The Spectrum of Open String Field Theory at the Stable Tachyonic Vacuum

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S. Giusto and C.I. Nucl. Phys. B 677 (2004) 52

Outline

The tachyon of Bosonic Open String Theory

- Open bosonic string theory has a tachyonic mode. (This motivated the introduction of susy and superstrings...)
- With the advent of D-branes it has been understood that open strings are excitations of solitonic objects of closed string theory.
- Natural interpretation of the tachyon of open bosonic string theory: the underlying solitonic object is unstable.
- This idea can be tested in a concrete way within a string field theory formulation of bosonic open strings.

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Witten's Open String Field Theory

- Bosonic Open String Field Theory action around the perturbative vacuum in 26d (Witten '86)

$$\Gamma[\Psi] = \frac{1}{2}(\Psi, Q_{BRS}\,\Psi) + \frac{1}{3}(\Psi, \Psi \star \Psi)$$

  - Ψ is the classical open string field, a state in the open string Fock space of ghost number 0.
  - $\star$ is Witten's associative and non-commutative open string product.
  - Q_{BRS} is the BRS operator of the CFT world-sheet theory.
  - Gauge invariance

$$\delta\,\Psi = Q_{BRS}\,C + \left[\Psi \stackrel{\star}{,} C\right]$$

  where C is a ghost number -1 gauge parameter.

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Sen's conjectures about OSFT

- Sen ('99) proposed a sort of Higgs mechanism for bosonic OSFT. He conjectured that the non-linear classical equations of motion of OSFT

$$Q_{BRS}\,\phi + \phi \star \phi = 0$$

possess a translation invariant solution whose energy density exactly cancels the D25 brane tension:

$$\Gamma[\phi] \equiv \frac{1}{2}(\phi, Q_{BRS}\,\phi) + \frac{1}{3}(\phi, \phi \star \phi) = -\frac{1}{2\,\pi^2}$$

Sen's conjectures about OSFT

- The existence of a solution with such a property has been demonstrated first numerically (Sen&Zwiebach '99, Moeller &Taylor '00, Gaiotto &Rastelli '00) and more recently analytically (Schnabl '05).
- This solution, the tachyonic vacuum, is believed to be the (classically) stable non-perturbative vacuum of OSFT representing the closed string vacuum with no open strings.

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Sen's third conjecture

- The closed string interpretation requires that the spectrum of quadratic fluctuations around this classical solution be not only tachyon-free but also gauge-trivial.
- Expand the string field around the tachyonic vacuum

$$\Psi = \phi + \tilde{\Psi}$$

The new action has the same form as the perturbative action

$$\widetilde{\Gamma}[\tilde{\Psi}] = \frac{1}{2}(\tilde{\Psi}, \widetilde{Q}\,\tilde{\Psi}) + \frac{1}{3}(\tilde{\Psi}, \tilde{\Psi} \star \tilde{\Psi})$$

with the modified kinetic operator $\widetilde{Q}$

$$\widetilde{Q}\tilde{\Psi} \equiv Q_{BRS}\,\tilde{\Psi} + [\phi \stackrel{\star}{,} \tilde{\Psi}]$$

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- The Fourier transformed linearized e.o.m's around the tachyonic vacuum are:

$$\widetilde{Q}(p)\,\psi^{(0)}(p) = 0$$

  $\psi^{(0)}(p)$ are the polarization vectors of the open string field.

- Perturbative physical states (i.e. string vertex operators) are solutions of these equations modulo linearized gauge transformations. (i.e. "transverse" polarization vectors modulo "longitudinal" ones)
- Mathematically, the cohomology

$$\mathcal{H}^{(0)}(\widetilde{Q}(p)) = \frac{\widetilde{Q}\,\text{closed states}}{\widetilde{Q}\,\text{trivial states}}$$

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- In conclusion, Sen's expectation is that $\mathcal{H}^{(0)}(\widetilde{Q})$ vanishes for all p^2.
- The theory should have no "local" particle-like states. In this sense this is a "topological" field theory.
- Closed string states should manifest themselves as composites, solitons,...?
- OSFT in the tachyonic vacuum provides a description of closed string in terms of open string (brane) degrees of freedom: sort of holography without supersymmetry?
- Closed string tachyon should appear as a 1-loop instability?

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Physical states in gauge theories

To count physical states in gauge theories one can use two methods.

- The “canonical” counting. No gauge- fixing:

  # of phys states at $p^2 = -m^2$=
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- Let $d_{matter,ghosts}$ be order of poles at $p^2 = -m^2$ of the propagators of matter and ghosts fields

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Example 1: Electrodynamics in 3+1 Dimensions

- "Canonical" counting: at $p^2 = 0$

  # of *transverse* photons $\{p^\mu \, \epsilon_\mu(p) = 0\}$ -
  - # of *longitudinal* photons $\{\epsilon_\mu(p) = p_\mu \, \chi(p)\}$=
  $= \quad 3 \quad - \quad 1 \quad = \quad 2$

- "Lagrangian" counting:

$$L_{g.f} = \frac{1}{2} A_\mu p^2 \, A^\mu(p) + \bar{c}(p) p^2 \, c(p)$$

and
number of phys states = $4 \quad - \quad 2 \times 1 \quad = \quad 2$

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Example 2: Chern-Simons in 2+1 Dimensions

- “Canonical” counting for any p^2

  # physical states= # sols of lin e.o.m. - # trivial sols=1-1=0

- “Lagrangian” counting in Landau gauge:

$$\det K_{boson}(p) = \det \begin{pmatrix} \epsilon_{\mu\nu\rho}p^{\rho} & p_{\mu} \\ -p_{\nu} & 0 \end{pmatrix} = p^4 \Rightarrow d_{matter} = 2$$

$$\det K_{ghost}(p) = p^2 \Rightarrow d_{ghost} = 1$$

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Non-gauge invariant approximations/regularizations

- "Canonical" counting is problematic if your approximation and/or regularization is not gauge-invariant.
- The problem is that one looses the concept of gauge-trivial solutions (i.e. "longitudinal" polarizations)
- The (only) known approximation scheme for OSFT is called Level Truncation (LT): it includes a finite number of open string states in the expansion of the string field, those whose level is less than a given number L.
- LT was used quite successfully to show that Sen's classical tachyon solution does exists.

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Non-gauge invariant approximations/regularizations

- LT respects gauge-invariance in the perturbative vacuum since the level operator commute with Q_{BRS}.
- LT does not respect gauge-invariance in the tachyonic vacuum since the level operator does not commute with $\widetilde{Q}$.
- In the gauge-fixed framework breaking of gauge-invariance shows up as "spurious" matter field propagators poles not being exactly degenerate with ghost propagator poles. As the regularization/approximation parameter is removed these poles should smoothly come together.
- In the exact theory all matter propagator poles should be exactly cancelled by ghost propagators poles.

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Gauge-fixed OSFT

- Pick the Siegel gauge:

$$b_0 \, \Psi_0 = 0$$

and define the associated operators

$$\widetilde{L}_0 \equiv \{b_0, \widetilde{Q}\}$$

- Fadeev-Popov procedure leads to an infinite number of ghost-for-ghost fields (Bochicchio ‘87, Thorn ‘87):

$$\widetilde{\Gamma}^{(2)}_{g.f.} = \frac{1}{2}(\phi_0, c_0 \, \widetilde{L}_0 \, \phi_0) + \sum_{n=1}^{\infty} (\phi_n, c_0 \, \widetilde{L}_0 \, \phi_{-n})$$

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OSFT Physical States Counting

- Introduce the determinants of the kinetic operators $\widetilde{L}_0^{(n)}(p)$ in momentum space:

$$\Delta^{(n)}(p^2) \equiv \det \widetilde{L}_0^{(n)}(p)$$

- Were not for gauge-invariance, physical states would correspond to zeros of $\Delta^{(0)}(p^2)$: if

$$\Delta^{(0)}(p^2) = a_0\,(p^2 + m^2)^{d_0}(1 + O(p^2 + m^2))$$

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- The number of physical states of mass m is given by the Fadeev-Popov index (Giusto & C.I. '04):

$$I_{FP}(m) = d_0 - 2\,d_1 + 2\,d_2 + \cdots = \sum_{n=-\infty}^{\infty} (-1)^n\, d_n$$

where the d_n are degrees of the poles of the second quantized ghost fields:

$$\Delta^{(n)}(p^2) = \Delta^{(-n)}(p^2) =$$
$$= a_n\,(p^2 + m^2)^{d_n}(1 + O(p^2 + m^2))$$

- This formula applies to the general case of an arbitrary number of ghost generations.
- The numbers d_n are gauge-dependent.
- The index $I_{FP}(m)$ is gauge-invariant

$$I_{FP}(m) = \dim \mathcal{H}^{(0)}(\widetilde{Q}(p)) \qquad \text{with } -p^2 = m^2$$

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- In the exact theory physical states of mass m^2 are in general associated to a multiplet of determinants $\Delta^{(n)}(p^2)$ with different n's that vanish simultaneously at $-p^2 = m^2$.
- Since level truncation breaks BRS invariance we expect that the zeros of the determinants in the same multiplet, when evaluated at finite L, would be only approximately coincident.
- Using the index formula to compute the number of physical states is meaningful when the splitting between approximately coincident determinant zeros is significantly smaller than the distance between the masses of different multiplets.
- It is expected that matter and ghost propagators poles begin to cluster into well-defined approximately degenerate multiplets for levels L that are increasingly large as $m^2 = -p^2 \to \infty$. One can probe reliably only up to masses with $m^2 \sim L$.

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The numerical evaluation

- In the theory truncated at level L, the operators $\widetilde{L}_0^{(n)}(p)$ reduce to finite dimensional matrices.
- For a given L, the $\widetilde{L}_0^{(n)}(p)$ vanish identically for n greater than a certain n_L which depends on the level. Thus only a finite number of Fadeev-Popov determinants enter the analysis at any given level L.
- The tachyon solution is a Lorentz scalar: thus one can restrict the LT matrices $\widetilde{L}_0^{(n)}(p)$ to sectors with definite Lorentz indices: scalars, vectors, ...

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- This $SU(1,1)$ symmetry is not broken by LT since its generators commute with the level
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Dimensions of scalar matrices $\widetilde{L}_0^{(n,\pm)}(p)$

Table: Number of b_0-invariant scalar states at up to level 10.

Level	ghost # 0	ghost # -1	ghost # -2	ghost # -3	ghost # -4
3 (odd)	9	6	1	0	0
4 (even)	24	13	2	0	0
5 (odd)	45	30	7	0	0
6 (even)	99	61	14	1	0
7 (odd)	183	125	35	2	0
8 (even)	363	240	68	7	0
9 (odd)	655	458	145	15	0
10 (even)	1216	841	272	36	1

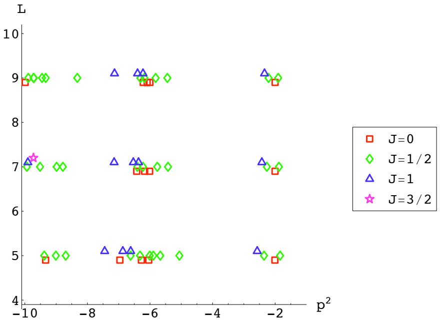
Dimensions of vector matrices $\widetilde{L}_0^{(n,\pm)}(p)$

Table: Number of b_0-invariant vector states up to level 10.

Level	ghost # 0	ghost # -1	ghost # -2	ghost # -3
3 (odd)	7	3	0	0
4 (even)	16	9	1	0
5 (odd)	40	22	3	0
6 (even)	85	52	10	0
7 (odd)	184	113	24	1
8 (even)	367	238	59	3
9 (odd)	730	478	127	10
10 (even)	1385	936	272	25

Location of FP zeros: Scalars, Odd Sector

Location of the zeros of FP determinants $\Delta_{-}^{(n)}(p^2)$ for $n = 0, -1, -2$ at levels $L = 4, \ldots, 9$ up to $p^2 = -10$, in the odd scalar sector



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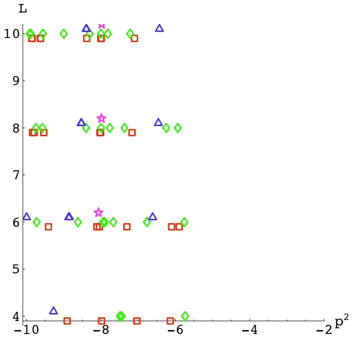

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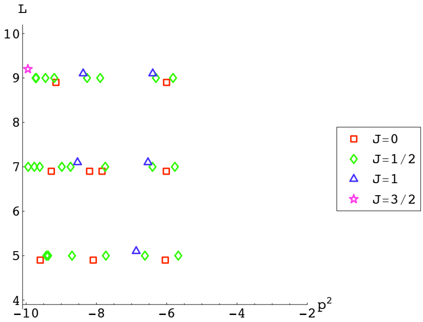

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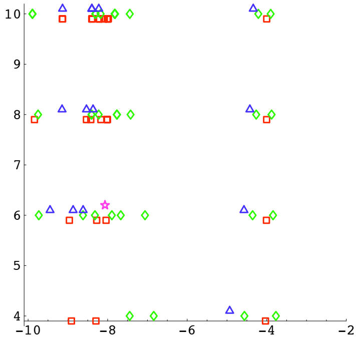

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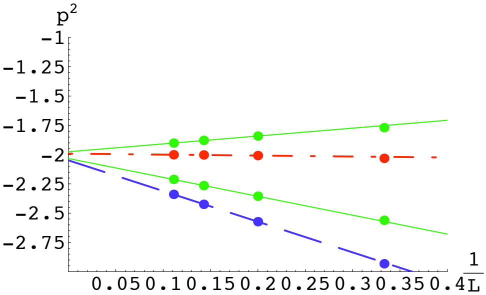

The first group of zeros of $\Delta_{-}^{(n)}(p^2)$ at $p^2 \approx -2.0$ as the Level L varies $L = 3, 5, 7, 9$.

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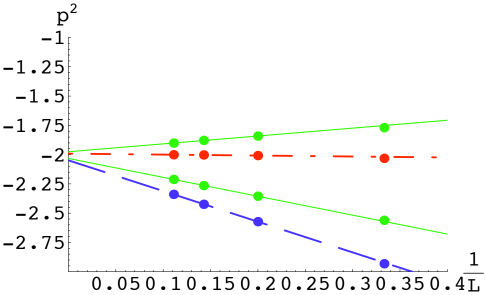

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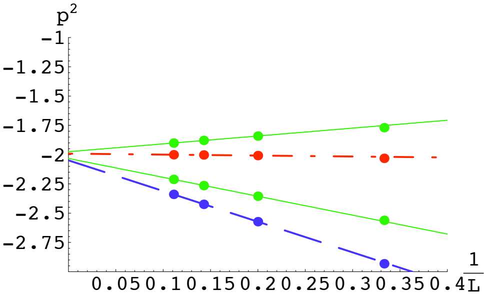

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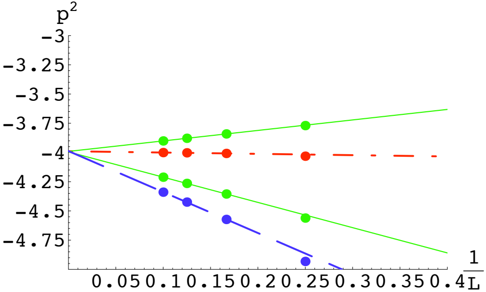

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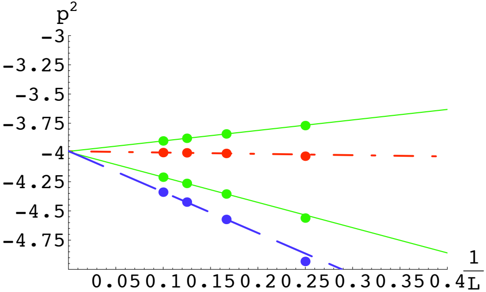

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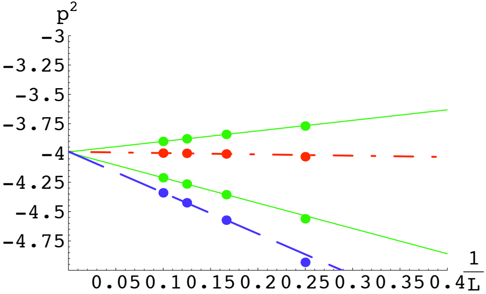

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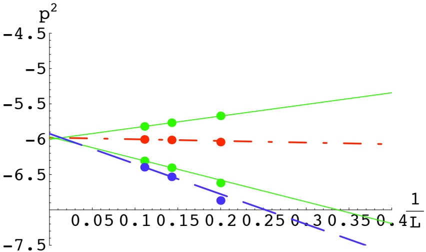

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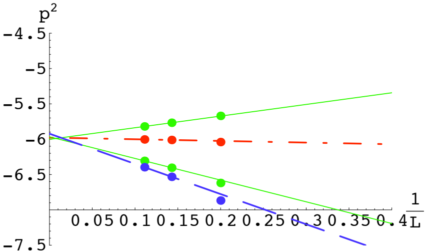

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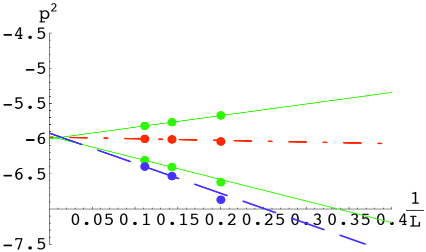

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Confirmations

- For all these multiples of zeros FP-index does vanish:

$$I_{FP}(m) = d_0 - 2\,d_1 + 2\,d_2 = 2 - 2 \cdot 2 + 2 \cdot 1 = 0$$

in agreement with Sen's conjecture.

- The first multiplets of zeros on the p^2 axis appear at $-p^2 = m^2 \approx 2.0$ for scalars and $-p^2 = m^2 \approx 4.0$ for vectors. These multiplets of zeros are approximately degenerate with good accuracy. This means that in the region to the right of $p^2 \approx 0$ the LT approximation is certainly trustworthy.
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- Extrapolated zeros with different J agree with remarkable accuracy.

Table: Determinant zeros extrapolated at $L = \infty$

Sector	J=0	J=1/2	J=1
scalar odd	-1.99172	-2.03279; -1.97541	-2.04905
vector even	-3.98938	-3.99494; -3.99087	-3.98803
vector odd	-5.97751	-5.96576; -6.00275	-5.78701

- It is tempting to conjecture from these data that the *exact* values for the degenerate zeros are integers !

$$m^2{}_{scalar,-} = 2.0 \qquad m^2{}_{vector,+} = 4.0 \qquad m^2{}_{vector,-} = 6.0$$

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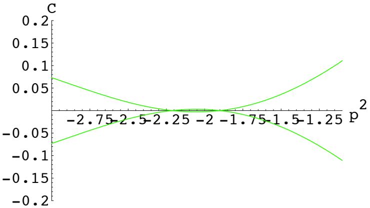

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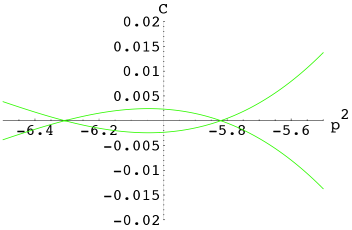

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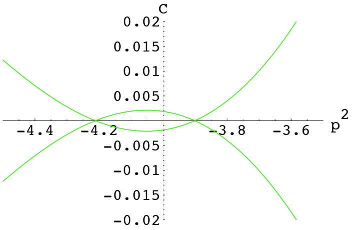

Surprise 2

- This means that for $p^2 \approx -m_i^2$ with $m_i^2 = 2.0, 4.0, 6.0, \ldots$ the OSFT quadratic action has the form

$$\Gamma^{(2)} = \frac{1}{2}\psi_s^{(0)}(-p)(p^2 + m_i^2)\psi_s^{(0)}(p) + \\ + \frac{1}{2}\psi_t^{(0)}(-p)(p^2 + m_i^2)\psi_t^{(0)}(p) + \\ + \psi^{(-1)}(-p)\left[p^2 + m_i^2\right]^2\psi^{(1)}(p) + \\ + \psi_t^{(-2)}(-p)(p^2 + m_i^2)\psi_t^{(2)}(p)$$

BRS cohomologies at non-standard ghost numbers

- Let v_0^s be the $SU(1,1)$ singlet, $\{v_{\pm 1}\}$ the doublet and $\{v_0^t, v_{\pm 2}\}$ zero modes of the kinetic operators with $-p^2 = m_i{}^2$ of the exact theory

$$\widetilde{L}_0^{(\pm 2)}(p)\, v_{\pm 2} = \widetilde{L}_0^{(\pm 1)}(p)\, v_{\pm 1} = \widetilde{L}_0^{(0)}(p)\, v_0^s = \widetilde{L}_0^{(0)}(p)\, v_0^t = 0$$

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- All but one of the 4 actions of $\widetilde{Q}$ on $\widetilde{W}$ are excluded by the exact sequences, assuming $\mathcal{H}^{(0)}(\widetilde{Q}) = 0$ (i.e. Sen's conjecture):

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- This means that the cohomologies of $\widetilde{Q}$ are not all empty

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- Vacuum String Field Theory approach to OSFT in the stable vacuum (Rastelli&Sen&Zwiebach '02) which assumes a trivial $\widetilde{Q}$ might be missing some aspect of the theory in the tachyonic vacuum
- Formal "exact" (i.e. not numerical) proof of Sen's conjecture ($\mathcal{H}^{(0)}(\widetilde{Q}) = 0$) (Ellwood&Schnabl '06) also implies that $\mathcal{H}^{(n)}(\widetilde{Q}) = 0$ for $n \neq 0$. Our results indicate that the tools involved (the identity state) might be not well defined.
- The (apparent) integer values of $-p^2 = 2, 4, 6$ we found for the zeros modes is possibly understood since we showed that these zeros modes do correspond to gauge-invariant quantities. This (possible) integrality is a hint that the full spectrum might be accessible to exact analysis.

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- Numerical findings confirm that

$$\dim \mathcal{H}^{(0)}(\widetilde{Q}) = 0 \Leftrightarrow \quad \text{no open string physical states}$$

but they also indicate that

- The cohomology of $\widetilde{Q}$ is not empty at “exotic” ghost numbers, for integer values of $-p^2 = 2, 4, 6, \ldots$.
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