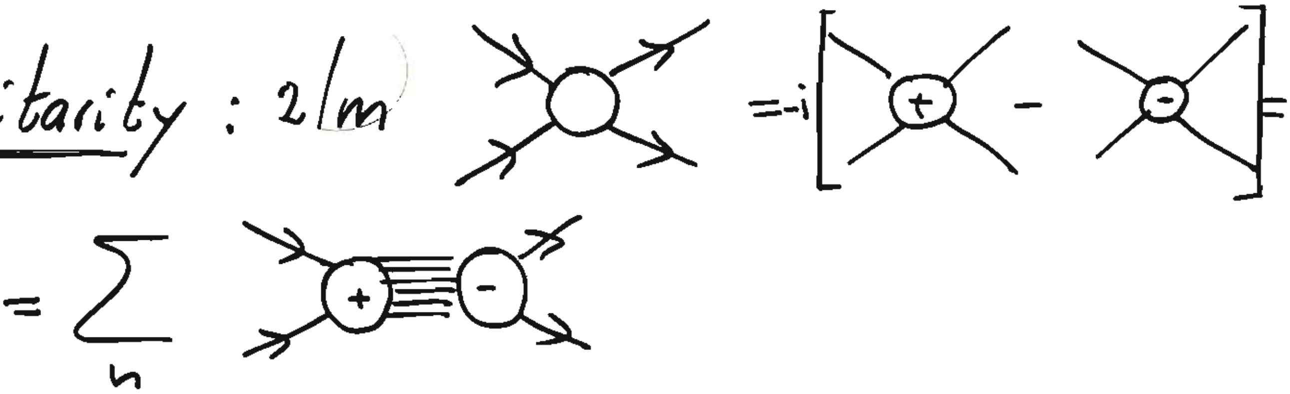
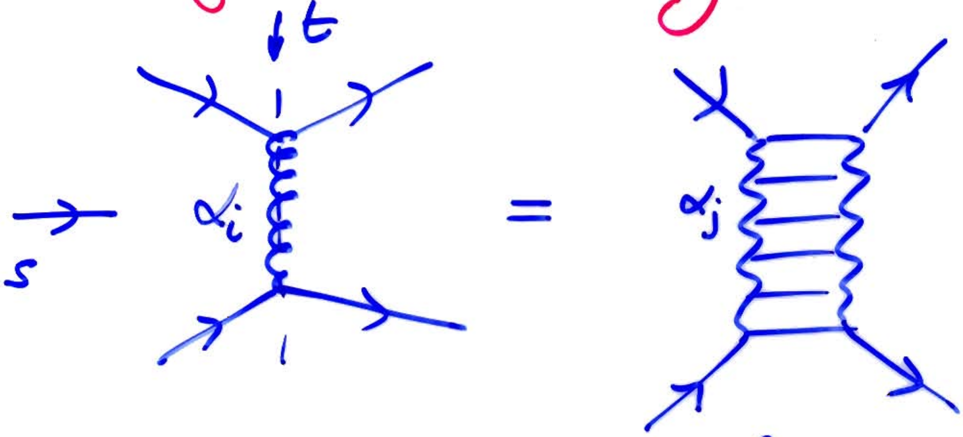
# A CHEAP BOOTSTRAP & ITS SOLUTION

## ① Chew's expensive bootstrap

**Unitarity** : 2 Im



At high-E assuming (multi) Regge behaviour




$$\sum_i \beta_i \, s^{\alpha_i(t)} = \sum_n \int d\Phi_n \prod_j \beta_j \, s_j^{2\alpha_j(t_j)}$$

$\Longrightarrow$ self-consistent determ. of $\alpha$'s & $\beta$'s ?

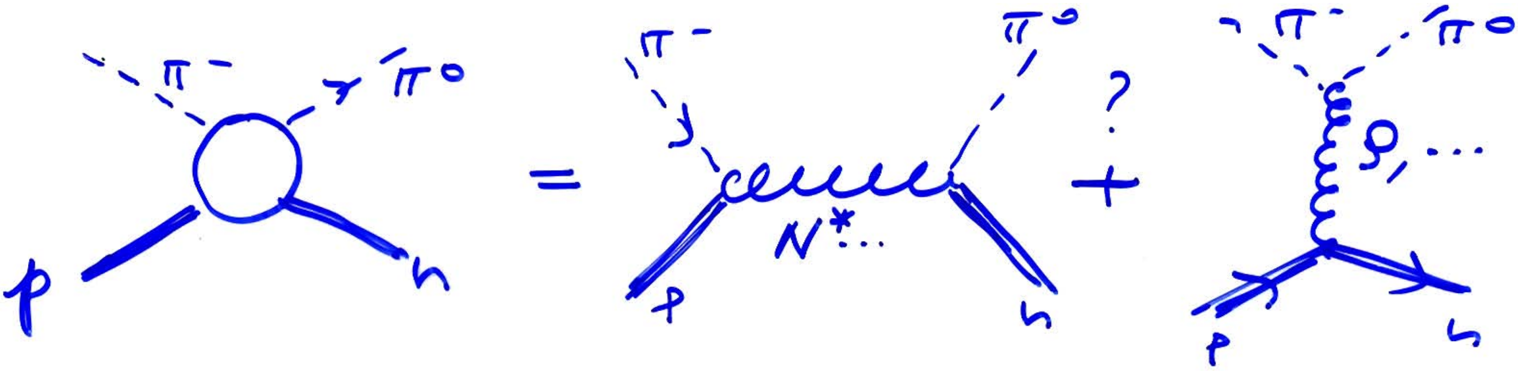
N.B. 1) Bootstrap among Regge poles in the same (t)-channel

2) QFT for strong interactions looked out of question (# of fields, large J)

....

# 2 Erice 1967, DHS duality, M. Gell Mann

- M. Gell Mann reported "en passant" on very recent work by Dolen, Horn & Schmit




Until then people thought that the two contributions had to be added, the 1st being dominant at low E, the 2nd at large E

D.H.S. discovered that, in some intermediate E region, $\mathrm{Im}\, A_{Res} \sim \mathrm{Im}\, A_{Regge}$

Adding both contribution would give $\sim 2 \times \mathrm{Im}\, A$

The "cheap bootstrap" consisted in writing FESR of the type:

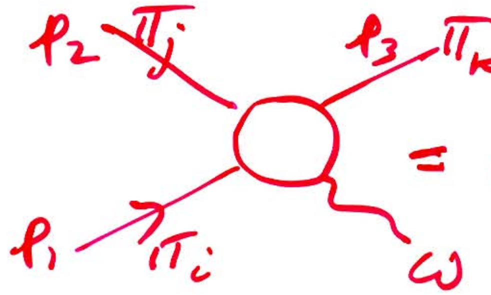
$$\int_{E-\Delta}^{E+\Delta} dE' \, \mathrm{Im}\, A_{Res}(E') = \int_{E-\Delta}^{E+\Delta} dE' \, \mathrm{Im}\, A_{Regge}(E')$$

$N^* \ldots$ $\qquad$ $\rho \ldots$

③ $\pi\pi \to \pi\omega$ (Ademollo, Rubinstein, V., Virasoro + Bishari, Schwimmer '67-'68)

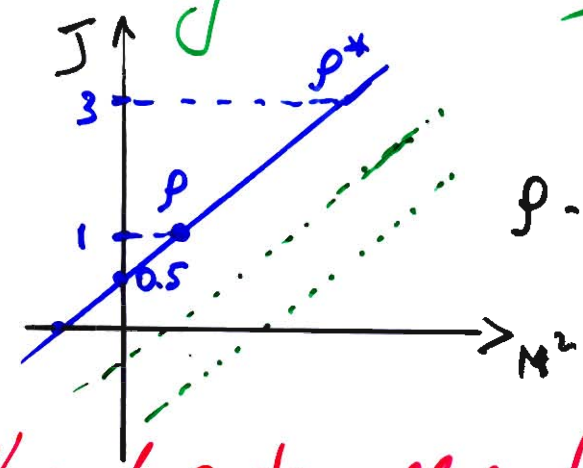
- Gell Mann's cheap bootstrap was nice & simple but, unlike Chew's, connected different objects, baryons on one side, mesons on the other

⇒ find a better (poss. gedanken) process
After some thinking a candidate came out




$$= \epsilon_{ijk}\, \epsilon_{\mu\nu\rho\sigma}\, p_1^\mu p_2^\nu p_3^\rho \epsilon^\sigma A(s,t,u)$$

- By Bose stat. $A(s,t,u) = A(t,s,u) = A(u,t,s)$
- QN's in any channel: $J=$ odd, $P=C=-1$, $I=1$, leaving basically $\rho$ & its Regge exc.'ns




FESR now relates $\rho$-Regge-pole properties in space-like & time-like regions

Worked quite well in limited range of $t$ w/ $\begin{cases} \alpha_0 \sim 0.5 \\ \alpha' \sim 0.8\,\text{GeV}^{-2} \end{cases}$

$\alpha = \alpha_0 + \alpha' t$

Worked better and better in larger & larger region of $t$ by adding parallel daughter trajectories

# 4 The Beta-function (1968)

- The simple ansatz that worked was

$$\mathrm{Im}\, A(s,t) = \frac{\beta(t)}{\Gamma(\alpha(t))} (\alpha' s)^{\alpha(t)-1} \left(1 + O(1/s)\right)$$

$$= \mathrm{Im}\left[\beta \Gamma(1-\alpha(t)) (-\alpha' s)^{\alpha(t)-1} + \cdots \right]$$

w/ $\beta \sim$ const, $\boxed{\alpha = \alpha_0 + \alpha' t}$ Linear trajectories!

- The crucial steps were:

i) Concentrate on $A$ rather than on $\mathrm{Im}\, A$

ii) Emphasize Resonances rather than Regge

iii) Impose xing symmetry

$$(-\alpha' s)^{\alpha(t)-1} = \lim_{\substack{s \to \infty \\ t\ \text{fixed}}} \frac{\Gamma(1-\alpha(s))}{\Gamma(2-\alpha(s)-\alpha(t))}$$

($\hookrightarrow$ $\frac{\Gamma(a+b)}{\Gamma(a)} \xrightarrow[a \gg b]{} a^b$)

$$A = \frac{\Gamma(1-\alpha(t))\Gamma(1-\alpha(s))}{\Gamma(2-\alpha(s)-\alpha(t))} = B(1-\alpha(s), 1-\alpha(t))$$

$$= \int_0^1 dx\, x^{-\alpha(s)} (1-x)^{-\alpha(t)}$$

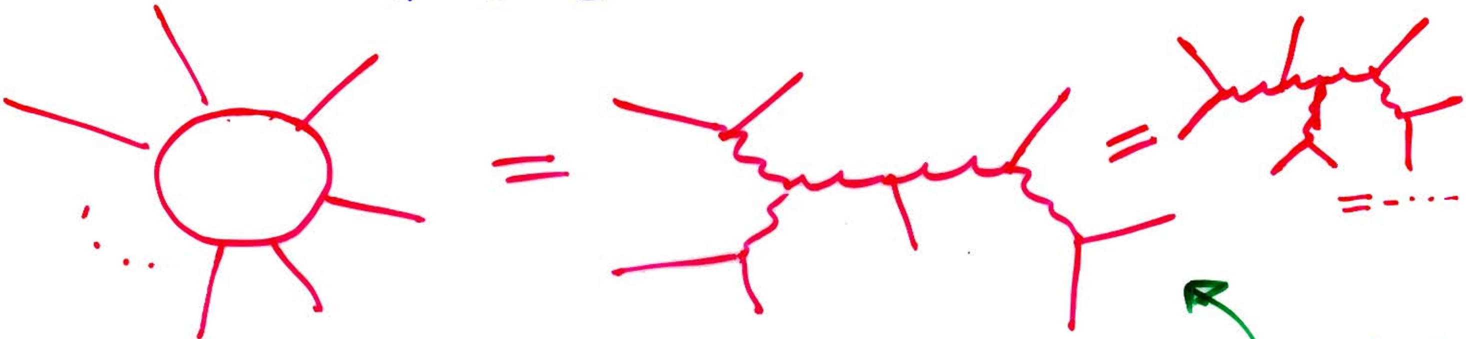
Full xing symm: $+ \; t \to u \quad + \; s \to u$

# DRM years @ MIT

## 1 Multiparticle generalizations (1968)

(Bardakci-Ruegg, Virasoro, Chan, ... Koba-Nielsen)

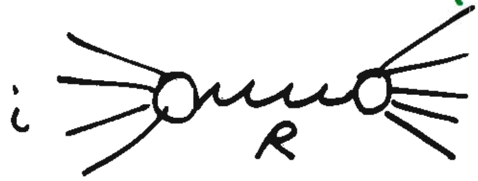
$$A_N^{(KN)} = \int \frac{\prod_i^N dx_i \, J_{abc}}{dx_a \, dx_b \, dx_c} \prod_{i \neq j} |z_i - z_j|^{2\alpha' k_i \cdot k_j}$$



N=8
5 poles

- Poles come from $z_i \to z_j$
- N-3 int. variables = max # of sim. poles

## 2 Counting states (Fubini & G.V. '69, Bardacki & Mandelstam)

Factorization :  $= g_{iR} \frac{1}{s - m_R^2} g_{Rf}$

Counting states by writing for each pole $\sum_{R=1}^{d_N} g_{iR} \frac{1}{s - m_N^2} g_{Rf}$
and for any i, f

Surprising (?) result $d_N \sim e^{c\sqrt{N}} = e^{m/T_H}$

$\Rightarrow$ Hagedorn temperature in DRM

Counting procedure was cumbersome ....

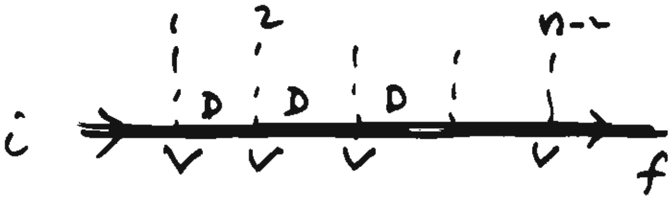
Operator formalism much simpler (FGV, Nambu)

⑥

$$|R\rangle = \frac{1}{\mathcal{N}} \prod_{n,\mu} \left(a^{+}_{n,\mu}\right)^{N_{n,\mu}} |0\rangle$$

$$\alpha' M_R^2 = \sum_{n,\mu} n\, N_{n,\mu} = \sum_{n,\mu} n\, a^{+}_{n,\mu} a_{n,\mu} \equiv H.$$

Amplitude could be written as $\langle\ \ \rangle$ of product of Vertices & Propagators




$$= \langle i | V_1 D V_2 D \cdots V_{n-1} | f \rangle$$

$$V_i = \exp\left(i k_i \cdot \sum_n \frac{a^{+}_n}{\sqrt{n}}\right) \exp\left(i k_i \sum_n \frac{a_n}{\sqrt{n}}\right)$$

$$D = \frac{1}{\alpha' s - H}$$

This huge Hilbert space was sufficient to factorize but, fortunately, not necessary

In fact some states had negative norm (basically, $a^{+}_{n,0}$ gives ghosts)

Q: Could they possibly decouple from all $i, f$ ?

This was obviously a crucial test for the theoretical consistency of DRM's

# ③ Hunting ghosts, $\alpha_0 = 1$, $D \leq 26$



For generic $\alpha_0$, FV had found a whole set of decoupled states:

$$L_{-1} |X\rangle = (p \cdot a_1^+ + \cdots) |X\rangle \quad \text{decouples}$$

(a part from ... in c.o.m. this is $a_{1,0}^+ |X\rangle$ ..)

This could remove ghosts created by $a_{1,0}^+$ but was hardly enough

In 1970 Virasoro made a crucial discovery: Iff $\alpha_0 = 1$, $\exists$ an $\infty$ set of operators that, acting on any state, give a spurious (dec.) state

$$L_{-n} |X\rangle = (p \cdot a_n^+ + \cdots) |X\rangle \quad \text{decouples} \quad (\alpha_0 = 1)$$

Now, complete ghost cancellation was possible

Proof (by Brower & Goddard + Thorn) had to wait till 72

Needs some technical development, e.g. construction of positive-norm physical states (DDF states) s.t. $|X\rangle = |DDF\rangle + L_{-n} |Y\rangle$

Surprises: i) $D \leq 26$ needed (for $D > 26$ ghosts are there!)

ii) For $D = 26$ DDF exhaust physical states

## (4) Algebras & their interpretation



Around end of '69 Gliozzi and Chiu-Matsuda-Rebbi noticed that $L_0 = \alpha' p^2 - H$, $L_{-1}$ & $L_1 = (L_{-1})^\dagger$ obeyed an $SO(2,1)$ algebra:

$$[L_0, L_{\pm 1}] = \mp L_{\pm 1}, \quad [L_{+1}, L_{-1}] = 2L_0$$

On the other hand FV + Gervais had introduced vertex operators:

$$\langle V_1 D V_2 D V_3 D \cdots \rangle = \langle T \prod_i : \int dx_i\, e^{ik_i Q(x_i)} : \rangle$$

$$Q(x_i) = q + p \ln x_i + \sum_n \frac{a_n}{\sqrt{n}} x_i^n + \sum_n \frac{a_n^\dagger}{\sqrt{n}} x_i^{-n}$$

$$V(k) \equiv \int dx : \exp(i k_\mu \cdot Q^\mu(x)) :$$

It was then relatively easy to see how $L_n$ $(n = 0, \pm 1)$ acted on $Q(x)$

$$[L_n, Q(x)] = x^{n+1} \frac{d}{dx} Q(x) \qquad (x \to x + \epsilon x^{n+1})$$

Because of N.O. action on $V(k)$ was slightly different & s.t. $[L_{-1}, V] = \int dx \frac{d}{dx} x e^{ikQ(x)} = 0$

etc. $\Longrightarrow$ Explains decoupling of $L_{-1}|x\rangle$ + duality ...

After Virasoro had introduced $L_n$ ($|n|>1$)
F & V looked at how these acted on
$Q(x)$ & $V(k)$

They were just the trivial generalization
corresponding to $x \to x + \epsilon x^{n+1}$

They appeared to satisfy the algebra

$$[L_n, L_m] = (n-m) L_{n+m}$$

and this is what FV wrote down...
... without bothering to check (it was O.K. for $n=0,\pm1$)

Before our paper was published in Ann. of Phys.
J. Weis pointed out to us that we had
forgotten a c-number in the algebra

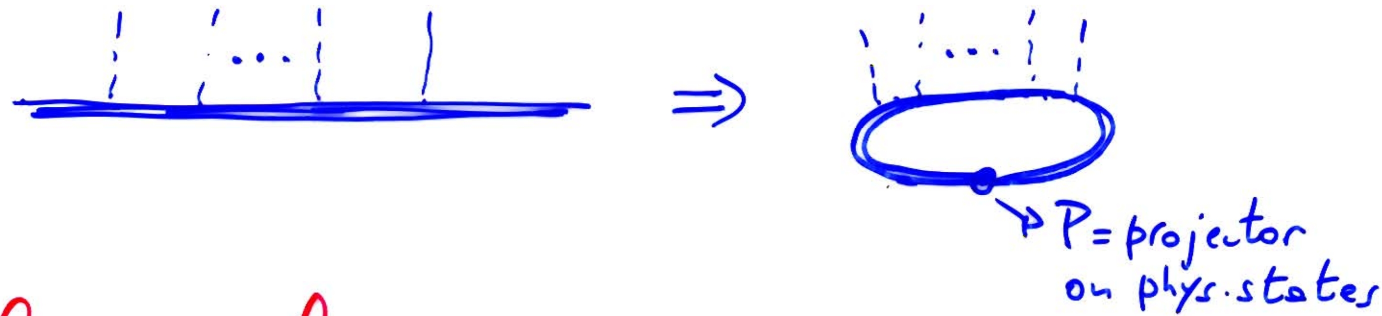
$$[L_n, L_m] = (n-m) L_{n+m} + \frac{D}{12} n(n^2-1)\delta_{n+m,0}$$

This became known as the Virasoro algebra

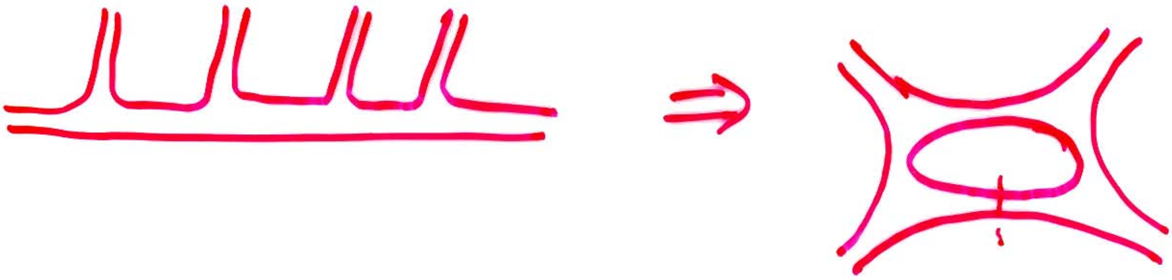
The D-dep. of the c-number (anomaly due
to N.O. of Vir. op.'s) is what makes proof of
no-ghost thrm. fail for $D>26$

# 5 Loops, $D=26$

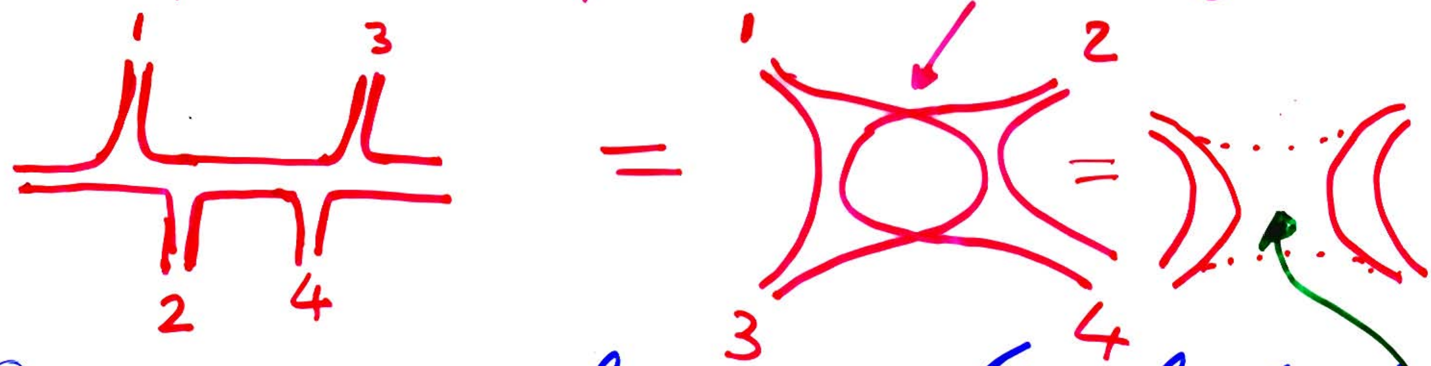
After having identified phys. states, constructing loops was (almost) a technical problem. Use e.g. the sewing procedure:




Planar loop:



Non planar loop:

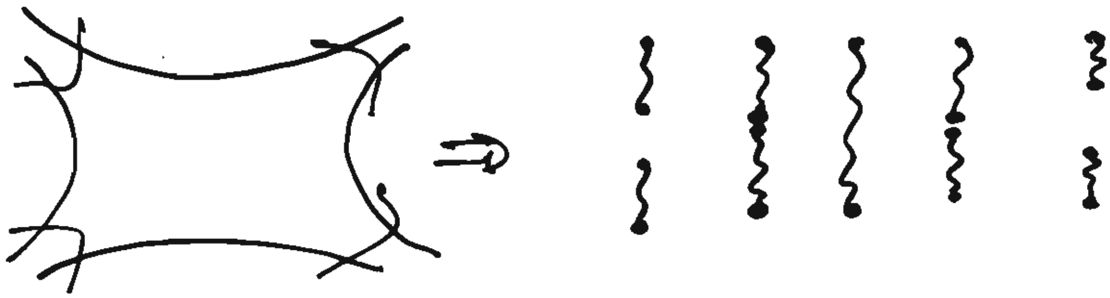



Gave nonsense unless $D=26$ (Lovelace '71) where it gives new spectrum of states in vacuum channel

- There was actually another peculiarity of $D=26$ already at tree-level:
- DDF states (corresponding to $(D-2)$ h. osc.) where complete only for $D=26$
- Remaining states had positive norm for $D<26$ and indefinite norm for $D>26$
- One characteristic of a string is that its physical d.o.f. correspond to "transverse" vibrations, hence to vibrations in $(D-2)$ directions
- This is the reason why $D=26$ ($D=D_{cr}$ in general) will come out automatically if we consider, from the start, a string theory

# Early hints of underlying string

- Duality & duality diagrams



- Linear Regge trajectories:

$$J = \alpha' M^2 \qquad \alpha' = E \cdot \ell / E^2 = \ell / E \quad (c=1)$$

$1/\alpha'$ has (classically!) dimensions of a Tension

- The set of harmonic oscillators w/ a fundamental frequency + higher harmonics

- The field $Q(z)$ hints a low-dimensional QFT ($D=1$, $D=2$?)

- The correlator $\langle Q(x)\, Q(y) \rangle \sim \ln(x-y)$ smells of Green's functions in $D=2$

etc. etc.

# GOOD and BAD NEWS

**Good** (theoretical) news:

- NS & R generalizations
- GSO projection & tachyon elimination
- A theoretically consistent SUPERSTRING IN $D=10$

**BAD** (phenomenological) news:

- $D=10$ IS STILL TOO LARGE!
- $m=0$ states w/ $J=0 \cdots 2$, some apparently protected by gauge inv.
- SOFTNESS, a real killer!

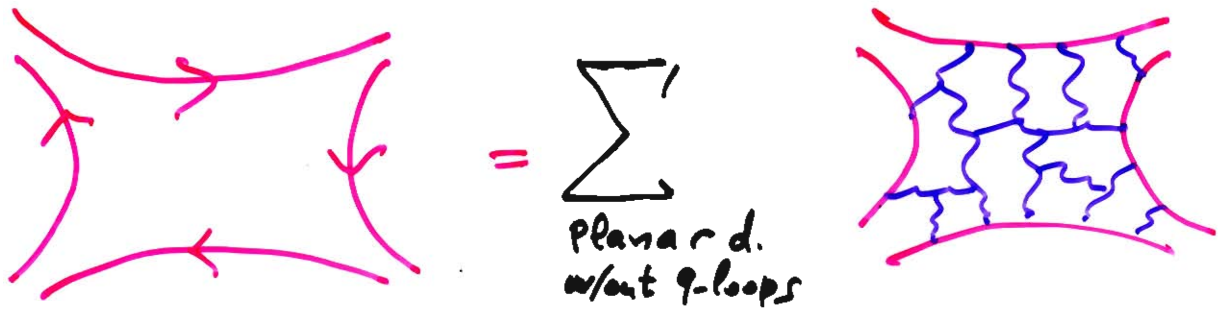
- SCALING in $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$
- Bj-scaling in DIS
- Large $P_\perp$ events at ISR

Point-like structure inside hadrons!

FINALLY, AF put QCD on spotlight & DRM/STRINGS OUT OF IT!

Finally, in 1974, 't Hooft argued that QCD gives an effective string theory via a $1/N_c$ expansion (w/ $g^2 N_c$, $N_f$ fixed)

Duality diagrams reinterpreted, filled...




$\Rightarrow g_s^2 = O(1/N_c) \Rightarrow \infty^{ly}$ narrow resonances

I had been playing for a number of years ('70-'74) with an approach to unitarization of DRMs based on topology of higher-loop diagrams

This became a $1/N_f$ expansion (w/ $g_s^2 N_f$ fixed)

Within QCD it became the topological expansion where $g_s^2 N_f \to N_f/N_c$ is held fixed (basis of the dual parton model)

ALL(?) STRING THEORISTS GAVE UP ON STRINGS AS A FUNDAMENTAL THEORY OF HADRONS

- A HANDFUL OF THEORISTS KEPT WORKING ON STRING THEORY
- TOO BEAUTIFUL TO THROWN AWAY?
- IN 1974 SCHERK & SCHWARZ MADE THE DARING PROPOSAL THAT STRING THEORY SHOULD BE REINTERPRETED AS A UNIFIED QUANTUM THEORY OF ALL FUNDAMENTAL PARTICLES & INTERACTIONS (AFTER AN APPROPRIATE RESCALING OF THE TENSION BY ~18 ORDERS OF MAGNITUDE)

SUDDENLY OUR PROBLEMS DISAPPEAR

- $m=0$ particles: needed for gauge & gravity
- softness: cures UV divergences of Q-gravity
- $D>4$: allows for "realistic" theories if (D-4) dimensions are of string size