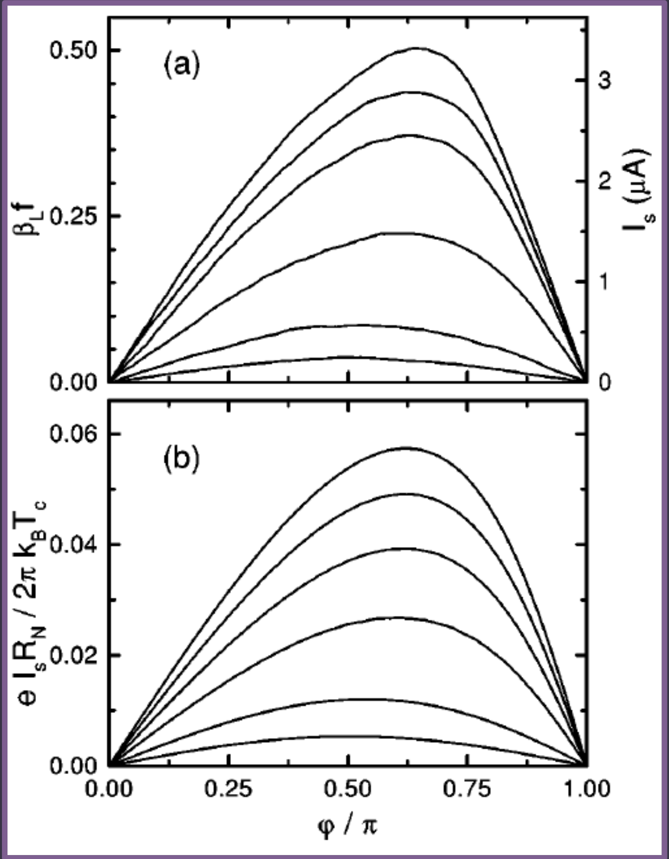
Experimental agreement



le Sueur et al., PRL **100**, 197002 (2008)



D'Ambrosio et al. , Appl. Phys. Lett. **107** 113110 (2015)



Götz et al., PRB. **62** R14645 (2000)

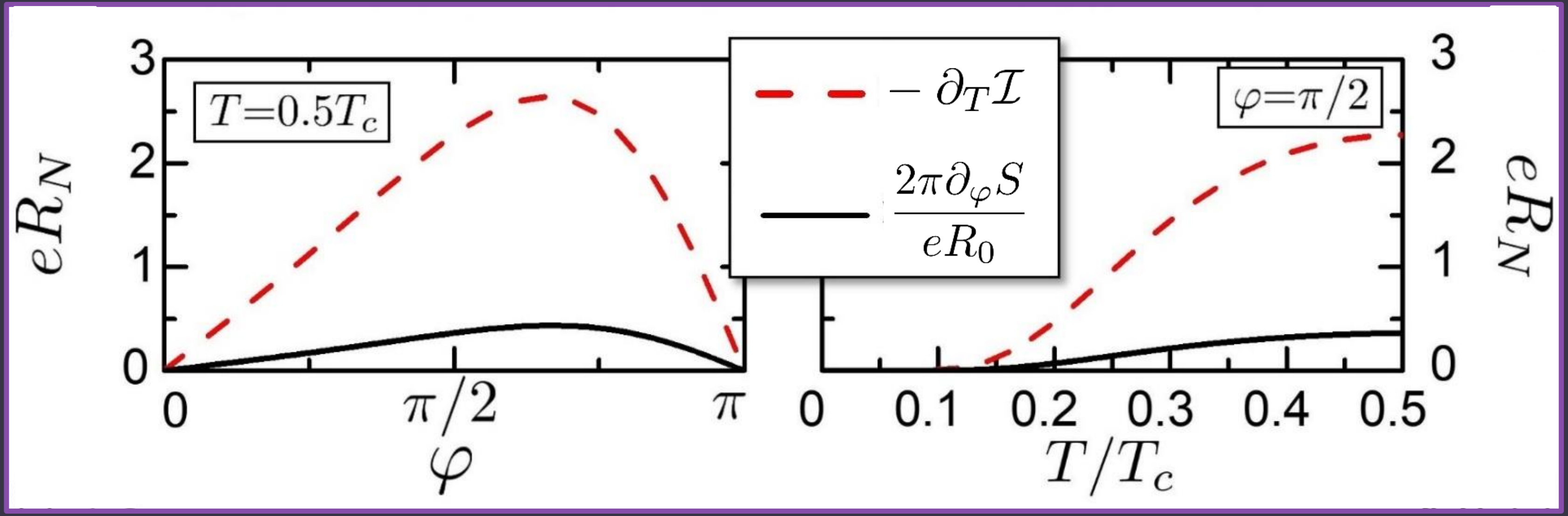
The thermodynamic inconsistency

The rigid boundary conditions results **are not** consistent with the Maxwell relation

$$\mathcal{I}(\varphi, T) \propto \frac{1}{R_N} \propto \frac{1}{L_N}$$

$$\delta S(\varphi, T) \propto V_N \propto L_N$$

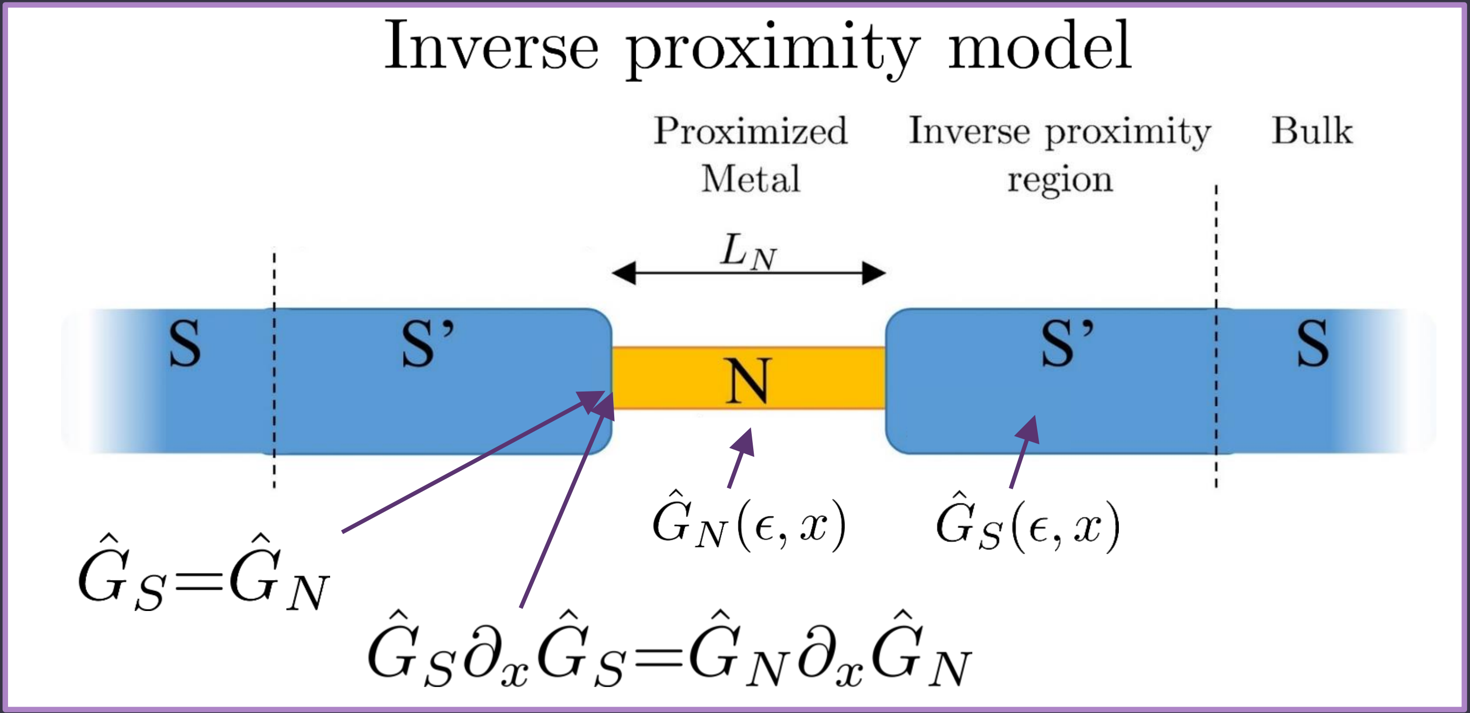
$$-\partial_T \mathcal{I} \neq \frac{2\pi}{eR_0} \partial_\varphi S$$

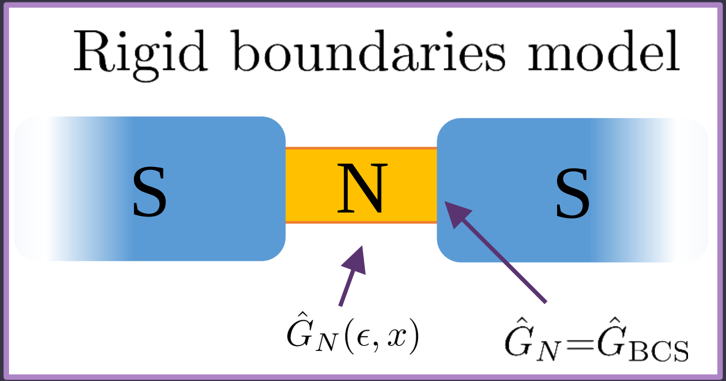


Ref. 2

Inverse proximity model

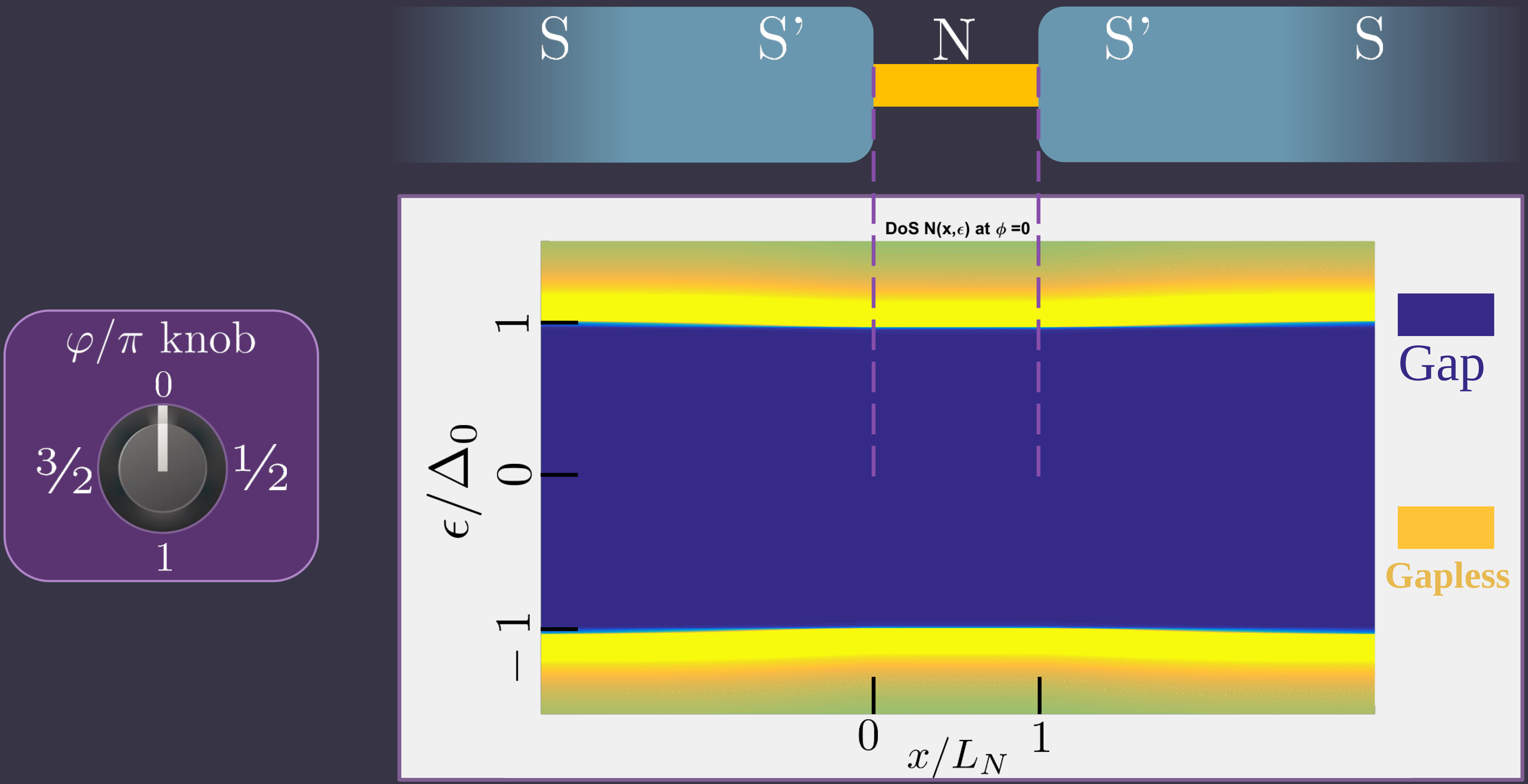
- Inverse proximity effect : the normal region affects the superconductor
- Part of the entropy variation comes from the superconducting part





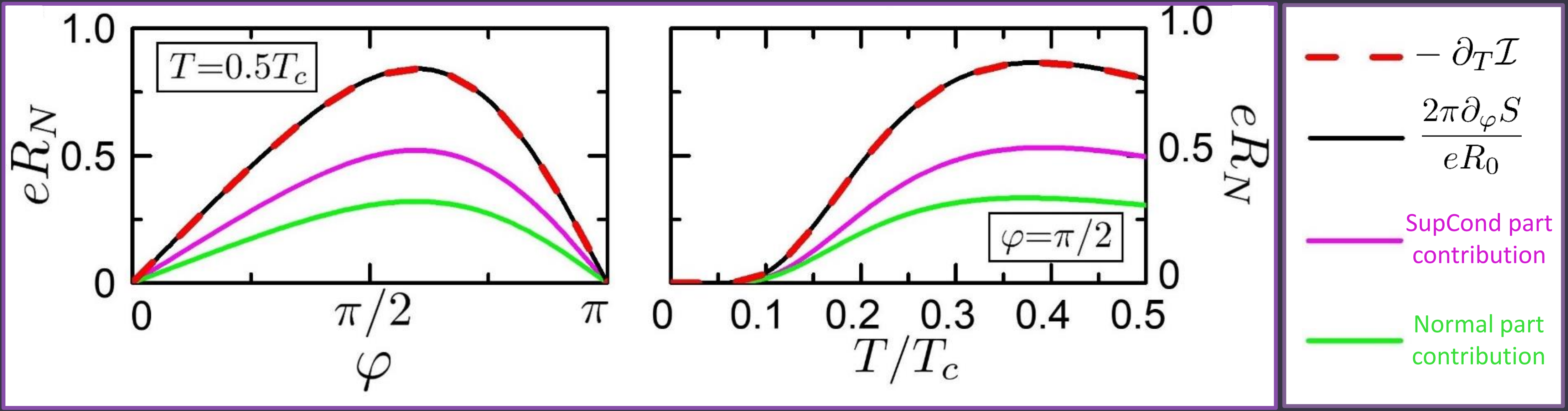
INVERSE PROXIMITY RESULTS:

- Superconductor and metal are mutually influenced
- The DoS in the superconductor is phase-dependent



Thermodynamic consistency of the result

- Including the inverse proximity effect : **Maxwell consistence recovered**
- **Inverse proximity effect** is the **main mechanism** of **entropy modulation**



Entropy and heat capacity variations

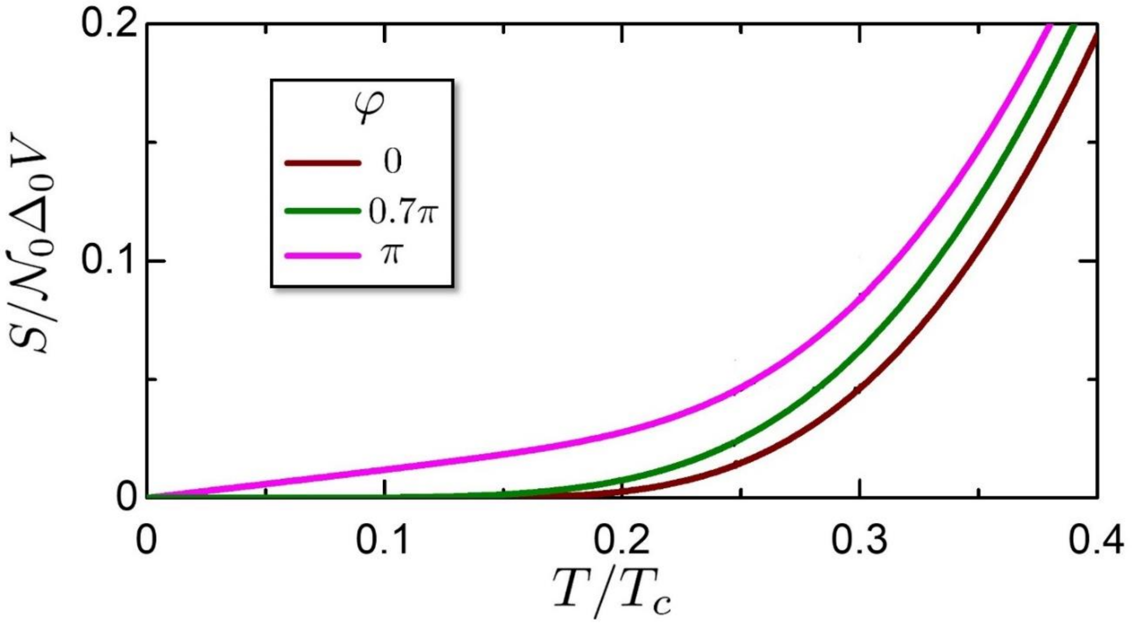
Inverse proximity contribution -> greater amount of tuned-entropy -> enhanced thermodynamic effects

$$S(\varphi{=}0, T \to 0) \propto \sqrt{\frac{\Delta_0}{T}} e^{-\Delta_0/T} V \mathcal{N}_0 \Delta_0$$

$$S(\varphi{=}\pi, T \to 0) \propto \frac{eR_0\mathcal{I}^c}{\Delta_0} \frac{T}{\Delta_0}$$

$$C(\varphi{=}0, T \to 0) \propto \left(\frac{\Delta_0}{T}\right)^{3/2} e^{-\Delta_0/T} V \mathcal{N}_0 \Delta_0$$

$$C(\varphi{=}0, T \to 0) \propto \frac{eR_0\mathcal{I}^c}{\Delta_0} \frac{T}{\Delta_0}$$



V: device volume

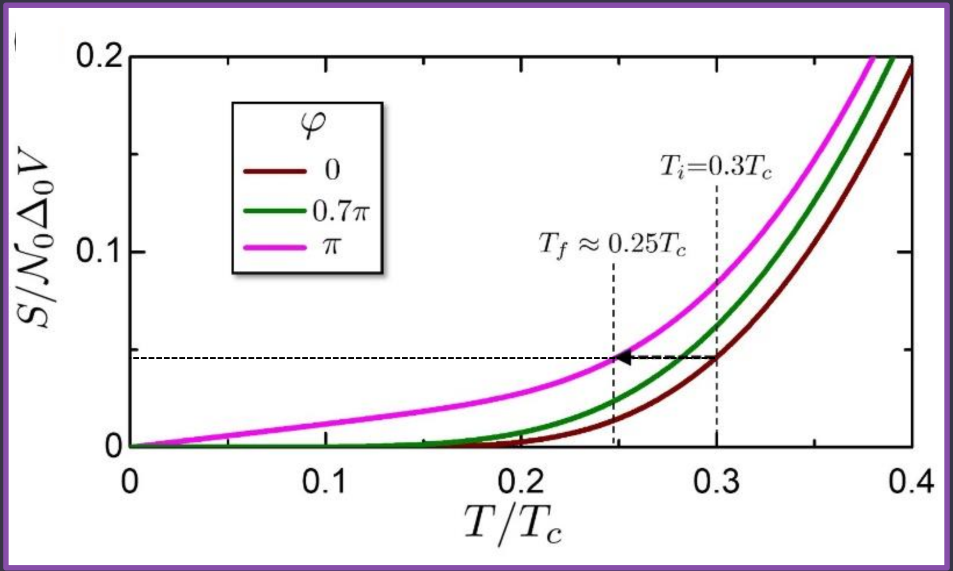
Δ_0: BCS gap

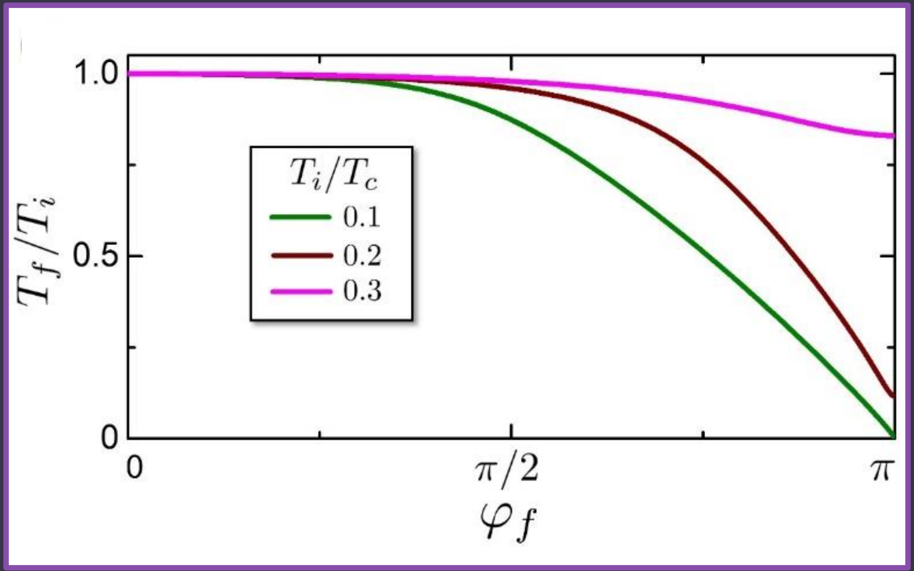
$\mathcal{N}_0$: density of states at Fermi energy

Iso-entropic process

- A quasi-static iso-entropic process does not exchange heat with the external world
- Thermal isolation **is not an easy task**: required small volumes and low temperatures to suppress phonons heating
- An iso-entropic process gives an exponential decrease of temperature. At low temperatures:

$$\frac{T_f}{T_i} \approx \frac{eR_0 I^c}{\mathcal{N}_0 \Delta_0^2 V} \left(\frac{\Delta_0}{T_i}\right)^{3/2} e^{-\Delta_0/T_i}$$



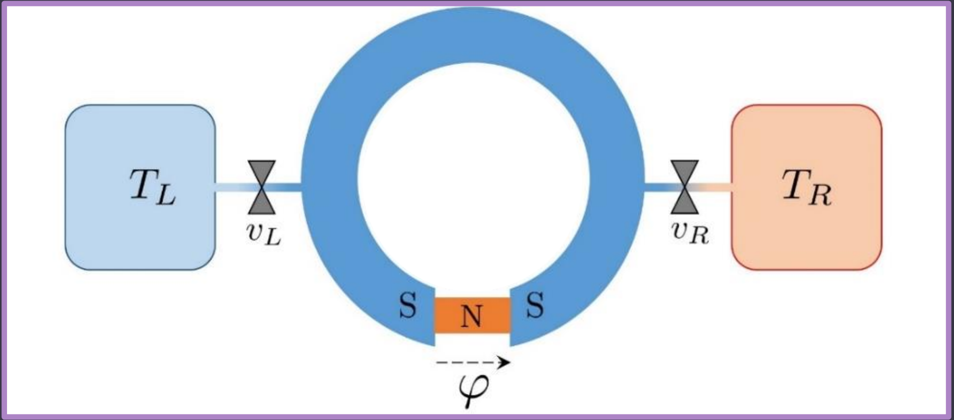


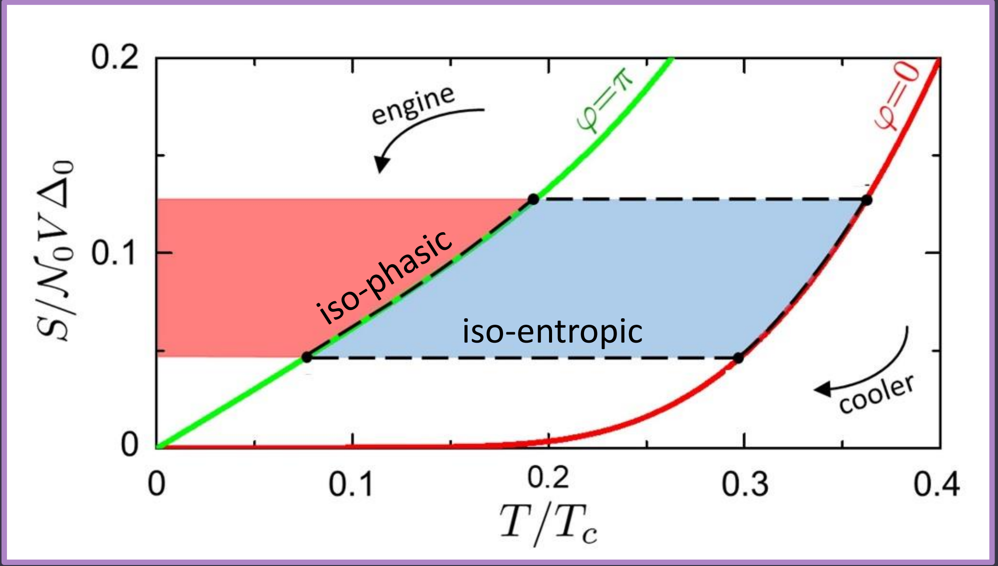
SNS thermodynamic machine

- Thermodynamic cycle composed of iso-entropic and iso-phasic processes.
- The phase drop is tuned by an external magnetic field,

$$\varphi = 2\pi\Phi/\Phi_0$$

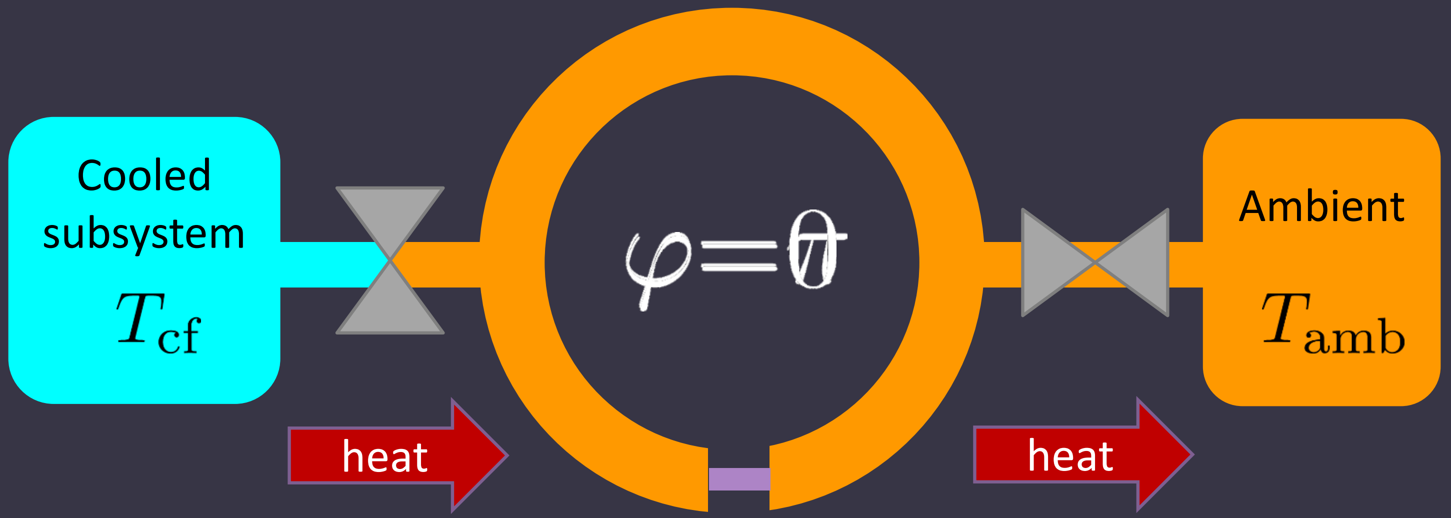
- Junction connected to thermal reservoirs by mean of valves.

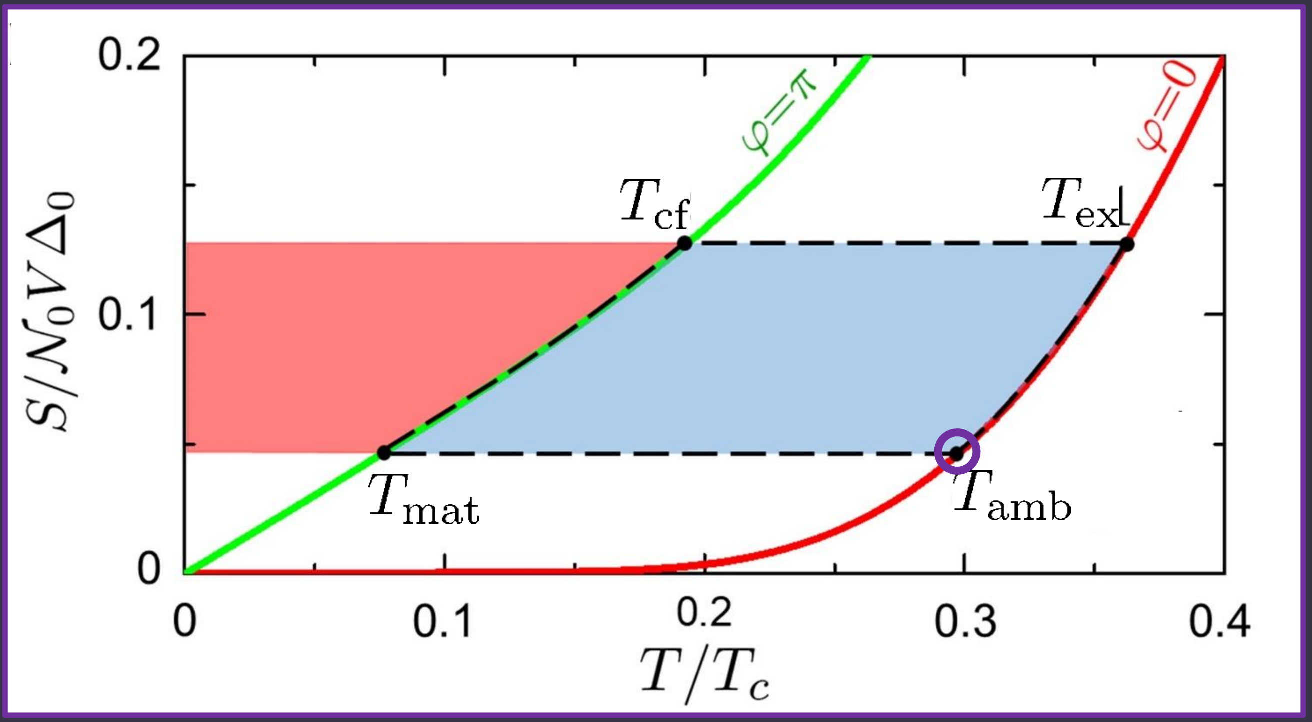




Cooling cycle

- Iso-entropic temperature decrease
  $(\varphi{=}0, T_{\mathrm{amb}}) \rightarrow (\varphi{=}\pi, T_{\mathrm{mat}})$
- Iso-phasic heat absorption
  $(\varphi{=}\pi, T_{\mathrm{mat}}) \rightarrow (\varphi{=}\pi, T_{\mathrm{cf}})$
- Iso-entropic temperature increase
  $(\varphi{=}\pi, T_{\mathrm{cf}}) \rightarrow (\varphi{=}0, T_{\mathrm{ex}})$
- Iso-phasic heat release
  $(\varphi{=}0, T_{\mathrm{ex}}) \rightarrow (\varphi{=}0, T_{\mathrm{amb}})$





Experimental feasibility

An experimental observation of these effects is very challenging: requires small device volumes and high junction current.

In order of difficulty in experimental realization:

- The variation of specific heat is the simplest to observe. A variation of 70% is predicted at temperature $30\,\mathrm{mK}$, volume $10\,\mu\mathrm{m}^3$ and critical current $1\,\mu\mathrm{A}$
- The isoentropic cooling is more complex to observe, due to the phonon heating. In this case, the iso-entropic process must be very fast (less than 10ps)
- The cooler requires the thermal valves and sincronisation of the parts -> very hard to realise. Calculated that at $T = 0.2T_c, \mathcal{I}^c = 1\,\mathrm{mA}, \nu = 100\,\mathrm{MHz}$ the cooling power is $2\,\mathrm{pW}$

Conclusions

- Quasi-classical treatment in rigid boundaries yields a thermodynamic inconsistency with Maxwell relations
- Inverse proximity effect is the main mechanism of the entropy dependence on phase across the junction
- The phase dependence of the entropy can be exploited to implement iso-entropic cooling of the junction
- A thermodynamic machine can be realized by the combination of iso-entropic and iso-phasic processes

Further developments

- Consider different kinds of process: current-biased, isothermal, different cycles
- Explore different kind of junctions: balistic SNS, ferromagnet, semiconductor
- Explore different kind of external variables: current, electric field

The work presented has been developed in collaboration with Matteo Carrega, Pauli Virtanen, Alessandro Braggio, Elia Strambini and Francesco Giazotto, from superconductivity group in NEST laboratories in Pisa.

I thank them for the efforts they put in this work.

Thank you for the attention!

References

1. G. Grosso, G. Pastori Parravicini, *Solid State Physics,* Academic Press (2000)
2. P. Virtanen et al. , PRB 96, 245311 (2017)
3. F. Vischi et al. , arXiv 1806.01568 (2018)
4. W. Belzig et al. , Superlatt. and Microstruct. 25, 1251 (1999)
5. J. C. Hammer et al. , PRB 76, 064514 (2007)
6. le Sueur et al., PRL **100**, 197002 (2008)
7. D’Ambrosio et al. , Appl. Phys. Lett. **107,** 113110 (2015)
8. Götz et al., PRB. **62** R14645 (2000)

Backup slides

Quasi-classical theory

The statistical theory for a hybrid diffusive system in equilibrium: QUASI-CLASSICAL THEORY OF THE SUPERCONDUCTIVITY

In this theory, the information about quasi-particle DoS and supercurrent transport is stored in a momentum-averaged Green function, that is a matrix in the Nambu space

$$\hat{G}^R(\mathbf{r},\epsilon) = \begin{pmatrix} g^R(\mathbf{r},\epsilon) & f^R(\mathbf{r},\epsilon) \\ \tilde{f}^R(\mathbf{r},\epsilon) & \tilde{g}^R(\mathbf{r},\epsilon) \end{pmatrix} \qquad \hat{G}^R(\mathbf{r},\epsilon)^2 = \mathbb{I}$$

and obeys the Usadel equation $\hbar D(\mathbf{r})\nabla\left(\hat{G}^R(\mathbf{r},\epsilon)\nabla\hat{G}^R(\mathbf{r},\epsilon)\right) + \left[i\epsilon\hat{\tau}_3 - \hat{\mathbf{\Delta}}(\mathbf{r}), \hat{G}^R(\mathbf{r},\epsilon)\right] = 0$

$$\hat{\mathbf{\Delta}}(\mathbf{r}) = \begin{pmatrix} 0 & \mathbf{\Delta}(\mathbf{r}) \\ \mathbf{\Delta}^*(\mathbf{r}) & 0 \end{pmatrix} \qquad \mathbf{\Delta}(\mathbf{r}) = \frac{\lambda}{4i}\int_{-E_c}^{+E_c} \tanh\left(\frac{\epsilon}{2T}\right)[f^R(\mathbf{r},\epsilon) - (\tilde{f}^R)^*(\mathbf{r},\epsilon)]\mathrm{d}\epsilon$$

Further boundary conditions describe the geometry of the problem.

From the Green function it can be EXTRACTED QUASI-PARTICLE DOS AND FLOWING SUPERCURRENT

Example: entropy in bulk superconductor

According the quasi-classical theory (already in BCS theory) the Density of States of the quasi-particles is

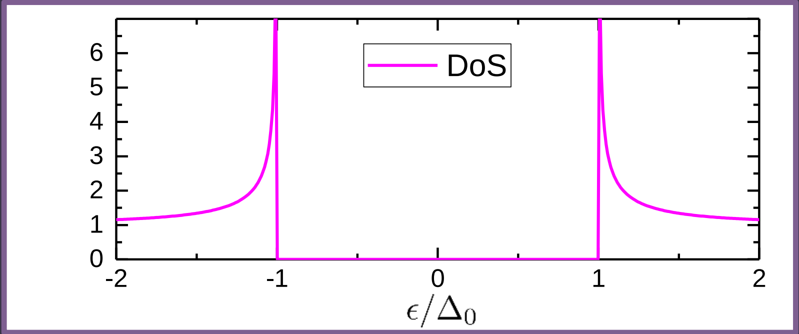
$$\mathcal{N}(\epsilon, T) = \mathcal{N}_0 \frac{|\epsilon|}{\sqrt{\epsilon^2 - \Delta^2(T)}}$$

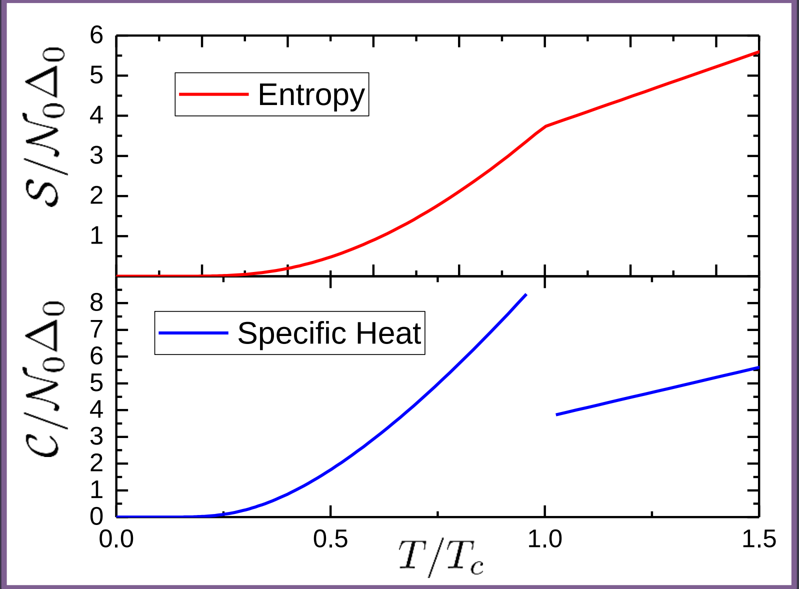
The entropy is

$$\mathcal{S}(x, T, \varphi) = -4 \int_{-\infty}^{+\infty} \mathcal{N}(\epsilon, T) f(\epsilon, T) \log f(\epsilon, T) \mathrm{d}\epsilon$$

The specific heat is

$$\mathcal{C} = T \frac{\partial \mathcal{S}}{\partial T}$$





Analytical solutions for SNS junction

$$\mathcal{I}(\varphi, T) = \frac{\pi\Delta(T)\cos(\varphi/2)}{eR_N}\int_{\Delta(T)|\cos(\varphi/2)|}^{\Delta(T)} \frac{1}{\sqrt{\epsilon^2 - \Delta(T)^2\cos^2(\varphi/2)}}\tanh\left(\frac{\epsilon}{2T}\right)\mathrm{d}\epsilon$$

$$N(\tilde{x}, \epsilon, T, \varphi) = \mathrm{Re}\left\{\cosh(\theta(\tilde{x}, \epsilon, T, \varphi))\right\}$$

$$\theta(\tilde{x}, \epsilon, T, \varphi) = \operatorname{arccosh}\left\{\alpha(\epsilon, \varphi, T)\cosh\left[2(\tilde{x} - 1/2)\operatorname{arccosh}(\beta(\epsilon, \varphi, T))\right]\right\}$$

$$\alpha = \sqrt{\frac{\epsilon^2}{\epsilon^2 - \Delta^2(T)\cos^2(\varphi/2)}}$$

$$\beta = \sqrt{\frac{\epsilon^2 - \Delta^2(T)\cos^2(\varphi/2)}{\epsilon^2 - \Delta^2(T)}}$$