A Solitonic Approach to Holographic Nuclear Physics

Based on the recent work: S.Baldino, S.Bolognesi, S.B.Gudnason, D.Koksal, Phys.Rev.D 96 (2017) 034008 [arXiv:1703.08695v2 [hep-th]].

Pre-thesis

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Sakai-Sugimoto Model

QCD **dual** String theory (in a certain curved background)

4 dim **10 dim**

- Holographic dual of QCD = flows to QCD at low energies.

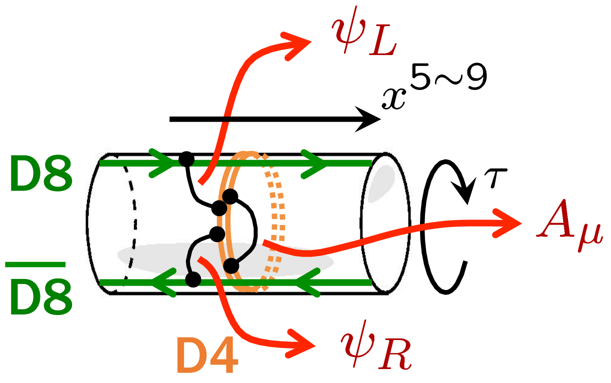
	x^0	x^1	x^2	x^3	τ	x^5	x^6	x^7	x^8	x^9
D4 $\times N_c$	○	○	○	○	○	–	–	–	–	–
D8-$\overline{\text{D8}}$ $\times N_f$	○	○	○	○	–	○	○	○	○	○

4 dim (x^0–x^3)　　S^1 (τ)

[Sakai-S.S. 2004]

"Gauge/String duality"

[Maldacena 1997]

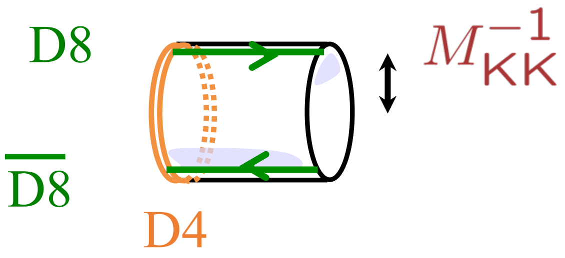


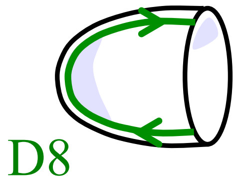
(low energy) →

A_μ : gluon

$\begin{pmatrix}\psi_L \\ \psi_R\end{pmatrix}$: quark $\times N_f$

$SU(N_c)$ QCD with N_f massless quarks.





N_c D4-branes on S^1
+ N_f D8-$\overline{\text{D8}}$ pairs

parameters :

$$\begin{cases} M_{\rm KK} \sim \text{cut off scale} \\ \lambda = g_{\rm YM}^2 N_c \end{cases}$$

('t Hooft coupling)

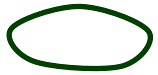
$$\begin{cases} \text{string coupling} \quad \propto 1/N_c \\ \text{string length} \quad \propto \lambda^{-1/2} \end{cases}$$

good description for

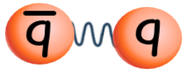
$$\begin{cases} \text{large} \quad N_c \\ \text{large} \quad \lambda \leftrightarrow \text{low energy} \end{cases}$$

Recipe:

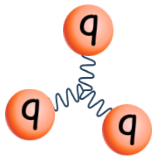
- Closed strings: glueballs,



- Open strings on D8: mesons,



- D4 wrapped on S^4 : Baryons.
(with Nc strings attached)



An effective theory of mesons and hadrons

- Probe approximation: (Nc >> Nf) → weakly coupled gravity
- Large t'Hooft coupling: (λ >>1) → Low- enery effective theory on the conformal boundary.

} Leading terms are considered in: $1/N_c$ & $1/\lambda$ expansions.

- Kaluza-Klein scale: M_{KK} = 1 GeV, in order to include the physical meson spectrum.

  (UV cut-off for the boundary theory)

- Global chiral symmetry is realized between the adjacent flavor brane configurations. Their meeting on the flat space limit demonstrates spontaneous breaking of chiral symetry:

$$U(N_f)_L \times U(N_f)_R \rightarrow U(N_f)_D \qquad \text{with} \qquad N_f = 2\,.$$

- At ow energy, quarks + pion are massless.

5D Yang-Mills + Chern-Simons Action

Global gauge symmetry:

U(2) = SU(2) x U(1)

$$S_{5\text{dim}} \simeq S_{\text{YM}} + S_{\text{CS}} \qquad k(z) = 1 + z^2 \quad h(z) = (1+z^2)^{-1/3} \quad \kappa = \frac{\lambda N_c}{216\pi^3}$$

$$S_{\text{YM}} = \kappa \int d^4x dz \, \text{Tr}\left(\frac{1}{2}h(z)F_{\mu\nu}^2 + k(z)F_{\mu z}^2\right) = -\frac{\kappa}{2}\int_{5\text{dim}} \text{Tr}(F \wedge *F)$$

$$\delta_\Lambda \omega_5(A) = d\omega_4^1(\Lambda, A)$$

: WZW term on the boundary. [Witten 1998]

$$S_{\text{CS}} = \frac{N_c}{24\pi^2}\int_5 \omega_5(A)$$

$$\omega_4^1(A) = \text{Tr}\left(\Lambda d\left(AdA + \frac{1}{2}A^3\right)\right)$$

Reproduces the chiral anomaly:

$$\omega_5(A) = \text{Tr}\left(AF^2 - \frac{1}{2}A^3F + \frac{1}{10}A^5\right)$$

$$\delta_\Lambda S_{\text{CS}} = \frac{N_c}{24\pi^2}\int_{M_4}\left(\omega_4^1(\Lambda, A)|_{z=+\infty} - \omega_4^1(\Lambda, A)|_{z=-\infty}\right)$$

Not gauge invariant!

Meson Theory

$$(f,g) = \int_{-\infty}^{+\infty} \frac{f(z)g(z)}{H^{\frac{1}{2}}} dz, \qquad < f,g > = \int_{-\infty}^{+\infty} H^{\frac{3}{2}} f(z)g(z)dz.$$

- 4D Effective action is extracted from the gauge in the bullk by expanding the fields by complete sets of eigenfunctions on the holographic direction = Kaluza-Klein expansion:

$$H^{\frac{1}{2}}\partial_z(H^{\frac{3}{2}}\partial_z\psi_n(z)) + k_n^2\psi_n(z) = 0$$

$$\partial_z(H^{\frac{1}{2}}\partial_z(H^{\frac{3}{2}}\phi_n(z))) + k_n^2\phi_n(z) = 0.$$

$$A_\mu(x^\mu, z) = \sum_{n=1}^{\infty} B_\mu^{(n)}(x^\mu)\psi_n(z),$$

$$A_z(x^\mu, z) = \varphi^{(0)}(x^\mu)\phi_0(z) + \sum_{n=1}^{\infty} \varphi^{(n)}(x^\mu)\phi_n(z).$$

$$(\psi_n, \psi_m) = \frac{< \psi_n', \psi_m' >}{k_n^2},$$

From the motion equations, k_n are interpreted as meson masses

$$\phi_n(z) \propto \partial_z\psi_n(z)$$

$$S = -\frac{1}{2}\int d^4x \, \mathrm{tr}\Big[(\partial_\mu\varphi^{(0)}\partial^\mu\varphi^{(0)}) + \sum_{n=1}^{\infty}\Big(\frac{1}{4}F_{\mu\nu(n)}F^{\mu\nu(n)} + \frac{1}{2}k_n^2 B_\mu^{(n)}B^{\mu(n)}\Big)\Big]$$

$\varphi^{(0)} \sim$ pion $B_\mu^{(1)} \sim \rho$ meson $B_\mu^{(2)} \sim a_1$ meson $\cdots$

$$S_{\text{5dim}}(A) = S_{\text{4dim}}(\pi, \rho, a_1, \rho', a_1', \cdots)$$

- Gauge fields in the bulk are related to the residual gauge symmetry on the infinity boundary by:

$$\mathcal{A}_\alpha(x^\mu, z) \mapsto g(x^\mu, z)\mathcal{A}_\alpha(x^\mu, z)g^{-1}(x^\mu, z) - ig(x^\mu, z)\partial_\alpha g^{-1}(x^\mu, z)$$

$$\lim_{z\to\pm\infty} g(x^\mu, z) = g_\pm \ , \qquad \lim_{z\to\pm\infty} \partial_\alpha g(x^\mu, z) = 0 \ ,$$

$$(g_+, g_-) \in G^{\text{glob}} = U(N_f)_L \times U(N_f)_R \ .$$

- Which give us the boundary source terms:

$$\lim_{z\to+\infty} A_\mu(x, z) = A_{\mu,L}(x)$$
$$\lim_{z\to-\infty} A_\mu(x, z) = A_{\mu,R}(x)$$

- The pion is given by the holonomy between the chiral branes placed at infinity:

$$\mathcal{U}(x^\mu) = \mathcal{P}\exp\left(-i\int_{-\infty}^{\infty} \mathrm{d}z\, \mathcal{A}_z(x^\mu, z)\right) \equiv \exp\left(\frac{2i}{f_\pi}\pi^a(x^\mu)T^a\right)$$

$$\mathrm{Tr}\left(T^aT^b\right) = \tfrac{1}{2}\delta_{ab}$$

and scales with...

$$\phi_0 = \frac{1}{\sqrt{\kappa\pi}}\frac{1}{k(z)}$$

which is the nomalizable/massless Nambu-Goldstone mode of χSB.

- $\psi_0 \propto \arctan z$; on the other hand is non-normalizable and not included in the expansion.

- Reproduces the Skyrme model, vector meson dominance, hidden local symmetry results at the relative limits.

Classical Baryon Solution

- Gauge fields respresent the homotopy group: $\mathcal{A} : \mathbb{R}^{4|1} \to U(N_f)$, with the metric:

$$g = H(z)\eta_{\mu\nu}dx^{\mu}dx^{\nu} + \frac{1}{H(z)}dz^2, \qquad H(z) = \left(1 + M_{\mathrm{KK}}^2 z^2\right)^{\frac{2}{3}}$$

Taking the static ansatz : $A_I = A_I(x_J), \quad A_0 = 0, \quad \hat{A}_I = 0, \quad \hat{A}_0 = \hat{A}_0(x_I)$ $\quad \Lambda = \frac{8\lambda}{27\pi}$

$$\mathcal{S} = \int \left(\frac{1}{2H^{\frac{1}{2}}}(\partial_i \hat{A}_0)^2 + \frac{H^{\frac{3}{2}}}{2}(\partial_z \hat{A}_0)^2 - \frac{1}{2H^{\frac{1}{2}}}\mathrm{tr}(F_{ij}^2) - H^{\frac{3}{2}}\mathrm{tr}(F_{iz}^2) \right) d^4x dz$$
$$+ \frac{1}{\Lambda}\int \hat{A}_0 \mathrm{tr}(F_{IJ}F_{KL})\epsilon_{IJKL} d^4x dz,$$

- For the finite action instanton, gauge fields must approach pure gauge configuration on the world-sphere at infinity :

$$A_I(x^I)|_{S^3_\infty} = g^\dagger \partial_I g, \qquad g : S^3_\infty \to SU(2)$$

$$\pi_3(SU(2)) = \mathbb{Z}$$

→ We have a discrete but infinite # of topological sectors, labeled by the topological charge:

$$\mathrm{B} = \int B^0(x,z) d^3x dz = -\frac{1}{32\pi^2}\int \mathrm{tr}(F_{IJ}F_{KL})\epsilon_{IJKL} d^3x dz$$

- We make the self-dual t'Hooft ansatz for the gauge field: [Hata, 2007]

$$A_I = \frac{1}{2}\sigma_{IJ}\partial_J \log\left(1 + \frac{\mu^2}{\rho^2}\right)$$

linearize for the instanton tail...

$$A_I^{(1)} = -\sigma_{IJ}\frac{x_J\,\mu^2}{\rho^4} = \frac{\mu^2}{2}\sigma_{IJ}\partial_J\frac{1}{\rho^2}$$

and approximate with 4D-spherically symmetric radial ansatz for the B=1 instanton: [Bolognesi, 2007]

$$A_I = -\sigma_{IJ}\partial_J b(\rho), \qquad \hat{A}_0 = a(\rho)$$

$$b(\rho) = \frac{1}{\Lambda(\rho^2 + \mu^2)}$$

with b.c.'s. $\lim_{\rho\to\infty} \rho^2 b(\rho) = 1, \qquad b'(0) = 0$

- Gives us the BPST instanton from the classical action, which scales as Λ^0. The rescaled self-energy is given by;

$$\mathcal{E} = 2\pi^2\left(4 + \frac{2}{3}\mu^2 + \frac{256}{5\Lambda^2\mu^2}\right)$$

which is minimized by the classical instanton size:

$$\mu = \frac{4}{\sqrt{\Lambda}}\left(\frac{3}{10}\right)^{\frac{1}{4}}$$

- Λ^{-1} corrections scale the stabilizing effects on the instanton configuration = μ is not a modulus!

- Form of the electric field from the minimization of the energy functional to the flat-space BPST instanton

$$a(\rho) = \frac{8}{\Lambda} \frac{\rho^2 + 2\mu^2}{(\rho^2 + \mu^2)^2}$$

- Rescaled energy:

$$E = M = \frac{N_c \Lambda}{8} + \sqrt{\frac{2}{15}} N_c$$

Soliton rest mass

- Linear expansion: $A_I^{(n)} \sim 1/\Lambda^n$

(linear zone: $\rho > 1/\sqrt{\Lambda}$)

$$A_I = A_I^{(1)} + A_I^{(2)} + \dots$$

Flat-space B=1 / Curvature+electrostatic field.

= Large Λ validates the small instanton (linearization).

Linear regime

- Equations of motion from the 5D YM-CS Lagrangian + linearized: $(D_\mu \longrightarrow \partial_\mu)$

$$\frac{1}{H^{\frac{1}{2}}} D_j F_{ji} + D_z(H^{\frac{3}{2}} F_{zi}) = \frac{1}{\Lambda}\epsilon_{iJKL} F_{KL} \partial_J \hat{A}_0,$$

$$H^{\frac{3}{2}} D_j F_{jz} = \frac{1}{\Lambda}\epsilon_{ijk} F_{jk} \partial_i \hat{A}_0,$$

$$\frac{1}{H^{\frac{1}{2}}} \partial_i\partial_i \hat{A}_0 + \partial_z(H^{\frac{3}{2}}\partial_z \hat{A}_0) = \frac{1}{\Lambda}\mathrm{tr}(F_{IJ}F_{KL})\epsilon_{IJKL},$$

$$\left\{ \begin{array}{ll} \partial_I A_I^{(1)} = 0 & \partial_J \partial_J A_I^{(1)} = 0. \\ F_{IJ}^{(1)} = & \partial_I A_J^{(1)} - \partial_J A_I^{(1)} \end{array} \right\}$$

$$\begin{aligned} \frac{\partial_i\partial_i}{H^{\frac{1}{2}}}\hat{A}_0 + \partial_z(H^{\frac{3}{2}}\partial_z \hat{A}_0) &= \text{source1}\ , \\ \frac{\partial_j\partial_j}{H^{\frac{1}{2}}} A_i^+ + \partial_z(H^{\frac{3}{2}}\partial_z A_i^+) &= \text{source2}\ , \\ H^{\frac{3}{2}}(\partial_i\partial_i A_z^+ - \partial_i\partial_z A_i^-) &= \text{source3}\ , \\ \frac{\partial_j\partial_j A_i^- - \partial_j\partial_i A_j^-}{H^{\frac{1}{2}}} - \partial_z(H^{\frac{3}{2}}(\partial_i A_z^+ - \partial_z A_i^-)) &= \text{source4}\ , \end{aligned}$$

- Source terms are derived from the boundary theory currents, which in the linear approximation are delta functions for the localization of the fields...

- Separation of variables + Fourier expansion:

$$\frac{1}{\rho^2} = \frac{1}{r^2 + z^2} = \int_0^\infty \frac{e^{-kr}}{r} \cos(kz)\, dk$$

- Can express delta functions in terms of our KK-expansion modes:

$$\left.\begin{aligned} \sum_{n=1}^{\infty} \frac{\psi_n(z)\psi_n(z')}{H^{\frac{1}{2}}(z) c_n} &= \delta(z - z'), \\ \sum_{n=1}^{\infty} H^{\frac{3}{2}}(z) \frac{\phi_n(z)\phi_n(z')}{d_n} &= \delta(z - z'). \end{aligned}\right\}$$

$$G(x, z, x', z') = -\frac{1}{4\pi} \sum_{n=1}^{\infty} \frac{\psi_n(z)\psi_n(z')}{c_n} \frac{e^{-k_n|x-x'|}}{|x - x'|},$$

$$L(x, z, x', z') = -\frac{1}{4\pi} \sum_{n=0}^{\infty} \frac{\phi_n(z)\phi_n(z')}{d_n} \frac{e^{-k_n|x-x'|}}{|x - x'|}.$$

$$H^{1/2}\partial_z(H^{3/2}\partial_z\psi^{\pm}_{(k)}) + k^2\psi^{\pm}_{(k)} = 0$$

$$\partial_z(H^{\frac{1}{2}}\partial_z(H^{\frac{3}{2}}\phi_n(z))) + k_n^2\phi_n(z) = 0.$$

$$\left\{\begin{aligned} \frac{\partial_i\partial_i G}{H^{\frac{1}{2}}(z)} + \partial_z(H^{\frac{3}{2}}(z)\partial_z G) &= \delta^3(x - x')\delta(z - z')\ , \\ \partial_i\partial_i L - \partial_z\partial_{z'} G &= \delta^3(x - x')\delta(z - z')\ , \\ \partial_z(H^{\frac{3}{2}}(z)L) + H^{-\frac{1}{2}}(z)\partial_{z'} G &= 0\ . \end{aligned}\right.$$

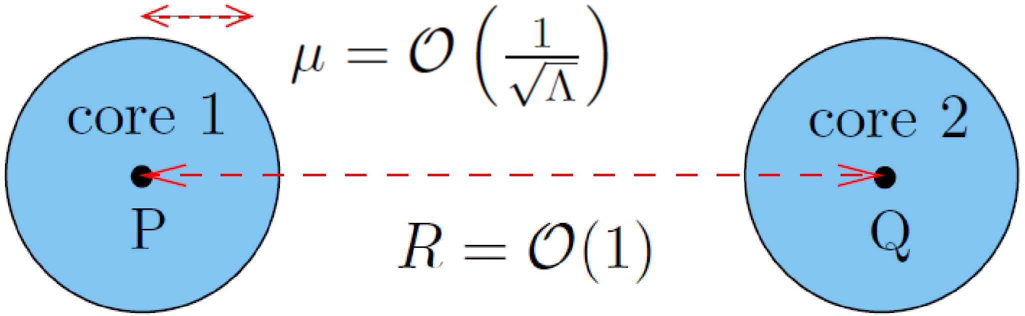
$$\hat{A}_0(x,z) = -\frac{32\pi^2}{\Lambda} G(x,z,0,0) \,,$$
$$A_i^+(x,z) = -2\pi^2\mu^2 \epsilon_{ijk}\sigma_k \partial_j G(x,z,0,0) \,,$$
$$A_i^-(x,z) = -2\pi^2\mu^2 \sigma_i \partial_{z'} G(x,z,0,z')|_{z'=0} \,,$$
$$A_z^+(x,z) = -2\pi^2\mu^2 \sigma_i \partial_i L(x,z,0,0) \,.$$

- Gauge field are generalized by moving the instanton center on $\mathbb{R}^3$:

$$G(x,z,0,0) \longrightarrow G(x,z,X,0)$$
$$\sigma_i \longrightarrow G\sigma_i G^\dagger$$

$$\frac{\partial_i\partial_i}{H^{\frac{1}{2}}}\hat{A}_0 + \partial_z(H^{\frac{3}{2}}\partial_z\hat{A}_0) = -\frac{32\pi^2}{\Lambda}\delta^3(x)\delta(z) \,,$$
$$\frac{\partial_j\partial_j}{H^{\frac{1}{2}}}A_i^+ + \partial_z(H^{\frac{3}{2}}\partial_z A_i^+) = -2\pi^2\mu^2\epsilon_{ijk}\sigma_k\partial_j\delta^3(x)\delta(z) \,,$$
$$H^{\frac{3}{2}}(\partial_i\partial_i A_z^+ - \partial_i\partial_z A_i^-) = -2\pi^2\mu^2\sigma_i\partial_i\delta^3(x)\delta(z) \,,$$
$$\frac{\partial_j\partial_j A_i^- - \partial_j\partial_i A_j^-}{H^{\frac{1}{2}}} - \partial_z(H^{\frac{3}{2}}(\partial_i A_z^+ - \partial_z A_i^-)) = 2\pi^2\mu^2\sigma_i\delta^3(x)\partial_z\delta(z) \,.$$

Nucleon-nucleon Potential



$$\mu = \mathcal{O}\left(\frac{1}{\sqrt{\Lambda}}\right)$$

$$R = \mathcal{O}(1)$$

- B=1

$$\mathcal{E} = \int \left(\frac{1}{2H^{\frac{1}{2}}} \operatorname{tr}(F_{ij}^2) + H^{\frac{3}{2}} \operatorname{tr}(F_{iz}^2) - \frac{1}{2}\hat{A}_0 \underbrace{\left(\frac{\partial_i \partial_i}{H^{\frac{1}{2}}} + \partial_z (H^{\frac{3}{2}} \partial_z) \right)}_{} \hat{A}_0 \right) d^3x dz$$

- B=2 / Neglecting self energies and linearly superimposing the two monopole terms (localized at the opposing nuclei cores):

- Electro-static monopole term:

$$\mathcal{E}_2 = -\int \frac{1}{2} \hat{A}_0 \Box \hat{A}_0 d^3x dz$$

$$\hat{A}_0^p \Box \hat{A}_0^q = -32\pi^2 \Lambda^{-1} \hat{A}^p B^{0,q}. \quad B^{0,q} \simeq \delta^3(x - R)\delta(z)$$

$$\mathcal{V}_{mp} = \frac{16\pi^2}{\Lambda} (\hat{A}_0^p(R,0) + \hat{A}_0^q(0,0)) = \frac{256\pi^3}{\Lambda^2} \sum_{n=1}^{\infty} \frac{1}{c_{2n-1}} \frac{e^{-k_{2n-1}R}}{R}$$

B = 2 / Dipole Interaction

1. Divide the topological sector of B=2 into three sectors: 2 core + linear zone:

$$\int_{\mathcal{R}} = \int_P + \int_Q + \int_{LZ}$$

2. Add distant instanton effects as a perturbation:

$$\delta A_I^p = A_I^q$$

3. Integrals over the core contribute to self-energies/ we are iterested in the variations wrt. the field perturbations:

$$\delta \int \mathrm{tr}(F_{IJ}^p F_{IJ}^p) d^3x dz = 4 \int \mathrm{tr}(F_{IJ}^p D_I^p A_J^q) d^3x dz$$

4. Using Stokes', we integrate the field strenghts in the linear zone + come back into core region using the Green functions:

$$\partial P = -\partial(Q \cup LZ)$$

$$\begin{aligned}
\mathcal{V}_d \;=\; & 2\int_P \mathrm{tr}\left(A_i^{+,q}\left(\frac{\partial_j\partial_j A_i^{+,p}}{H^{\frac{1}{2}}} + \partial_z(H^{\frac{3}{2}}\partial_z A_i^{+,p})\right)\right) d^3x dz \\
& + 2\int_P H^{\frac{3}{2}} \mathrm{tr}\left(A_z^{+,q}\left(\partial_i\partial_i A_z^{+,p} - \partial_i\partial_z A_i^{-,p}\right)\right) d^3x dz \\
& + 2\int_P \mathrm{tr}\left(A_i^{-,q}\left(\frac{\partial_j\partial_j A_i^{-,p} - \partial_j\partial_i A_j^{-,p}}{H^{\frac{1}{2}}} - \partial_z(H^{\frac{3}{2}}(\partial_i A_z^{+,p} - \partial_z A_i^{-,p}))\right)\right) d^3x dz\,.
\end{aligned}$$

- Composite SU(2) configuration: $M_{ij}(G) = \frac{1}{2}\text{tr}\left(\sigma_i G \sigma_j G^\dagger\right)$ → $B^\dagger C$

- Along with the spatial rotation tensor:

$$P_{ij}(r,k) = \delta_{ij}((rk)^2 + rk + 1) - \frac{r_i r_j}{r^2}((rk)^2 + 3rk + 3)$$

- Quaternionic respresentation on S^4:

$$B = \exp\left(iB_i \frac{\sigma_i}{2}\right)$$

$$C = \exp\left(iC_i \frac{\sigma_i}{2}\right)$$

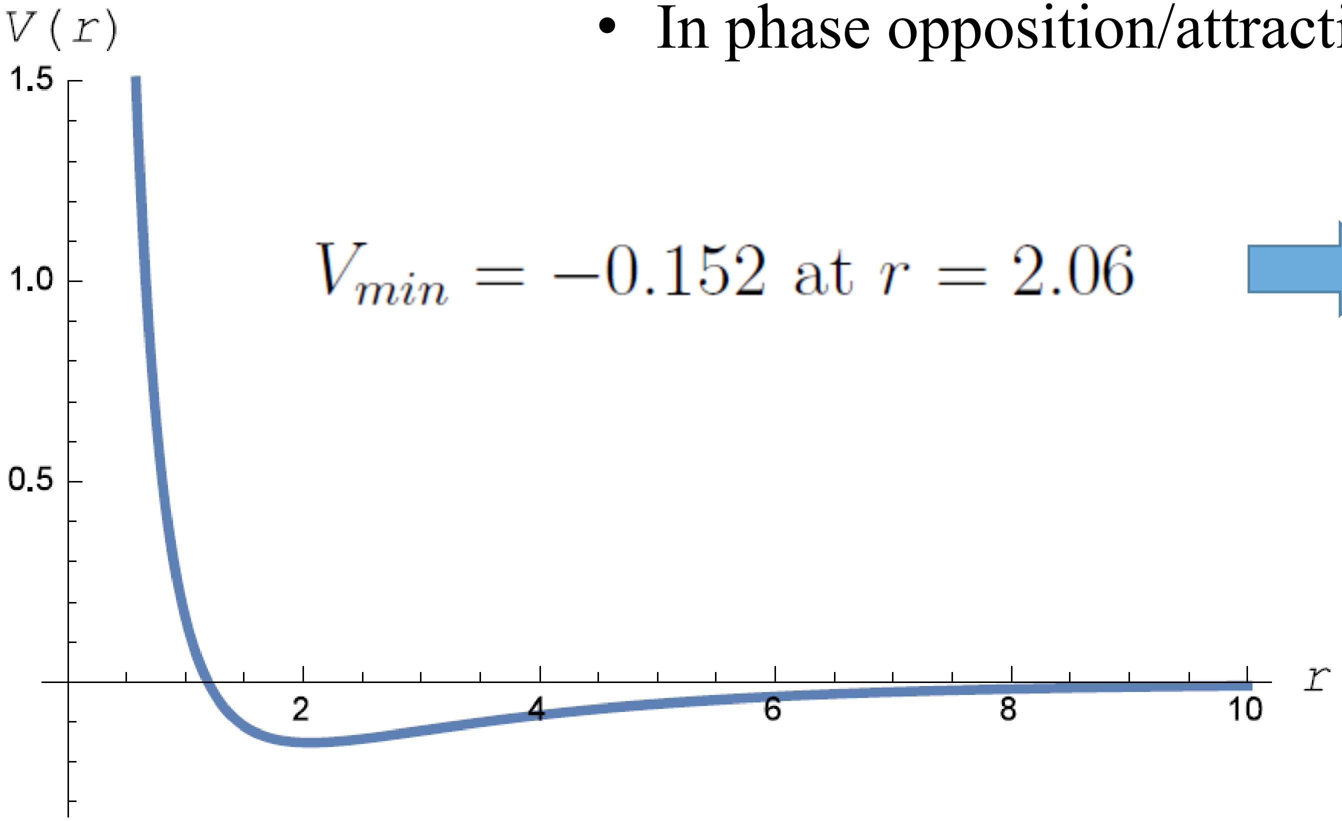
$$V(r, B^\dagger C) = \frac{4\pi N_c}{\Lambda}\Big[\sum_{n=1}^{\infty}\Big(\frac{1}{c_{2n-1}}\frac{e^{-\sqrt{k_{2n}^2+k_0^2}r}}{r} + \frac{6}{5}\frac{1}{c_{2n-1}}M_{ij}(B^\dagger C)P_{ij}\left(r, \sqrt{k_{2n}^2 + k_0^2}, r\right)\Big)\frac{e^{-\sqrt{k_{2n}^2+k_0^2}r}}{r^3}\Big) - \sum_{n=0}^{\infty}\frac{6}{5}\frac{1}{d_{2n}}\frac{e^{-\sqrt{k_{2n}^2+k_0^2}r}}{r^3}M_{ij}(B^\dagger C)P_{ij}\left(r, \sqrt{k_{2n}^2 + k_0^2}\right)\Big]$$

$$d_0 = \pi$$

$$P_{ij}(r,0) = \delta_{ij} - 3\left(\frac{r_i r_j}{r^2}\right)$$

B = 2, Bound state

- In phase opposition/attractive channel: $B^{\dagger}C = \pm i\sigma_3$



$V_{min} = -0.152$ at $r = 2.06$ $\Rightarrow$ $E_{2,c} = 2M - 0.152\frac{N_c}{\Lambda}.$

Nucleon-nucleon potential in the attractive channel.

- Which means that in the $\Lambda \to \infty$ limit, we have weakly bound baryons of small size $(\mathcal{O}(\lambda^{-1/2}))$ and large separation $(\mathcal{O}(\lambda^{0}))$.

Moduli space for B=2

- Linear approximation allows to combine two single charge fields as:

$$BA_I\left(x - X_1\right)B^\dagger + CA_I\left(x - X_2\right)C^\dagger$$

- This gives us the manifold of the zero mode symmetries:

$$\mathcal{L} = \mathbb{R}^3 \times SU(2)_I \times SU(2)_J \times \mathcal{P}$$

(Parity: x → -x.)

- Acts on the fields by: $(M(U)_{ij})$ and $(M(E)_{ij})$ where: $U \in SU(2)_I,\ E \in SU(2)_J$

$$\begin{aligned}\mathcal{A}_I = \quad & UE^\dagger A_I\left(x - (-)^P M(E)\frac{R}{2}, z\right)(UE^\dagger)^\dagger \\ & + Ui\sigma_3 E^\dagger A_I\left(x + (-)^P M(E)\frac{R}{2}, z\right)(Ui\sigma_3 E^\dagger)^\dagger,\end{aligned}$$

→

- Gives us the stabilizing topolgy:

$$\mathcal{Z} = SU(2)_I \times SU(2)_J / \{1, \mathcal{O}_{11}, \mathcal{O}_{12}, \mathcal{O}_{03}\}$$

Right/Left handed transformations coincide due to spherical symmetry.

- Zero-manifold metric: $g|_{\mathcal{M}} = dX_1^i dX_1^i + 2\mu^2 d\Omega_{SU(2),B} + dX_2^i dX_2^i + 2\mu^2 d\Omega_{SU(2),C}$

- Kinetic energy and the zero manifold Lagrangian:

$$\omega_{B,i} = -i\mathrm{tr}(B^\dagger \dot{B}\sigma_i)$$

$$T = \frac{1}{4}M\left(\dot{r}^i\dot{r}^i + \mu^2\omega_{B,i}\omega_{B,i} + \mu^2\omega_{C,i}\omega_{C,i}\right) \qquad L|_{\mathcal{Z}} = T|_{\mathcal{Z}} - V_{\min} - 2M.$$

- For the static configuration, we fix the SU(2) axes to the combined spin/isospin moduli:

$$\begin{cases} r_i = M(E)_{ij}R_j, \\ B = UE^\dagger, \\ C = Ui\sigma_3 E^\dagger. \end{cases}$$

- Spherical harmonics on 4D + FR constraints:

$$|\psi\rangle = |k, k_3, i_3, l, l_3, j_3\rangle$$

'Deuteron' as a minimum config. in phase opposition.

$$|D\rangle = |0,0,0,1,0,j_3\rangle\,, \quad |I_0\rangle = |1,0,i_3,0,0,0\rangle\,,$$
$$|I_1\rangle = \frac{1}{\sqrt{2}}(|1,1,i_3,0,0,0\rangle + |1,-1,i_3,0,0,0\rangle).$$

- First three min. energy states are degenerate due to spherical symmetry!

Further Remarks

- Quantization on the moduli space using harmonic approximation adds subleading order corrections in Λ and N_c,
- $N_c \to \infty$ and $\Lambda \to \infty$ limits do not commute.
- For the classical baryon solution, we need: $N_c \gg \sqrt{\Lambda}.$
- Calculated classical binding ratios/masses are two orders of magnitude higer than the experimenta values.
- We do find classical bound states for up to B=8, with the correct geometries + sensible binding ratios.
- We need higher order Lambda/quantum corrections (within the range of validity) in order to extrapolate to physical results ($N_c = 3$, $\Lambda_{ss} = 1.569$ from ΛQCD.)

Adding pion mass (in progress)

- In the SS-model, it is proposed as an excitation of the flavor branes, given by the holonomy:

$$S_m = \frac{\Lambda^{\frac{3}{2}}}{16\sqrt{2}\pi^{\frac{3}{2}}} \int P\,\mathrm{tr}[(M \exp(-i \int_{-\infty}^{+\infty} A_z dz) - \mathbf{1}) + c.c.\,]\, d^3x dz$$

- Pion matrix: $$U(x,z) = P\exp(-i\int_{-\infty}^{+\infty} A_z dz) = \exp[2i\pi(x)/f_\pi]$$

- Quark (degenerate) mass as a chiral perturbation (symmetry breaking identical to the Skyrme term):

$$\delta S = \int d^4x\, \delta L\ , \quad \delta L \equiv c\,\mathrm{tr}\left[M(U + U^\dagger - 2\mathbf{1}_2)\right]$$

Open questions

- How to include the pion mass without changing the gauge symmetry?
- Self-energy/potential modifications due to quark masses in the degenerate/ non-degenerate cases...
- Quantization of the approximate moduli (μ and Z) and the consequent finite Λ corrections.

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